Effective Teaching for the Development of Skill and Conceptual Understanding of Number: What is Most Effective?

Documenting which instructional methods are most effective for students' learning continues to be one of the great challenges for educational research. Should teachers use Method A or Method B?

No single study can *prove* that one method or feature of teaching is better than another for helping students achieve a particular learning goal because too many factors affect the results. But by detecting patterns across studies, especially across a set of studies that used different research designs and procedures, educators can identify robust features of teaching that seem to produce similar effects related to particular learning goals.

We select here two learning goals around which a substantial amount of data point to effective features of mathematics instruction. These goals are (1) skill efficiency—the rapid, smooth, and accurate execution of mathematical procedures (Gagne 1985)—and (2) conceptual understanding—the construction of relationships among mathematical facts, procedures, and ideas (Brownell 1935; Davis 1984; Hiebert and Carpenter 1992).

Skill Efficiency

A large set of studies conducted in the 1970s and 1980s within the process-product paradigm (identifying relationships between what teachers do in the classroom—the process—and what students learn as a result of this instruction—the product) supplies the best evidence from which to induce patterns for links between teaching and skill efficiency. Good and Grouws (1977) examined the teaching performance of more than 100 third- and fourth-grade teachers over a two-year period. Results from the study indicated that teaching effectiveness was associated with the following behavioral clusters: whole-class instruction with demonstrations by the teacher; a task-focused environment; faster paced lessons and more homework; and classrooms relatively free of behavioral problems. Evertson and her colleagues (1980) studied junior high school teachers of mathematics (N = 29). More effective mathematics teachers asked more questions than less effective teachers did, with most of the questions lower-order product questions. The more effective teachers also ran well-organized classrooms focused on academic tasks and emphasized whole-class instruction with some time devoted to seatwork and practice.

Findings from a number of additional studies, summarized in Brophy and Good (1986), reinforce the following claim: mathematics teaching that facilitates skill efficiency is rapidly paced, includes modeling by the teacher with many teacher-directed, product type of questions, and displays a smooth transition from demonstration to substantial amounts of error-free practice. The teacher plays a central role in organizing, pacing, and presenting information to meet well-defined learning goals.

Conceptual Understanding

Two features of instruction emerge from the literature as especially likely to help students develop conceptual understanding of the mathematics topic they are studying: attending explicitly to connections among facts, procedures, and ideas; and encouraging students to wrestle with the important mathematical ideas in an intentional and conscious way.

Making important mathematical relationships explicit has been shown to support students' understanding of the relationships in primary-grade arithmetic (Brownell and Moser 1949; Fuson and Briars 1990; Hiebert and Wearne 1993), middle school mathematics (Good and Grouws 1977; Good, Grouws, and Ebmeier 1983), secondary school geometry and algebra (Boaler 1998; Fawcett 1938), and college calculus (Heid 1988). The importance of this instructional feature has been found in classrooms where the teacher plays the central role of demonstrating the relationships (Brownell and Moser 1949; Fuson and Briars 1990; Good, Grouws, and Ebmeier 1983) and in classrooms where students do more demonstrating and explaining (Fuson and Briars 1990; Fawcett 1938; Hiebert and Wearne 1993). Similar findings have been reported with instruction lasting for just days or weeks (Brownell and Moser 1949; Hiebert and Wearne 1993) and instruction lasting two to three years (Boaler 1998; Fawcett 1938).

The particular ways that important relationships are made explicit during instruction vary from study to study. In some instances, students were asked to examine carefully the differences and similarities between concrete and

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symbolic representations of the same quantities and operations (Brownell and Moser 1949; Fuson and Briars 1990; Hiebert and Wearne, 1993). In some situations, teachers explained in detail why the arithmetic procedures worked as they did (Brownell and Moser, 1949). In other situations, students were asked to develop their own solution methods and justify their validity (Fawcett 1938; Hiebert and Wearne 1993).

It is striking that given the robustness of the link between instructional attention to important relationships and students' level of understanding, typical classrooms in the United States focus on low-level skills and rarely attend explicitly to the important mathematical relationships (Hiebert et al. 2003; National Advisory Committee on Mathematics Education 1975; Rowan, Harrison, and Hayes 2004; Stigler et al. 1999; Weiss et al. 2003).

The second feature of instruction that consistently facilitates students' conceptual understanding is the engagement of students in wrestling with, or struggling with, important mathematical ideas. We use the word *struggle* to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent.

The phenomenon of benefiting from struggling with the significant mathematical ideas can be inferred from a number of studies in which the students were presented with challenging tasks and asked to work out new solution methods on their own (Carpenter et al. 1989; Cobb et al. 1991; Fawcett 1938; Hiebert and Wearne 1993; Lampert 2001; Schoenfeld 1985; Stein, Grover, and Henningsen 1996; Stein and Lane 1996). As an example, Stein and colleagues examined the mathematical tasks presented to middle school students. Results clearly indicate that students attending schools in which teachers presented *and* faithfully implemented more challenging problems were more likely to develop increased conceptual understanding of the mathematics. Implementing "challenging" problems means that the teacher does not step in and do the mathematical work but rather allows students to do the work by struggling, if needed, to complete the problems.

Not a One-to-One Correspondence

The features of teaching we have described for promoting skill efficiency are quite different from those that promote conceptual understanding. Certainly the teacher's demonstrations, fast pacing, and error-free practice features seem to constitute a different method or system of instruction from those that present challenging problems and encourage students to wrestle or struggle with the important ideas. But this apparent correspondence between one set of features for skill efficiency and a completely different set of features for conceptual understanding breaks down when it is observed that many of the studies that focused on conceptual development also reported that students' skills increased at a level equal to or greater than those of students in the control groups (Boaler 1998; Fawcett 1938; Fuson and Briars 1990; Good, Grouws, and Ebmeier 1983; Hiebert and Wearne 1993; Stein and Lane 1996).

One way to explain this finding is that the *nature* of skill learning might be somewhat different under the two instructional systems. In one system, instruction is fast paced, teachers ask short-answer targeted questions, and students complete relatively large numbers of problems during the lesson with high success rates. This appears to yield skill efficiency. In the other system, instruction is more slowly paced, teachers ask questions that require longer responses, and students complete relatively few problems in each lesson. At least under some conditions, this appears to yield skill efficiency coupled with conceptual understanding. The cognitive mechanisms that students are likely to use in response to these different systems are different (Wittrock 1986), leading to different skill competencies.

Old Dichotomies Are Not Helpful

The features of teaching that facilitate skill efficiency and conceptual understanding do not fall neatly into categories frequently used to contrast methods of teaching, such as expository versus discovery, direct instruction versus inquiry-based teaching, student-centered versus teacher-centered teaching, and traditional versus reform-based teaching. Although the features of teaching identified earlier that promote skill efficiency fit some of these labels better than others, the features of teaching that promote conceptual understanding (and perhaps skill efficiency) cut across these common labels. In particular, attending explicitly to important mathematical relationships can be done within any of these methods.

By James Hiebert and Douglas A. Grouws
Judith Reed, Series Editor

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