

An Administrator's Guide to High School Mathematics:

Making Reasoning and Sense Making the Focus

What kinds of mathematical experiences should high schools offer students to ensure their future success? The purpose of this pamphlet is to give administrators a better understanding of how they can support effective teaching and learning in every part of the high school mathematics program. The recommendations that follow are based on a new set of guidelines for improving high school mathematics presented in *Focus in High School Mathematics: Reasoning and Sense Making*—a landmark publication from the nation's leading advocate for more and better mathematics, the National Council of Teachers of Mathematics (NCTM). This pamphlet is organized around a series of questions that can form the basis for extended reflection on what is currently happening—and what needs to happen—in today's high schools. Such reflection can spur administrators to collaborate with others to achieve the ambitious goals of this new publication from NCTM.

Do school districts, schools, mathematics departments, and teachers ensure that students receive a high-quality high school mathematics curriculum that promotes their future success?

A strong preparation in high school mathematics is essential for preparing students for future success, whatever they may aspire to do. Although mathematics is clearly central to preparation for scientific and technical careers, it is also increasingly necessary for a wide range of careers, from finance to sports journalism. The advent of the Internet has produced an explosion of new careers in mathematics and statistics based on harnessing the huge amount of data at people's fingertips. Success in mathematics opens doors for students. All too often, however, an inadequate preparation in high school mathematics means that doors will be closed to some students on graduation—certain careers will not be accessible without years of make-up work in mathematics.

In addition, all citizens need a solid background in mathematics to make informed and reasoned decisions about their lives and society, including decisions about their personal finances, which public policies deserve their support, and which insurance or health plans to select. In today's highly technological world, having a solid preparation in mathematics is a tremendous asset.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS



Those without a solid mathematical background risk being taken advantage of or being left behind. Moreover, high school mathematics must focus on reasoning and sense making to ensure that students have the preparation that they need to succeed in all aspects of their future lives.

What do reasoning and sense making mean?

Simply exposing students to mathematical topics is not enough. Nor is it enough for students to know only how to perform mathematical procedures or recall facts. They must learn to reason and make sense of mathematics so that they are able to use math in meaningful ways. Students today need to develop critical thinking skills to succeed in school and in life. For example, in high school literature courses, students are often asked to analyze, interpret, or think critically about books that they are reading. Reasoning is important in fields such as literature, but it is particularly important in mathematics.

In mathematics, *reasoning* involves drawing logical conclusions based on assumptions and definitions. *Sense making* involves developing an understanding of a situation, context, or concept by connecting it with existing knowledge. Reasoning and sense making are closely interrelated and are the foundation for a solid preparation in mathematics.

What does a curriculum focused on reasoning and sense making look like?

Many groups have offered recommendations of topics to include in high school mathematics. Among these, the Standards for grades 9–12 recommended by NCTM in *Principles and Standards for School Mathematics* have been influential. However, *Focus in High School Mathematics: Reasoning and Sense Making* goes one step further, recommending that no matter what curriculum is adopted, reasoning and sense making be made the central foci of high school mathematics. To ensure that this occurs, teachers must incorporate worthwhile tasks that engage all students in thinking about and making sense of mathematics, not just practicing concepts they have already been taught. Consider the problem scenario “Tidal Waves,” shown below.

Although there is no set procedure for solving this problem, as students explore what is happening, they will develop deeper insights into aspects of trigonometric functions and see the usefulness of mathematics as a tool for deciding, for example, at what times the water will be deep enough to allow the passage of a large ship into the port.

However, simply providing interesting tasks is not sufficient. Careful planning is necessary to develop sequences of tasks that fit together and achieve important mathematical goals. Classroom teachers must maintain a focus on reasoning and sense making throughout the curriculum.

“TIDAL WAVES”

The captain of a shipping vessel must consider the tides when entering a seaport because the water depth can vary greatly from one time of day to another. Suppose that high tide in a certain port occurs at 5:00 AM, when the

water is 10.6 meters deep, and the next low tide occurs at 11:00 AM, when the water is 6.5 meters deep. Develop a mathematical model that will predict the water depth as a function of the elapsed time since midnight.

As an administrator, you are ultimately responsible for ensuring that your high school mathematics program provides students with experiences that will help them build the mathematical understanding that they need to succeed in life. Achieving this outcome will require reordering priorities in a number of areas. It will not only affect curriculum choices but also call for—

- standards for classroom observations that support effective instruction;
- mandated assessments that are in alignment with the goals of enhancing students' critical thinking;
- a focus on providing all students—even those who have not been successful in mathematics in the past—with the resources that they need to succeed;
- professional development opportunities for teachers; and
- collaboration with other stakeholders in high school mathematics.

What should administrators expect in classroom observations of mathematics teachers who focus on reasoning and sense making? How can administrators provide useful formative feedback focusing on reasoning and sense making?

Administrators may need to adjust their expectations when observing classrooms that focus on reasoning and sense making. Consider the following “tips” for teachers provided in *Focus in High School Mathematics*:

- Provide tasks that require students to figure things out for themselves.
- Ask students to restate the problem in their own words, including any assumptions that they have made.
- Give students time to analyze a problem

intuitively, explore the problem further by using models, and then proceed to a more formal approach.

- Resist the urge to tell students how to solve a problem when they become frustrated; find other ways to support students as they think and work.
- Ask students questions that will stimulate their thinking—for example, “Why does this work?” or “How do you know?”
- Provide adequate wait time after a question for students to formulate their own reasoning.
- Encourage students to ask probing questions of themselves and one another.
- Expect students to communicate their reasoning to their classmates and the teacher, orally and in writing, using proper mathematical vocabulary.
- Highlight exemplary explanations, and have students reflect on what makes them effective.
- Establish a classroom climate in which students feel comfortable sharing their mathematical arguments and critiquing the arguments of others in a productive manner.

Clearly, such classrooms will be active and sometimes noisy places in which students have an opportunity to work with one another and share their ideas. Administrators should not expect to see students sitting in rows, quietly working on sheets of problems. Although no learning can occur in a classroom that does not have a concentrated focus on learning mathematics, the standard for evaluating instruction should be the quality of the students' reasoning rather than merely classroom behavior. In a classroom emphasizing reasoning and sense making, students typically discover multiple ways of solving a given problem, depending on their experiences and ways of thinking. Rather than showing students the “correct” steps for solving a problem, teachers should encourage them to share their different approaches, thereby deepening students'

understanding and making the content accessible to more students.

Finally, teacher evaluations should incorporate feedback that will be useful to teachers—not just summative ratings based on a single observation. Evaluations of the teaching of mathematics should be based on information collected from a variety of sources in an ongoing process. Teachers should be participants in the evaluation of teaching, with an emphasis on self-reflection and collaboration. The ultimate goal of any evaluation should be to help teachers provide better support for their students’ development of reasoning and sense making in the context of teaching important mathematics.

Are district and school assessments focused on promoting mathematical reasoning and sense making?

What we assess should reflect what we value. We will not be able to gauge whether we are meeting our goals if we do not assess progress toward those goals. Moreover, high-stakes testing that concentrates primarily on procedural skills *without* assessing reasoning and sense making sends a message that is contrary to the vision set forth in *Focus in High School Mathematics: Reasoning and Sense Making*. Assessment that focuses primarily on students’ ability to do algebraic manipulations, apply geometric formulas, and perform basic statistical computations can adversely influence instruction and students’ learning by leading students to believe that reasoning and sense making are not important.

Although summative assessments of students’ understanding of and ability to use mathematics are important, all too often high-stakes tests assess minimal standards that do not reflect the content that students need for their future success. When students, educators, and the community focus on students’ success in learning lower-level content, everyone may lose sight of the importance of pushing students to succeed with higher-level content. Further progress is needed in the design

of standardized test items that provide better measures of students’ reasoning and sense making.

Ultimately, formative assessment needs to be a focus, since it is an integral part of the learning process for the student. Formative assessment involves providing students with learning activities and, on the basis of feedback from those activities, adjusting teaching to meet the students’ needs. Teachers need encouragement to include ongoing assessments that can furnish information about their students’ reasoning and sense making. Such assessments include teachers’ observations from classroom discussions, students’ journals, presentations, homework, and work on in-class tasks, as well as tests and quizzes that ask students to explain their thinking.

Are support systems in place to provide high school students with the necessary assistance to meet high expectations in mathematics?

Focus in High School Mathematics: Reasoning and Sense Making takes the strong stand that “reasoning and sense making must be evident in the mathematical experiences of all students.” Often, well-meaning teachers and administrators cause inequity through practices that create biases that they do not intend. Administrators can monitor equity by focusing attention on phenomena that pose potential barriers to engaging every student in reasoning and sense making, including the following:

- The *courses* that students take. For example, students in “informal” geometry deserve the same experiences with reasoning and sense making as those in “honors” geometry. Instruction should provide a focus on multilevel tasks that engage a wide range of students in exploring significant mathematics.
- *Students’ demographics*. All too often, students in some racial or socioeconomic groups or with perceived learning or behavioral problems are deemed difficult to teach and so receive less

rich experiences in mathematics. Teachers (and administrators) need to reflect on any implicit biases that may unintentionally hinder students' learning.

- *Expectations, beliefs, and biases.* Having or communicating high expectations can motivate students to perform at high levels, just as holding or conveying low expectations can lower students' confidence in their mathematical abilities and inhibit student performance. If schools and teachers operate on the assumption that all students can engage in reasoning and sense making, the programs that they design can make these expectations a reality.

How can the needs of a wide range of students be effectively met, including students who need additional support to ensure that they succeed in mathematics, as well as those who show particular mathematical talent?

Many schools rely on “tracking” to meet the needs of different students by offering different levels of courses, such as “honors” courses for students deemed to be higher performing or courses that spread the learning of algebra over a longer time for those deemed to be lower performing. With the tracking of students, high schools run the risk of affording inequitable opportunities for mathematical learning for those students placed in “remedial” classes or in classes on “low” or “regular” tracks. For example, research has shown that teacher quality often varies across tracks, with more out-of-field teachers assigned to teach low-track classes. High schools that use tracking or ability grouping can be proactive by diligently ensuring that at every level, all mathematics courses offer students opportunities for reasoning and sense making. In addition, with proper school support and access to such opportunities,

students do not have to be locked into one track for their entire high school career. Successful high schools make sure that students are both offered opportunities to move to different tracks and actively supported in making any transition. Furthermore, although experienced teachers may view teaching more advanced classes as a reward for their continued, successful service, their expertise is needed to ensure the continued growth of students at all levels.

In any given high school classroom, students—both those who need extra support and those who are ready to go further than the rest of the class—manifest varied levels of readiness for mathematical content. Schools may need to provide additional support structures, such as after-school tutoring or Saturday programs, for students who have less preparation for some courses. Relegating such students to “remedial” courses may have the twofold effect of keeping them from exploring more advanced topics and destroying their interest in pursuing mathematics.

Often, students are deemed “gifted” or “mathematically talented” on the basis of a series of tests. However, teachers and schools should recognize that mathematical “talent” is a fluid quality in students; a student who shows great promise in geometry, for example, might need extra support in understanding algebraic content. For students who demonstrate readiness to move beyond the core curriculum, teachers can provide opportunities to delve more deeply into the content that the class is currently studying. Those opportunities should help students maximize their mathematics learning experiences by extending their reasoning and sense making. Although acceleration is definitely one component of appropriate mathematics experiences for such students, opportunities for enrichment, advanced content, and more in-depth study can all be part of their mathematical education.

Does the school's and school district's counseling infrastructure promote perseverance in studying higher-level mathematics, thereby supporting the goal of increasing students' mathematical reasoning and sense making?

Principles and Standards for School Mathematics enunciates NCTM's conviction that students should "study mathematics each of the four years that they are enrolled in high school" and that they should experience reasoning and sense making across all four years of a high-quality high school mathematics program. These expectations are reinforced in NCTM's Position Statement on Time: "Evidence supports the enrollment of high school students in a mathematics course every year, continuing beyond the equivalent of a second year of algebra and a year of geometry."

Many of the differences in mathematics achievement among groups of students can be traced to the quality and number of mathematics courses that they take. Counselors need to promote perseverance in taking higher-level mathematics. Students will not excel in mathematics if they are not "stretched" mathematically.

Do school districts support the growth of their mathematics teachers' knowledge and skill to meet their students' needs? Do teachers have access to long-term professional development, including ample opportunities for reflection on their practice and their work to improve it?

Professional development is necessary to ensure that teachers have the tools that they need to build a classroom culture that promotes student engagement in reasoning and sense making. Simply providing teachers with isolated workshops will not achieve this goal. Rather, teachers need intensive, long-term experiences to help them transform their practice. Professional development opportunities should—

- engage teachers in thinking deeply about the their classroom practices and the impact that these practices have on students;
- allow teachers to explore new approaches that might be more effective than those that they currently use; and
- provide teachers with classroom support as they implement new methodologies with the potential to enhance reasoning and sense making.

Can professional development activities help teachers experience mathematics as reasoning and sense making for themselves?

Many teachers may not have personally experienced mathematics as a reasoning and sense-making endeavor. Consequently, they may have some difficulty in providing their students with the kinds of classroom experiences that are necessary for promoting critical thinking. Teachers should be encouraged to attend workshops and take mathematics courses that help them grow in their ability to reason about and make sense of mathematics. These experiences should help them increase their "content knowledge for teaching," enabling them to connect the mathematics that they are learning with the high school mathematics curriculum and assisting them in examining the ways in which they teach.

Do teachers have time during the school day to collaborate to improve mathematics instruction? Are they encouraged to work with colleagues teaching at the same level to develop tasks that promote reasoning and sense making? Do they work with those teaching at higher or lower levels to ensure that reasoning and sense making are being developed coherently across the grades?

Effective professional development must incorporate collaboration among mathematics teachers. Improvement of a high school's mathematics curriculum cannot be the responsibility of a single individual or small group but rather must be a shared responsibility of the entire mathematics department. Research suggests that teachers who engage in joint efforts to improve their practice make much more progress.

Moreover, schedules should be designed to facilitate collaboration, incorporating, for example, joint planning periods that are organized to enhance collaboration among teachers of a particular course, as well as professional development experiences that are embedded in the school day, such as peer coaching or “fishbowl” sessions in which teachers can observe a common lesson. Collaborative professional development opportunities might also include focused workshops on in-service days to address identified areas of concern.

Providing teachers with adequate time for work across courses and grade levels is also critical to students' success in mathematics. “Vertical alignment” of work in high school with work in the lower grades can help minimize discontinuities—both in content and in the focus on reasoning and sense making—as students move into and through the high school years. Furthermore, these discussions should focus on developing a common understanding across the levels rather than merely allowing high school teachers to dictate what should happen at the lower levels.

Do teachers analyze students' work collaboratively and improve the level of formative feedback for students on reasoning and sense making?

As stated earlier, formative assessment is a critical component of ensuring that high schools offer students an effective mathematics program based on reasoning and sense making. Collaboration

within high school mathematics departments should focus on students' work and how students are progressing in their mathematical reasoning and sense making. Analysis of what students are actually learning serves multiple purposes, including providing students with useful feedback for improving their performance, helping teachers make adjustments in their instruction, and steering efforts to improve the high school mathematics program at the school.

Is mentoring provided for novice teachers as they work to develop students' reasoning and sense-making skills?

As high school mathematics programs move toward the goal of focusing on reasoning and sense making, new teachers coming into the profession must be given adequate support to join the emergent community of practice. Schools should establish programs to provide this support. New mathematics teachers need mentoring by strong mathematics teacher leaders who can help them embark on successful careers focused on continued growth in promoting mathematical reasoning and sense making in their students.

Do schools and school districts develop strong partnerships with other stakeholders to promote continued improvement in the high school mathematics program?

Efforts to focus the mathematics curriculum on reasoning and sense making will require responsible efforts of—and considerable work from—all stakeholders to ensure that the high school mathematics program meets the needs of all students. Schools and school districts will need to enlist the help and support of the broader community.

Do schools and school districts work with nearby universities and draw on the expertise of academicians, including mathematicians, statisticians, and mathematics teacher educators, to develop effective programs and courses that address teachers' needs?

Collaboration with universities and members of university communities—including mathematicians, statisticians, and mathematics teacher educators—is important in providing effective professional development experiences for teachers. Schools and school districts should work closely with universities in developing and implementing effective programs and courses that focus the high school mathematics program on reasoning and sense making. A goal of this partnership should also be to ensure that preservice teachers are receiving the preparation that they need to promote reasoning and sense making when they arrive in the classroom. Preservice teachers need meaningful experiences with mathematical content that bolsters their ability to reason about and make sense of the mathematics that they need to teach high school. They also need experiences in working with teaching practices that encourage students' reasoning and sense making—including effective instructional methods, curriculum design and lesson planning, and formative and summative assessment methods.

Do schools and school districts work with families and the broader community to maintain a central focus on reasoning and sense making in high school mathematics?

Attaining the goals of *Focus in High School Mathematics: Reasoning and Sense Making* requires a culture of high expectations and a commitment to success for all students, shared not only by individual teachers but also by the entire school, including families and the community. Families need to understand why mathematics is important and why a focus on reasoning and sense making is needed for their students' future success. Without their support, the goals of *Focus in High School Mathematics: Reasoning and Sense Making* cannot be met. Other pamphlets in this series can help in opening conversations with families and other stakeholders.