Book Study Guide for

Catalyzing Change in High School Mathematics: Initiating Critical Conversations

Today, it seems as if nearly everyone agrees that high school mathematics needs to change. For far too long high school mathematics has not worked for far too many students: too many students leave high school unprepared for college or a career, particularly a STEM career; too many students do not see how math is useful in their lives; too many students leave high school without an affinity for doing math; too many students leave high school without the quantitative skills necessary to make sound decisions in their personal life and in our society which is increasingly quantitative in nature.

—Matt Larson, NCTM
President's Message, October 25, 2016

Although high school mathematics has changed somewhat in recent history, those changes are minor compared to the changes that society has undergone in the same period. The National Council of Teachers of Mathematics (NCTM) formed a task force to write *Catalyzing Change in High School Mathematics* (2017) with the intent to

- initiate the critical conversations needed to address the purposes of high school mathematics, including the guiding principles of access, equity, and empowerment;
- define math curricular pathways leading to college and career readiness, including active participation in our democratic society; and
- provide descriptions of course exemplars that could populate the pathways.

This study guide is a way to initiate these conversations in our own schools, districts, and states or provinces.

More information on *Catalyzing Change*, including information on ordering copies of the book, can be found at the following site: www.nctm.org/catalyzingchange/

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Audience: This book study surfaces issues relevant for all teachers across Pre-K–grade 12 and higher education faculty as well as administrators, specialists, and coaches. This book study guide can also be tailored to fit your audience.

Format: The book study guide is designed to support multiple formats including face-to-face, online, or hybrid settings. The nine sessions can be adapted to best meet your needs.

Process: This book study guide's format incorporates the following four steps in Sessions 2 through 9 to fully engage its participants in reflection as they read, discuss, and consider actions for next steps.

Before You Read	While You Read	After You Read	Action Steps

- **Before You Read:** In this section, we invite you to examine your own ideas about the content for that chapter by reflecting on the posed prompts, reading a related article, or engaging in conversations with other individuals.

- **While You Read:** In this section, we encourage you to record your thoughts and reactions while reading, including any questions that the book provokes.

- **After You Read:** In this section, we provide reflection questions to guide the group discussion. While many of the questions are from the NCTM More4U feature for *Catalyzing Change*, we have also added questions to many of the sessions to engage a broader audience. We recommend you select and discuss those questions that are most appropriate for your book study group.

- **Action Steps:** In this section, we invite you to continue your learning and sharing with others by creating and following an action plan related to the content from the chapter.

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Session 1

Launching the Book Study

We provide the following suggestions for launching the book study of *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM 2017).

PERSONAL CONTEXT

Ask each person in your group to share a reason for joining the book study.

- What is your personal context for teaching, examining, and/or leading high school mathematics?
- What are you hoping to gain through this book study?

DISCUSSION QUESTIONS

The following questions are intended to initiate your critical conversations about the teaching of high school mathematics and the related implications at the elementary and middle school levels. Select questions that are most applicable for your study group.

- What concerns you most about students' learning of high school mathematics?
- What jobs requiring mathematical skills exist today that didn't exist fifteen years ago?
- What do you want students to remember from their high school mathematics classes fifteen years from now when they may be in a mathematical profession that doesn't exist yet, doing a kind of mathematics that doesn't exist yet?
- Why, in your opinion, has there been so little change in high school mathematics over the past 30 years, given how much society has changed?
- What concerns you most about the teaching of high school mathematics in your school, district, or state or province?
- What beliefs, policies, and structures might need to be examined in your classroom, school, or district to improve high school mathematics?
- What barriers might exist to enacting change in your local context?
- What critical conversations, in your view, need to take place, and between which stakeholders?

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Session 2

Purpose of School Mathematics

Key Recommendation: Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.

| Before You Read | While You Read | After You Read | Action Steps |

What do you believe is the purpose of learning mathematics?

Brainstorm your own initial list. Then ask the same question of at least two or three other individuals and record their responses. Be sure to ask people not associated with education as well as educators.

My Ideas on the Purpose of Learning Mathematics	Ideas I Heard from Others

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| Before You Read | **While You Read** | After You Read | Action Steps |

Catalyzing Change: read the "Preface" (pp. xi–xii), "Introduction" (pp. 1–8), and "Purpose of School Mathematics" (pp. 9–14).

Record your thoughts, observations, reactions, and questions. Also log in the page number related to each of your responses so that you can easily find and reference the book passage during your study group discussion.

Consider	**Thoughts and Questions**
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

| Before You Read | While You Read | **After You Read** | Action Steps |

Use these four questions to frame the group discussion.

1. How would you describe your thoughts or beliefs about the purposes of high school mathematics?
2. In what ways can you help students connect their study of mathematics with their ability to make sense of and critique the claims that they encounter in public discourse?
3. What opportunities do you have to present mathematics as a triumph of human creativity, with a heritage and history that span every part of the globe?
4. How can you create opportunities for—and experiences of—wonder and joy in your classroom practices?

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| Before You Read | While You Read | After You Read | Action Steps |

Consider one or two teaching actions that you could immediately integrate that would support students in understanding and developing a purpose for learning mathematics.

Mini–Action Plan	
Who will you target?	
What will you do?	
When will you do this?	
How will you share the results of this action?	

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Session 3

Equitable Structures

> **Key Recommendation**: High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.

Before You Read	While You Read	After You Read	Action Steps

How does your school or school district address equity and access for students in mathematics? Select one or more of the following for your study group to read:

- "Access and Equity in Mathematics Education," NCTM Position Statement, April 18, 2014. https://www.nctm.org/Standards-and-Positions/Position-Statements/Access-and-Equity-in-Mathematics-Education/
- *The Opportunity Myth: What Students Can Show Us About How School Is Letting Them Down— and How to Fix It*. The New Teacher Project, 2018. https://tntp.org/assets/documents/TNTP_Opportunity-Myth_Executive-Summary_WEB.pdf (executive summary) https://opportunitymyth.tntp.org/ (full report)
- *Unlocking STEM Pathways for All Students*. Education Week, May 22, 2018. https://www.edweek.org/ew/articles/2018/05/23/unlocking-stem-pathways-for-all-students.html

Use these questions to reflect on your selected reading(s):

- What is the position or central message being advocated?
- What surprised you?

Consider your local context and examine whether your school or school district provides each and every student the opportunity to access high-quality and rigorous mathematics instruction. First, record your school's practices on grouping students for mathematics. Then describe the instruction and level of rigor provided students in the various settings. Next, record questions you have about the current student grouping practices in your school or school district.

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What We Do	Questions
Grouping: Rigor: 	

Catalyzing Change: read "Creating Equitable Structures" (pp. 15–24).

Record your thoughts, reactions, and questions along with the page number related to each response.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

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Before You Read ▸ While You Read ▸ **After You Read** ▸ Action Steps

Use these three questions to frame the group discussion.

1. What correlations can you discern between mathematics achievement and participation in your school or district and students' race, class, ethnicity, gender, beliefs, or proficiency in the dominant language?
2. What is one condition or systematic structure at your school or district that is currently a barrier to creating a positive mathematical experience for each and every student?
3. How are teachers in your local school or district currently assigned to teach mathematics courses at the different grade levels (e.g., 6–12) and course levels (e.g., standard, honors, and accelerated)?

Before You Read ▸ While You Read ▸ After You Read ▸ **Action Steps**

Which stakeholders from your mathematics education community need to be part of an ongoing conversation to address school or district practices on grouping students for mathematics learning and their access to high-quality mathematics instruction?

Mini–Action Plan	
Who are the stakeholders?	
How will you engage them in this conversation?	
When will you do this?	
How will you share the results of this action?	

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Session 4

Equitable Instruction

> **Key Recommendation**: Classroom instruction should be consistent with research-informed and equitable teaching practices.

| Before You Read | While You Read | After You Read | Action Steps |

In their book *The Impact of Identity in K–8 Mathematics Learning and Teaching: Rethinking Equity-Based Practices* (2013), Julia Aguirre, Karen Mayfield-Ingram, and Danny Martin defined mathematical identity as "the dispositions and deeply held beliefs that students develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics in powerful ways across the contexts of their lives" (p.14). Reflect on this definition, thinking about your current students or students whom you have taught in the past.

What are some of the mathematical identities held by these students?	How are you contributing to the development of positive mathematical identities for students?

| Before You Read | While You Read | After You Read | Action Steps |

Catalyzing Change: read "Implementing Equitable Instruction" (pp. 25–36).
Record your thoughts, reactions, and questions along with the page number related to each response.

Consider	**Thoughts and Questions**
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

| Before You Read | While You Read | After You Read | Action Steps |

Use these seven questions or select the questions that are most applicable to your discussion group to frame the group conversation.

1. Consider your responses in the Before You Read section. What did you learn from the reading that enhances your understanding of students' mathematical identities?
2. How are you contributing to positive mathematical identities for students?
3. Do you have students who need more support in building positive mathematical identities? Why is that the case?
4. Are you aware of practices, intended or unintended, that you or your colleagues implement that negatively affect students' identities? If so, what are these practices?
5. Does your school or district support all students? Are there gaps? If so, why?
6. To what extent do you, individually and collectively with your colleagues, implement equitable teaching practices consistently? Are there classes where this happens more than others? If so, why?
7. How do you engage students in reasoning and sense making?

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Before You Read > While You Read > After You Read > Action Steps

Review pages 25–36 in *Catalyzing Change*. Which mathematics teaching practice and corresponding equitable teaching practices might you begin to strengthen in your own teaching or to focus on in your professional work (e.g., coaching) to ensure that all students consistently experience equitable and supportive learning?

Mini–Action Plan	
What teaching practice and related equitable teaching practices did you select?	
How will you begin to implement these practices?	
When will you do this?	
How will you determine the effectiveness of this action?	

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Session 5

Mathematical Practices and Processes and Essential Concepts for Number, Algebra, and Functions

> **Key Recommendation**: Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.

The chapter on "Essential Concepts in High School Mathematics" includes discussions of developing mathematical practices and processes and of essential concepts in four key content domains—number, algebra and functions, statistics and probability, and geometry and measurement. The study of this chapter has been extended across Sessions 4, 5, and 6 to provide more time for the examination of its content and participation in in-depth conversations. Discussion questions appropriate for a broader Pre-K–12 audience and questions more specific to a high school audience are identified.

| Before You Read | While You Read | After You Read | Action Steps |

This session includes three options for Before You Read. Select the option that is most applicable to your book study group.

Option 1: What high school mathematics courses are offered in your school or district? How are the courses sequenced? Which courses include the study of number, algebra, and functions? List the high school mathematics courses taught in your school or district and record the key topics for number, algebra, and functions taught in each course.

List the high school course sequence.	Record the key topics for number, algebra, and functions taught in each course.

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Option 2: Individuals often have strong opinions about their learning of high school algebra. Conduct short interviews with four adults using the following questions and record their responses:

- What do you remember learning in high school algebra?
- Was it a topic you enjoyed studying? Why or why not?
- What are some examples in your current life in which you use the knowledge and skills from your study of high school algebra?
- What are some examples of how others use knowledge and skills of algebra in today's world? How does their work influence your life (e.g., insurance rates, supply and demand of goods and services)?

Option 3: Why is algebra an important topic of study for students? Select one of the following for your study group to read:

- "Algebra as a Strand of School Mathematics for All Students," NCTM Position Statement, April 2014.
  https://www.nctm.org/Standards-and-Positions/Position-Statements/Algebra-as-a-Strand-of-School-Mathematics-for-All-Students/
- "A Journey in Algebraic Thinking," President's Message by Cathy Seeley, September 2004.
  https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Cathy-Seeley/A-Journey-in-Algebraic-Thinking/
- "What Algebra? When?," President's Message by Skip Fennell, January/February 2008.
  https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Skip-Fennell/What-Algebra_-When_/

In reflecting on the reading, how do students develop algebraic ideas across Pre-K–grade 12? Select a grade band (e.g., Pre-K–2, 3–5, 6–8, 9–12) and list ways students experience and develop algebraic ideas across these grades.

Grade Band	What algebraic ideas are studied at this grade band?

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| Before You Read | While You Read | After You Read | Action Steps |

Catalyzing Change: Read "Essential Concepts in High School Mathematics" pages 37–55. These pages focus on mathematical practices and processes, number, and algebra and functions. Record your thoughts, reactions, and questions along with the page number related to each response.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

| Before You Read | While You Read | After You Read | Action Steps |

Select questions that are most applicable to frame the discussion for your study group.

Mathematical Practices and Processes

1. How do you plan for and implement instruction that engages students in developing the mathematical practices identified in the *Common Core State Standards for Mathematics* and the processes identified in *Principles and Standards for School Mathematics*?
2. How might opportunities for students to model mathematics be regularly integrated into and across mathematics courses?
3. What kinds of tasks invite students to model with mathematics?

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Essential Concepts in Number

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. How does the discussion of number change your perception about effective strategies for teaching number concepts across Pre-K–grade 12?
2. Elementary and middle school mathematics programs concentrate on number concepts. What strategies do you use to support students' reasoning about number?
3. How might elementary and middle school mathematics curricula better support students' understanding of units of measurement as part of problem solving?
4. What particular elementary, middle, and high school number ideas need to be bolstered? Why?

HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. The Essential Concepts in Number are addressed most effectively as part of the study of algebra and functions, statistics and probability, and geometry and measurement. High school mathematics programs do not need and are unlikely to offer an entire course, or even units, of study devoted specifically to the concept of number. What are effective strategies for addressing the Essential Concepts in Number within the high school curriculum?
2. Students can readily grasp the idea that $2x = 5$ cannot be solved without using a fraction and that $2x + 10 = 6$ cannot be solved without using a negative number. How can solving quadratic equations like $ax^2 = b$ demonstrate to students that rational numbers are not sufficient, suggesting a need for "other numbers" to "complete" the real number system?
3. Students often ignore the units in graphing and in describing slope as well as in doing calculations. How can attention to measurement units contribute to developing quantitative reasoning ability in high school students?
4. Ignoring or not attending to units of measurement as part of problem solving and mathematical modeling can lead to errors or unrealistic solutions. What might some of the consequences be if measurement units are ignored?
5. Because percentages, by definition, provide a common basis for comparison basis out of 100, their use may have both positive and potentially negative consequences on students' interpretations of results. Why are percentages often useful in contextual situations? What examples can you think of or find of situations in which conclusions are drawn solely on the basis of percentages, and what are the positive or negative consequences of doing so?

Essential Concepts in Algebra and Functions: Focus Area 1

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. How does the discussion of algebra change your perception about effective strategies for teaching and integrating algebraic concepts across Pre-K– grade 12?
2. What strategies can you use to foster algebraic reasoning in your classroom or school?
3. What are the some of the big ideas in elementary school that lay the foundation for algebraic reasoning in middle school and high school?

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4. How can you ensure that students are not using technology to replace thinking and reasoning?
5. What particular algebra ideas might need to be bolstered at the elementary, middle, and high school levels?

HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. Students should come to high school knowing how to solve linear equations that can be reduced to the form $ax + b = cx + d$, how to graph linear equations in two variables, and how to interpret the coefficients of those equations in relation to the graph. What can an algebra teacher in high school do to remind students of what they have learned previously without re-teaching all of the earlier material?
2. Using technology to build conceptual understanding is as important, or even more so, than using technology to draw graphs, crunch numbers, or make tables. What are some central algebraic concepts that technology can help students grasp or understand more deeply?
3. How can you ensure that students are not using technology to replace thinking and reasoning?
4. Which of the following forms of the equation of a line is the most fundamental and why?

$$y = mx + b$$
$$y - y_1 = m(x - x_1)$$
$$y = y_1 + m(x - x_1)$$
$$y = m(x - x_1)$$
$$ax + by = c$$

Essential Concepts in Algebra and Functions: Focus Area 2

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. How does the approach to functions in *Catalyzing Change* challenge your understanding of functions?
2. How can the approach to functions in *Catalyzing Change* connect to teaching practice at the elementary or middle school levels?
3. What strategies do you use to support students' opportunities to connect algebra to functions?
4. How can you provide more opportunities for students to model functions in your classroom or school?
5. How can elementary, middle, and high school teachers bolster ways to help students connect algebra to functions?

HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. Algebra typically deals with equations of the form $ax + by = c$. What is gained and what is lost by turning all equations of this form into equations of the form $y = (-a/b)x + c/b$ (comparable to the form $y = mx + b$)?
2. What is the difference, if any, between the concept of a parabola and the concept of a quadratic function?
3. How can graphs help students make sense of the algebraic ideas that they are working with, and how do those ideas connect to the concept of function?

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| Before You Read | While You Read | After You Read | Action Steps |

Which number, algebra, and functions concepts need more emphasis in your course sequence?

Which number, algebra, and functions concepts might you begin to strengthen in your own teaching or to focus on in your professional work (e.g., coaching) to ensure that all students consistently experience equitable and supportive learning?

Mini–Action Plan	
What are the number, algebra, and functions essential concepts you would like to strengthen? Select at least one concept, but no more than three.	
How will you strengthen the number, algebra, and functions essential concept(s)? Describe specific steps you will take. Consider individuals and resources you will need to support this work.	
What do you expect students to be able to do as a result of this change?	
How will you communicate the results of your action plan?	

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Session 6

Essential Concepts for Statistics and Probability

> **Key Recommendation**: Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.

Before You Read	While You Read	After You Read	Action Steps

This session includes two options for Before You Read. Select the option that is most applicable to your book study group.

Option 1: What high school mathematics courses are offered in your school or district? Which courses include study of statistics and probability? List the high school mathematics courses taught in your school or district and record the key topics for statistics and probability taught in each course.

List the high school course sequence.	Record the key topics for statistics and probability taught in each course.

Option 2: How are statistics used in different professions? What are some careers in statistics? Watch the short video "This is Statistics" with Roger Peng from Johns Hopkins University. Additional videos in the series are available at https://thisisstatistics.org/students/#statisticians

Use these questions to reflect on the video:

- What surprised you?
- What are some examples of careers in statistics or ways statistics are used in different professions?
- Do you remember studying statistics or probability in high school? Middle school? Elementary school?
- What are some implications for high school mathematics related to including more emphasis on statistics and probability?

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| Before You Read | While You Read | After You Read | Action Steps |

Catalyzing Change: Read "Essential Concepts in High School Mathematics" pages 56–66. These pages focus on statistics and probability.

Record your thoughts, reactions, and questions along with the page number related to each response.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

| Before You Read | While You Read | After You Read | Action Steps |

Select questions that are most applicable to frame the discussion for your study group.

Focus 1: Quantitative Literacy

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. What resonated with you from the discussion of quantitative literacy?
2. Which of these five dimensions of quantitative literacy are addressed in your school's mathematics program:
   a) practical (routine tasks encountered in daily life),
   b) civic (public policy matters),
   c) professional (employment skills),
   d) recreational (games, sports, and lotteries), and
   e) cultural (elements of human civilization)?
3. How does the discussion change your perception about the teaching of statistics and probability and integrating these concepts at all grade levels?

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HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. How might your school's mathematics program assess whether graduates have quantitative literacy?
2. Which of these five dimensions of quantitative literacy are addressed in your school's mathematics program and within which course or courses:
    a. practical (routine tasks encountered in daily life),
    b. civic (public policy matters),
    c. professional (employment skills),
    d. recreational (games, sports, and lotteries), and
    e. cultural (elements of human civilization)?

Focus 2: Visualizing and Summarizing Data

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. How does the discussion in *Catalyzing Change* inform your understanding of visualizing and summarizing data?
2. How might the discussion on visualizing and summarizing data connect to teaching practices at the elementary or middle school levels?
3. What are some basic misconceptions that students have in reading and interpreting graphs and discussing data? How can you help them confront and correct those misconceptions?
4. What are some strategies you might use to help students make sense of data they encounter in and outside of the mathematics classroom?
5. What understandings and capabilities related to visualizing and summarizing data need to be bolstered in elementary school? Middle school? High school?

HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. Many middle school statistics standards focus on graphing univariate data. What graphs should students be able to construct and interpret when they come to high school? What potential challenges might they have when they encounter these graphs in high school?
2. What are some basic misconceptions that students have about box plots? How can you help them confront and correct those misconceptions?
3. What strategies can you use to help students estimate the mean and standard deviation or the median and interquartile range (IQR) from a graphical representation of a distribution?
4. How can you help students progress from the median absolute deviation (MAD) developed in the middle grades as a measure of variability to the standard deviation as the measure of variability typically developed in high school?
5. Many different strategies can be used to find a linear model for the relationship between two variables. What are some of these strategies? Find an example that illustrates why one of these strategies can be advantageous over the other strategies.
6. Understanding the conditions necessary to infer a cause-and-effect relationship is a key element in thinking about the correlation between two variables. For example, data show that students who take algebra in eighth grade are successful in college, and some people infer from this that

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taking algebra in eighth grade is good. But what other factors might contribute to the association between taking algebra and success in college?

Focus 3: Statistical Inference

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. How does the discussion in *Catalyzing Change* inform your understanding of statistical inference? How might this understanding connect to your teaching practice at the elementary or middle school levels?
2. What strategies do you use to support students' opportunities to make statistical inferences?
3. What understandings and capabilities related to statistical inference need to be bolstered in elementary school? Middle school? High school?

HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. What is the role of questioning in the statistical investigative process? Give examples of questions that might be posed at different stages of the process.
2. Why is random sampling important for making inferences about a population proportion or mean?
3. Why is random assignment important in experimental design?
4. What are the key differences between observational studies and experiments? How do these differences affect the type of inferences that can made from the data?
5. What questions should consumers ask when confronted with a statistical analysis of data—for example, the results of a medical study or a survey of what are the most important priorities in high school education?
6. What is the advantage of using hands-on methods to simulate a sampling distribution? What is the advantage of using technology to simulate a sampling distribution?

Focus 4: Probability

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. How does the discussion in *Catalyzing Change* inform your understanding of probability? How might this understanding connect to your teaching practice at the elementary or middle school levels?
2. How might you construct opportunities for students to model real-world probability situations?
3. What strategies do you use to support students' opportunities to reason about probability?
4. What understandings and capabilities related to probability might need to be bolstered in elementary school? Middle school? High school?

HIGH SCHOOL DISCUSSION REFLECTION QUESTIONS

1. What does it mean that events are independent?
2. How is the probability of an outcome that results from a dependent event different from that of an independent event?
3. What are examples of situations that your students might encounter that involve conditional probability?
4. How does a contingency table help in computing conditional probability?

| Before You Read | While You Read | After You Read | Action Steps |

Which statistics and probability concepts might you begin to strengthen in your own teaching or to focus on in your professional work (e.g., coaching) to ensure that all students consistently experience equitable and supportive learning?

Mini–Action Plan	
What statistics and probability essential concepts would you like to strengthen? Select at least one concept, but no more than three.	
How will you strengthen the statistics and probability essential concept(s)? Describe specific steps you will take. Consider individuals and resources you will need to support this work.	
What do you expect students to be able to do as a result of this change?	
How will you communicate the results of your action plan?	

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Session 7

Essential Concepts for Geometry and Measurement

Key Recommendation: Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.

| Before You Read | While You Read | After You Read | Action Steps |

This session includes three options for Before You Read. Select the option that is most applicable to your book study group.

Option 1: What high school mathematics courses are offered in your school or district? Which courses include the study of geometry and measurement? List the high school mathematics courses taught in your school or district and record the key topics for geometry and measurement taught in each course.

List the high school course sequence.	Record the key topics for geometry and measurement taught in each course.

Option 2: Individuals often have strong opinions about their learning of high school geometry. Conduct short interviews with four adults using the following questions and record their responses:

- What do you remember about studying geometry in high school?
- Was it a topic you enjoyed studying? Why or why not?
- What are some examples in your current life in which you use the knowledge and skills from your study of high school geometry and measurement?
- What are some examples of how others use the knowledge and skills of geometry and measurement in today's world? How does their work influence your life?

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Option 3: Why are geometry and measurement important topics of study for students? Select one of the following NCTM President Messages for your study group to read:

- "Let's Not Forget Geometry!" by J. Michael Shaughnessy, October 2011. https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/J_-Michael-Shaughnessy/Let_s-Not-Forget-Geometry!/
- "Don't Forget Geometry" by Henry S. Kepner Jr., March 2009. https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Henry-(Hank)-Kepner,-Jr/Don_t-Forget-Geometry!/
- "Geometry: The Forgotten Strand" by Glenda Lappan, December 1999. https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Glenda-Lappan/Geometry_-The-Forgotten-Strand/
- "Measure by Measure" by Henry S. Kepner Jr., May/June 2009. https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Henry-(Hank)-Kepner,-Jr/Measure-for-Measure/

Use these questions to reflect on the reading:

- Why is geometry often a forgotten topic?
- How much emphasis is given to geometry and measurement at your grade level or in your school?
- What might be the value of giving greater emphasis to geometry and measurement across Pre-K–grade 12?

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| Before You Read | While You Read | After You Read | Action Steps |

Catalyzing Change: Read "Essential Concepts in High School Mathematics" pages 67–80. These pages focus on geometry and measurement.

Record your thoughts, reactions, and questions along with the page number related to each response.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

| Before You Read | While You Read | After You Read | Action Steps |

Select questions that are most applicable to frame the discussion for your study group.

Focus 1: Measurement

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. What resonated with you from the discussion of measurement in *Catalyzing Change*?
2. How might you connect the expectations for measurement at the high school level to teaching practice at the elementary or middle school levels?
3. How might you construct opportunities for students to model or engage in real-world measurement situations?
4. What particular measurement ideas might need to be bolstered at the elementary, middle, and high school levels?

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HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. In elementary and middle school, students might approximate the area enclosed within a curved figure, such as the shape of a hand, by using congruent squares. The smaller the squares, the better the approximation. How does this area approximation process relate to ways of justifying formulas for the area or circumference of a circle in high school geometry? How could these examples eventually lead to concepts underlying integration in calculus?

2. If two figures are known to be similar with ratio of similarity (or ratio of similitude) equal to k, then the ratio of any corresponding distances or lengths in the figures is k, the ratio of any corresponding areas is k^2, and the ratio of any corresponding volumes is k^3. How can this be explained in terms of the definitions of length, area, and volume? How does this relate to measurement approximations obtained by using unit segments, squares, and cubes? What sequence of activities might you use to help students understand these ideas, and what would be the key points for students to take away from each activity?

Focus 2: Transformations

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. How does the discussion in *Catalyzing Change* inform your understanding of transformations?
2. How might you connect the expectations for transformations at the high school level to teaching practice at the elementary or middle school levels?
3. How can you produce opportunities for students to build stronger understandings of congruence and similarity?
4. What particular understandings and capabilities related to transformations might need to be bolstered at the elementary, middle, and high school levels?

HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. Congruence and similarity can be defined from the perspective of transformations. In what ways does the definitions of *congruence* and *similarity* from a transformation perspective mirror each other? How might you sequence instructional activities for congruence and similarity to highlight for students how their structures mirror each other?

2. In elementary school, students learn about the symmetry of butterfly cutouts or heart cutouts: they can fold the paper and match sides to each other. How is symmetry related to reflection? Extending this idea, how many folds could match sides of a snowflake cutout? How could this be expressed in terms of reflections sending a figure back to itself? Are there rotations that could send the snowflake to itself? How could these examples be used to introduce students to the idea of symmetries coming from any congruence? What figures might have translational symmetry?

3. Below are three definitions of *congruence* that appear to be very different from one another on the surface. What are the advantages of each one, and how might you design congruence tasks differently for students depending on which definition you were using? Why are the three definitions mathematically equivalent?

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- ○ "Two figures in the plane are said to be congruent if there exists a finite composition of basic rigid motions [translations, rotations, and reflections] that maps one figure onto the other." (Eureka Math [2015], Geometry Module 1, Lesson 19, p. 161)
- ○ "Two figures F and G are congruent figures, written F ≅ G, if and only if G is the image of F under a reflection or composite of reflections." (Usiskin, Coxford, and Hirschhorn [1991], Lesson 6–5, p. 279)
- ○ "A set A is congruent to a set B (and written A ≅ B) if B is the image of A by a rigid motion" (Park City Mathematics Institute [2017], Geometry Transformed, Class 1 Facilitator's Guide, p. 3).

Focus 3: Geometric Arguments, Reasoning, and Proof

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. How does the discussion in *Catalyzing Change* inform your understanding of geometric arguments, reasoning, and proof?
2. Mathematics sometimes follows this cycle in high school:
   Experimentation → Discovery → Conjecture → Proof → Certification.
   What might this process look like in elementary or middle school classrooms?
3. Rather than giving students definitions, how can you support students in constructing their own geometric definitions across Pre-K–grade 12?
4. As an elementary or middle school teacher, how can you build opportunities for students to develop and engage in geometric reasoning?

HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. Mathematics sometimes follows this cycle:
   Experimentation → Discovery → Conjecture → Proof → Certification.
   What different types of questions and follow-up probes might you use with students in each stage of this cycle?
2. In what ways does the cycle in question 1 resemble or differ from the modeling cycle?
3. How might dynamic software contribute to the process of discovering a proof? How can you help students understand the purposes of and differences between convincing examples and deductive reasoning?
4. In what ways might communicating a geometric proof help a student later in life?

Focus 4: Solving Applied Problems and Modeling in Geometry

PRE-K–12 AUDIENCE DISCUSSION QUESTIONS

1. How does the discussion in *Catalyzing Change* inform your understanding of solving applied problems and modeling in geometry?

2. How might you connect the expectations for solving applied problems and modeling in geometry at the high school level to teaching practice at the elementary or middle school levels?

HIGH SCHOOL AUDIENCE DISCUSSION QUESTIONS

1. What is meant by the "complete modeling cycle?"
2. What would be required to support students in identifying and pursuing applied problems or short modeling activities that use geometry?

| Before You Read | While You Read | After You Read | Action Steps |

Which geometry and measurement concepts might you begin to strengthen in your own teaching or to focus on in your professional work (e.g., coaching) to ensure that all students consistently experience equitable and supportive learning?

Mini–Action Plan	
What are the geometry and measurement essential concepts you would like to strengthen? Select at least one concept, but no more than three.	
How will you strengthen the measurement and geometry essential concept(s)? Describe specific steps you will take. Consider individuals and resources you will need to support this work.	
What do you expect students to be able to do as a result of this change?	
How will you communicate the results of your action plan?	

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Session 8

Organizing High School Mathematics

Key Recommendation: High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts, to ensure the highest-quality mathematics education for all students.

| Before You Read | While You Read | After You Read | Action Steps |

How can we reimagine the way high school mathematics is organized? How would a reorganization of high school mathematics change the way elementary and middle school mathematics are organized? Select one of the following for your study group to read:

- "Opinion: How One City Got Math Right" by Jo Boaler, Alan Schoenfeld, Phil Daro, Harold Asturias, Patrick Callahan, and David Foster. *The Hechinger Report*, October 9, 2018. https://hechingerreport.org/opinion-how-one-city-got-math-right/
- "Calculus Is the Peak of High School Math. Maybe It's Time to Change That." *Education Week*, May 22, 2018. https://www.edweek.org/ew/articles/2018/05/23/calculus-is-the-peak-of-high-school.html

Use these questions to reflect on the reading:

- What surprised you about what you read?
- What pathway(s) does your school or district offer students to progress through the high school mathematics curriculum?

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Record your ideas for reimagining the way mathematics learning should be organized in high school. Then ask two other individuals for their ideas and record them.

My ideas for reimagining the way mathematics learning should be organized in high school.	Ideas I Heard from Others

<table>
<tr><td>Before You Read</td><td>While You Read</td><td>After You Read</td><td>Action Steps</td></tr>
</table>

Catalyzing Change: Read "Organizing High School Mathematics" (pp. 83–90). Record your thoughts, reactions, and questions along with the page number related to each response.

Consider	Thoughts and Questions
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

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Before You Read	While You Read	After You Read	Action Steps

Use these questions or select questions that are most applicable for your study group to frame the group discussion.

1. Look closely at the section "What Should Count as a Mathematics Course." Do you agree with the criteria given? If not, what would you change, and why?

2. Think about all the mathematics courses that you currently teach or that your school or district currently offers. Do these courses consistently meet these criteria?

3. As you consider the Essential Concepts in relation to your mathematics program, are you aware of any Essential Concepts that some students currently do not have access to or opportunities to learn?

4. What, if any, mathematics content do some students not encounter or have experiences with in your mathematics program? Are some students denied access to content learning opportunities?

5. What would be the benefits for students of the two sample pathways presented?

6. What challenges would exist for students? What challenges would exist for teachers and administrators?

Before You Read	While You Read	After You Read	Action Steps

Identify and invite a current student, former student, or graduate of your school or school district to be interviewed.

Ask the individual the following questions:

- Describe your experience learning mathematics.
- How would you describe yourself as a mathematics learner?
- What mathematics courses have you taken?
- Did the mathematics you learned in one course relate to the mathematics you learned in your other courses? Did the topics flow seamlessly from course to course?
- Do you feel as if the mathematics courses you took prepared you for your future goals?
- What mathematics topics do you wish you had studied or studied in more depth?

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Mini–Action Plan	
What will you do to identify a student to interview?	
What other questions will you ask?	
When will you conduct your interview?	
How will you share the results of this interview?	

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Session 9

Catalyzing Change: Next Steps

Key Recommendations of *Catalyzing Change*:

- Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.
- Classroom instruction should be consistent with research-informed and equitable teaching practices.
- High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.
- High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts, to ensure the highest-quality mathematics education for all students.

| Before You Read | While You Read | After You Read | Action Steps |

What information do you need to learn about decisions regarding high school mathematics in your school or district?

Who do you need to ask?

Information Needed	Who Can You Ask?

Before You Read	While You Read	After You Read	Action Steps

Catalyzing Change: read "Catalyzing Change: Next Steps" and "Conclusion" (pp. 91–94). Record your thoughts, reactions, and questions along with the page number related to each response.

Consider	**Thoughts and Questions**
What are you wondering? What resonates with you? What worries you? What do you want to learn more about?	

Before You Read	While You Read	After You Read	Action Steps

Use these questions to frame the group discussion.

- What strategies are in place in your classroom, district, or state or province to ensure that all students have access to equitable instruction?
- If you are an elementary or middle school teacher, how does the information in *Catalyzing Change* speak to the mathematics teaching, learning, policies and practices in your setting?

35

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Now is your chance to catalyze change! What steps will you take to initiate change in your school or school district?

1. What action would create the most positive change in realizing the goal of reaching and supporting each and every student?

2. Do you know of someone in another role who would be willing to engage with you on this effort? How could he or she work with you to move toward the goal of ensuring that each and every student has access to the mathematics they need on the basis of their interests and aspirations for whatever future path they may pursue in life?

Mini–Action Plan	
What will you do to create a positive change?	
Who will you need to contact to support you in making this change?	
What steps will you need to take to begin this change?	
When will you do this?	
How will you share the results of this action?	

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Resources

Additional resources for *Catalyzing Change* available at: www.nctm.org/catalyzingchange/
- Executive Summary of Catalyzing Change
- *Catalyzing Change* Infographic
- NCTM News Release
- Recorded webinar by Matt Larson, NCTM past president, on *Catalyzing Change*.
- Links to related news articles on high school mathematics.

NCTM President Messages on High School Mathematics
- "Bringing Needed Coherence and Focus to High School Mathematics" by Matt Larson, October 25, 2016.
  https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Matt-Larson/Bringing-Needed-Coherence-and-Focus-to-High-School-Mathematics/#
- "Catalyzing Change in High School Mathematics" by Matt Larson, March 21, 2018.
  https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Matt-Larson/Catalyzing-Change-in-High-School-Mathematics/#
- "Initiating Critical Conversations on the Discontinuation of Tracking" by Robert Q. Berry III, June 2018.
  https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Robert-Q_-Berry-III/Initiating-Critical-Conversations-on-the-Discontinuation-of-Tracking/
- "Positioning Students as Mathematically Competent" by Robert Q. Berry III, June 2018.
  https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Robert-Q_-Berry-III/Positioning-Students-as-Mathematically-Competent/

Education Week Articles
- "High Schoolers Should Take 4 Years of Leaner, More Relevant Math." *Education Week*: April 25, 2018.
  http://blogs.edweek.org/edweek/curriculum/2018/04/high_school_math_vision_nctm.html
- "Mathematics Education: Initiating Critical Conversations." *Education Week*: May 1, 2018.
  https://www.edweek.org/ew/articles/2018/05/02/mathematics-education.html?qs=catalyzing+change.html
- "Unlocking STEM Pathways for ALL students: Which Policies Open Doors for Students to Stem—and Which Slam Them Shut?" *Education Week*: May 22, 2018
  https://www.edweek.org/ew/articles/2018/05/23/unlocking-stem-pathways-for-all-students.html
- "Retooled Courses Help Students Avoid a Remedial-Math Roadblock to College." *Education Week*: May 22, 2018.
  https://www.edweek.org/ew/articles/2018/05/23/retooled-courses-help-students-avoid-a-remedial-math.html
- "Calculus Is the Peak of High School Math. Maybe It's Time to Change That." *Education Week*: May 22, 2018. https://www.edweek.org/ew/articles/2018/05/23/calculus-is-the-peak-of-high-school.html

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- "Don't Track Algebra." *Education Week*: July 17, 2018.
  https://www.edweek.org/ew/articles/2018/07/18/dont-track-algebra.html

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