**Lesson Plan – Candy Jar Task**

Common Core State Standards 7.RP:

2. Recognize and represent proportional relationships between quantities.

1. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
2. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
3. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
4. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

**Learning Goals:**

1. *Quantities that are in a proportional (multiplicative) relationship grow at a constant rate.*
2. *The unit rate specifies the relationship between the two quantities; it answers the question how many times larger one quantity is in comparison to the other (i.e., the number of JR is 2.6 times the number of JB; 2.6 is the unit rate).*
3. *The constant of proportionality (unit rate) can be identified in a table, graph, equation, and verbal descriptions of proportional relationships.*
4. *Proportional relationships are linear functions that go through the point (0,0); they can be written in the form of y=kx where k is the unit rate, the slope of the line, and the constant of proportionality.*
5. *The scale factor quantifies the relationship between the initial amount and the new amount of a quantity; it is the answer to the question how many times bigger the new amount is in comparison to the initial amount (i.e., the jar with 100 JR is 20 times larger than the jar with 5 JR; 20 is the scale factor).*

**Performance Goals:** Students will identify the relationship between quantities as multiplicative (proportional) and will use one of several strategies (e.g., unit rate, scale factor, scaling up, cross-multiplication) and/or representations (e.g., table, picture, procedures, verbal description) to solve the task.

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| The Candy Jar Task | |
| The candy jar contains Jolly Ranchers (the rectangles) and Jawbreakers (the circles). Solve each of the following problems:  Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers as shown above, but it contained 100 Jolly Ranchers. How many Jawbreakers would you have?    *If you finish, try solving the problem a different approach from the one you first used. Describe the relationship between the different approaches to solving the same problem.* |  |

Materials:

* + - copies of the task for each student
    - additional paper
    - candy jar with jolly ranchers and jawbreakers
    - red and green tiles
    - calculator
    - graph paper

(This is one task in a sequence of tasks that are intended to build students understanding of ratio and proportion. See page 8 to see where this task fits into the sequence.)

**Setting Up the Task**

* Introduce the task - have a student read the question; poll students on which of the two candies they prefer. Regardless of their answer, say that’s why in this jar there are more jawbreakers (either because that’s the favorite, or because that’s the ‘filler’ around the favorite—just like trick-or-treat bowls usually have filler candy)
* Ask students, “How many more JBs than JRs are there?” (Ans: 8). Ask, “How many *times* more JBs are there than JRs? Two times as many?” (Ans: more than that) “Three times as many?” (Ans: No, that would be 15 JBs)
* Ask students what a RATIO is in general and what it is in this candy jar (refer to an actual candy jar that has JB and JR in it); ask what it is they are trying to find in this task
* Tell students they willwork independently for 3-5 minutes; in groups/pairs for 10-15 minute; and then participate in a whole group discussion of a subset of selected solutions
* Tell students they use any of the resources available to solve the task and they can use any method they wish. BUT in addition to coming up with an answer to the question, they must justify their answer in writing and explain how they got it. Work will be shared using the document camera.

**Supporting Students Exploration of the Task**

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| Possible Student Approaches | | | | Questions to Ask |
| **Can’t get started** | | | | * Can you build a model of the candy jar using the red and green tiles? * What is the ratio of JR to JB in your candy jar? * What is the first problem asking you to do? * How could you make a jar that has 10 JR but has the same ratio of JR to JB? |
| **Finish Immediately and can answer all the questions asked about their approach** | | | | * Can you describe how you could use your strategy to find the number of JB in *any* jar: 1) given the number of JR? 2) given the number of JB? 3) given the total number of candies? * Can you draw a graph to represent the number of JB and JR? What does each point on the graph mean in terms of the problem? How does the graph relate to your other strategy for solving the problem? |
| **Incorrect Additive**  *100 JR is 95 more than the 5 I started with. So I will need 95 more JB than the 13 I started with.*  5 JR + 95 JR = 100 JR  13 JB + 95 JB = 108 JB | | | | * In the original ratio, about how many JB are there for every JR? In the new jar, about how many JB are there for every JR? * Are the two ratios (5:13 and 100:108) equal? Should they be? * Suppose you wanted to make a jar with 10 JR. How many JB would you need? Can you build the model? |
| Possible Student Approaches | | | | Questions to Ask |
| **Factor-of-Change (Scale Factor)**  *You had to multiply the five JR by 20 to get 100, so you’d also have to multiply the 13 JB by 20 to get 260.*  (x20)  5 JR 100 JR  13 JB 260 JB  (x20) | | | | * Why did you multiply the number of JR and JB by 20? * What does 20 mean in the context of this problem? * How much bigger is the new jar than the original jar? * Suppose the new candy jar had 35 JR. How many JB would it have? * Can you describe how you could use your strategy to find the number of JB in any jar given the number of JR? * Suppose the original ratio of JR to JB was 3:15. How many JB would you have in a jar that had 100 JR? |
| **Unit Rate**  *Since the ratio is 5 JR for 13 JB, for each JR you would have 2 JB; that would use up 10JB. So you have three JB left over. So I had to distribute the three JB to the 5 JR. 3 ÷ 5 = .6 so that would give the ratio of 1 JR to 2.6 JB. So then you just multiply 2.6 by 100.*  (x100)  1 JR 100 JR  2.6 JB 260 JB  (x100) | | | | * How did you get 2.6? * What does 2.6 mean in the context of the problem? * Suppose the new candy jar had 35 JR. How many jawbreakers would it have? * Can you describe how you could use your strategy to find the number of JB in any jar given the number of JR? * Suppose the original ratio of JR to JB was 3:15. How many jawbreakers would you have in a jar that had 100 JR? |
| **Cross Multiplication** | | | | * How did you set up the proportion? Why? * Why did you cross-multiply? * What does 5x=13 x 100 mean? * Why did you divide 1300 by 5? What does that mean? * What does your answer mean? * How is your initial ratio related to your new ratio? |
| **Scaling Up**  *I drew 100 JR in groups of 5. Then I put 13 JB with each group of 5 JR. I then counted the number of JB and found I had used 260 of them.* | | | | * Why did you put the 100 JR in groups of 5? * How many groups of 5 did you end up with? * How many groups of 13 did you end up with? * Why are the number of groups of 5 and 13 the same? * What does the number of groups of 5 and 13 mean? * Could you determine the number of groups you will have without drawing? Suppose you had a new jar with 150 JR. How many JB would there be? |
| Possible Student Approaches | | | | Questions to Ask |
| **Scaling Up**   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | JR | 5 | 10 | 20 | 40 | 80 | 100 | | JB | 13 | 26 | 52 | 104 | 208 | 260 |   *I started by doubling both the number of JR and JB. But then when I got to 80 JR I didn’t want to double it anymore because I wanted to end up at 100 JR and doubling 80 would give me too many. So I noticed that if I added 20 JR: 52 JB and 80 JR: 208 JB I would get 100 JR:260 JB.* | | | | * Why did you start by doubling? * Why did you stop doubling? * What does each of the entries in your table mean? * Does each entry have the same ratio as you started with? How do you know? * How is the initial ratio related to the final ratio? * How is the number of JR in any row related to the number of JB in that row? * Could you graph the entries you have in your table? What do you think you would get? Why? |
| **Scaling Up** | | | | * How did you build your table? |
| JR | JB | JR | JB | * What does each of the entries in your table mean? * Does each entry have the same ratio as you started with? How do you know? * How many rows do you have in your table? What do you think this means? * How is the initial ratio related to the final ratio? * How is the number of JR in any row related to the number of JB in that row? * Could you graph the entries you have in your table? What do you think you would get? Why? |
| 5 | 13 | 55 | 143 |
| 10 | 26 | 60 | 156 |
| 15 | 39 | 65 | 169 |
| 20 | 52 | 70 | 182 |
| 25 | 65 | 75 | 195 |
| 30 | 78 | 80 | 208 |
| 35 | 91 | 85 | 221 |
| 40 | 104 | 90 | 234 |
| 45 | 117 | 95 | 247 |
| 50 | 130 | **100** | **260** |

**Sharing and Discussing the Task**

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| Solutions to Share | Questions to Ask About This Strategy | Connections Between Strategies | KEY POINTS |
| Scaling Up Using a Table (5 and 13 are added on each time) | * What does each of the entries in the table mean? * Does each entry have the same ratio as we started with? How do we know? * How many rows are in the table? What does this mean? * How is the initial ratio related to the final ratio? * How are the numbers of JR and JB in one row related to subsequent entries in the table? |  | The ratios within the table are related to each other by multiplication. Adding a constant ratio is equivalent to multiplying.  The unit rate relates the number of JR to JB in a row. |
| Solutions to Share | Questions to Ask About This Strategy | Connections Between Strategies | KEY POINTS |
| Scale Factor | * Why multiply the number of JR and JB by 20? * What does 20 mean in the context of this problem? * How much bigger is the new jar than the original jar? * How could we use this strategy to find the number of JB in any jar given the number of JR? *(take the number of JR in new jar and divide by the number of original JR then multiply by the number of JB in the original jar e.g., 100÷5 x 13)* | How is this strategy related to the table?  *(the scale factor of 20 relates the initial ratio to the final ratio-it is the number of rows in the table)* | The scale factor tells you how much bigger the new jar is. |
| Unit Rate | * What does 2.6 mean in the context of the problem? * How could we use this strategy to find the number of JB in any jar given the number of JR? *(JB = JR x 2.6)* * Does every value you put in for JR actually make sense? | * How is this strategy related to the table?   *(the number of JR x the unit rate gives you the number of JB)*   * How is this strategy related to the scale factor? *(the factor 100 is 5 times bigger here because you are starting with a ratio that is one fifth the size of the original ratio)* | The unit rate is the constant of proportionality. It is the slope of the line and m in y=mx. |
| Cross Multiplication | * What does 5x=13 x 100 mean? * Why did you divide 1300 by 5? What does that mean? * Why does cross-multiplication work? | * How is this strategy related to the scale factor? *(5x=13x100 which is what you get when you cross multiply, could be rewritten as x = 13x100 ÷ 5 if you divide 100 by 5 you get the scale factor which you then multiply by 100.)* | Cross multiplication is a generalizable strategy that is related to scale factor. |
| \*Graph |  | * Could you create a graph from the table of values? * What would the graph look like? * What would each point on the graph mean? * Where do you find the unit rate in the graph? * Should we connect the points or not? Why or why not? * How is the equation (JB=JR x 2.6) related to the graph? | Proportional relationships are linear and contain the point (0,0).  Y = mx is the general equation for a proportional relationship.  Lines denote continuous variables—those that can take on any value in a given range; but in some contexts, variables are discrete. |

\*Students will probably not produce a graph. So start with the questions in the connections cell. Might provide a minute for students to create the graph after asking the first two questions.

Sequence of Tasks

This set of tasks asks students to reason about quantities that are multiplicative in nature. Task 1 asks students to write ratios based on the picture and to generate equivalent ratios. In both cases (A and B) the ratio has an integer unit rate. While Task 2 is similar to Task 1, it is more challenging because the unit rate is not an integer. Hence strategies that were used on Task 1 might not be easily applied to Task 2. Tasks 3 and 4 can be thought of as missing value problems. So instead of generating any equivalent ratio, as in Tasks 1 and 2, in Task 3 students need to generate the equivalent ratio with particular specifications (i.e., the ratio that has 100 JR). In Task 4 students have to shift from considering the part-to-part ratio to the part-to-whole ratio. Task 5 provides students with an opportunity to apply their understanding of ratio to a new situation which can’t be solved directly by using the previously methods.

Task 1

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| A.    ph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:30a09f3f4dc41320210f6a7067005d73.jpgph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:30a09f3f4dc41320210f6a7067005d73.jpgph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:30a09f3f4dc41320210f6a7067005d73.jpg | B.  ph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:images.jpegph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:1042191248364028.jpgph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:1042191248364028.jpgph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:1042191248364028.jpgph5515-smith:Domain:Education.Pitt.Edu:Users:pegs:Desktop:1042191248364028.jpg |
| 1. Look at each picture and write the ratio it suggests. 2. Write as many ratios as you can that are equivalent to the original ratio. | |

**Task 2**

This candy jar contains Jolly Ranchers (the green rectangles) and Jawbreakers

(the red circles).

1. What is the ratio of Jolly Ranchers to Jawbreakers in the candy jar?

b. Write as many ratios as you can that are equivalent to the first ratio that you wrote down.

Tasks 3-5

The candy jar shown contains Jolly Ranchers (the green rectangles) and Jawbreakers (the red circles). Solve each of the following problems:

1. Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers as shown in the picture, but it contained 100 Jolly Ranchers. How many Jawbreakers would you have?
2. Suppose you had a candy jar with the same ratio of Jolly Ranchers to Jawbreakers as shown above, but it contained 720 candies. How many of each kind of candy would you have?
3. Suppose that you are making treats to hand out to trick-or-treaters on Halloween. Each treat is a small bag that contains 5 Jolly Ranchers and 13 Jawbreakers. You have 50 Jolly Ranchers and 125 Jawbreakers. How many small bags could you make up?