Counting Cubes Task
Teacher: Peter Dubno
District: New York City School District
Grade: 8

Teacher: --come up and present what they came up with. Arden, come on. Now remember, one person speaks because we can't hear many people speaking at once.

Student: Okay, everybody, like, on the things there were arms, right, which are the extended parts, right? So our pattern was that every time you add one...a cube to each arm, the total volume increases by 5 cubic centimeters, which I think was 5 cubes or whatever you called it. And then on our whatever...

Student: Equation.

Student: --equation, was that $5n + 1$ equals the volume and $n$ equals the length of one individual arm. So that, like, there's the middle cube, excluding the middle cube, you would multiply that by 5 because there are 5 arms and then add 1 for the middle cube and that will give you the volume and number of cubes.

Student: For the 3rd question, it was, what's the volume of the 5th stage? Well, for the 5th stage, there will be 4 cubes on each arm so the equation will be $5 \times 4$, + 1 equals the volume. So $5 \times 4$ is 20, + 1 is 21, so the volume will be 21.

Student: Yeah.

Teacher: Any questions for the presenters? Did anyone come up with a different solution? All right, Cassie come on. Do you want to show us? Cassie, Deirdre?

Student: Originally, we got what Arden got, but we tried it out and it didn't always work.

Student: With the 1st one.

Student: With the first one, so what happened is we came up with $5n - 4$, so it's 5 arms and then $n$ would be the building number.

Student: Yeah, and then you subtract it.
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29  Student:  And then you subtract 4 and then you get the number of—
30  Student:  Cubes in the building.
31  Student:  --cubes in the building. So we wrote up an example and \( n = 2 \), so then
32  \[ 5 \times 2 - 4 = 10 - 4 \], and we got 6 cubes in the building.
33  Student:  And that's the number of the cubes in the building.
34  Student:  And it's on the sheet also.
35  Teacher:  So let me ask you. Of that formula, if I asked you how many cubes would
36  be in the 7\(^{th}\) building—
37  Student:  It would be 31.
38  Teacher:  And how would you get that?
39  Student:  You'd replace \( n \) with the 7 and then you do the work, you’d multiply 7
40  times 5 and then you get 35, and the you minus 4 and you get 31.
41  Teacher:  Okay. How is yours different or the same as what Arden did and Yoshio
42  did?
43  Student:  Both of us used 5\(n\), 5 times the building number for each arm but—
44  Student:  The only thing that was different was that we subtracted and he added.
45  Teacher:  Does that make it different or is it the same or what?
46  Student:  No...No, we did the middle square for each arm and then you subtracted
47  4 middle squares. But what we did is we just added the actual length of
48  the arm excluding the middle square.
49  Student:  But the reason why it didn't work for us was because we were trying to
50  figure out an equation that would fit all buildings instead of just each
51  every 1 except for the 1\(^{st}\) one, because it doesn’t really work for the 1\(^{st}\)
52  one.
53  Student:  Yea. Because there's only 1 cube.
54  Student:  Yeah, there's no—
55  Student:  \( 5 \times 0 = 0, + 1 = 1 \).
56  Student:  It’s the 1\(^{st}\) building, though.
Student: I have a question. What's 5 x 1? Ok, I think... Wait 1 second. I think what Arden is trying to... He defined \( n \) as the length of one arm, so for the 1\textsuperscript{st} building it would be 0, not 1. And you were trying to do the building number. So that's what was different about it. That's why you're having this quarrel.

Student: We did that while we were defining it as just the 1\textsuperscript{st} building number was the cube, so we were doing the building. Okay.

Teacher: What about this one? How does this one fit in with that? Is that... there a mathematical equivalence there somehow? Yoshio, do you think you can show us that or explain it or how is it different, how is it the same?

Student: Our definition of \( n \) is different from theirs. Ours is \( n \) equals the length of each arm. So the equation will change, will be different from the two.

Teacher: So what you're telling me is the definition of the variable is a very important idea in mathematics?

Students: Yea.

Teacher: Okay, it makes the whole difference of what the expression is?

Student: Yeah, because in there, they're multiplying the—they're considering each arm what we were considering plus the middle and then they were going to subtract 4 middles because they would have 4 extra middles. And that... But what we were doing is we were just multiplying each of the arms without the middle and then adding one middle. It's really just the same thing. It just depends on how you think of it.