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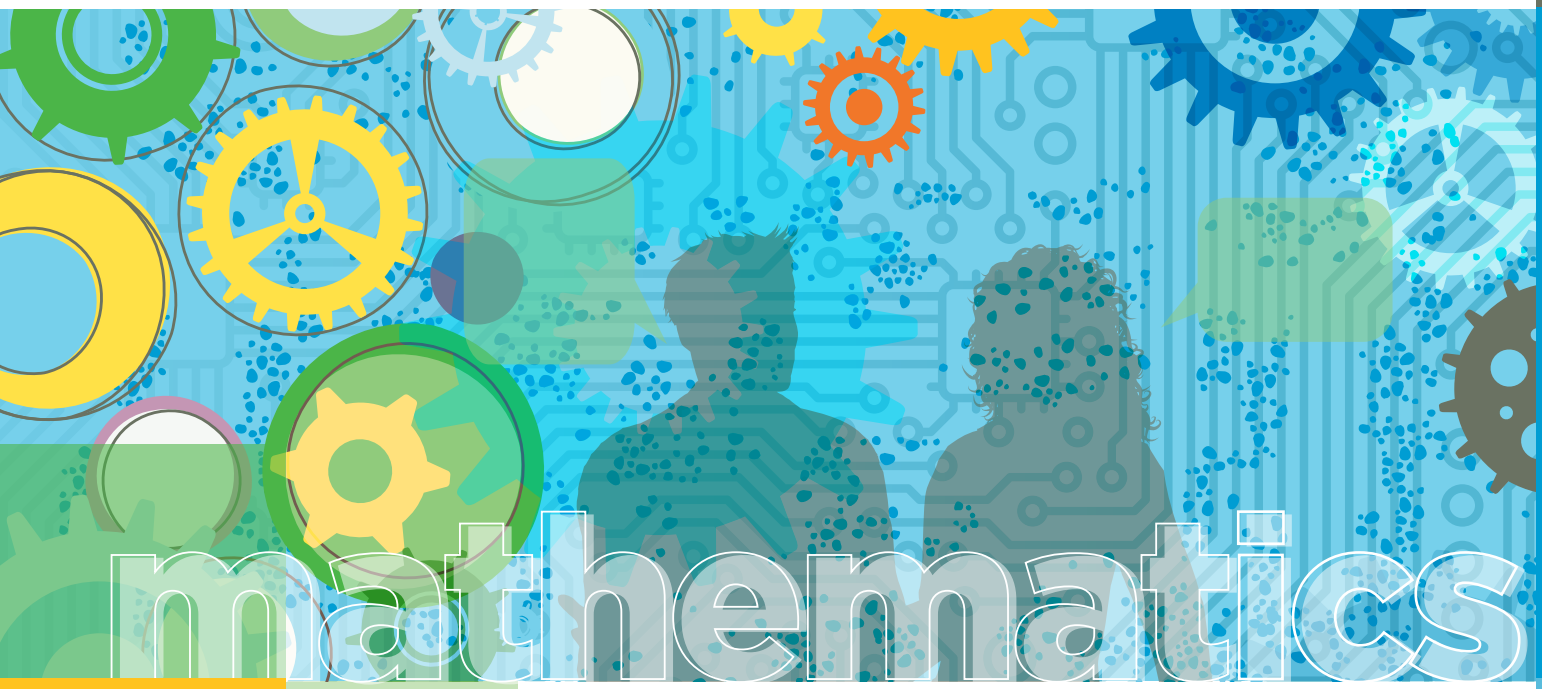

mathematics

Grade 3

Half of the Whole Task

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This is a single lesson. Full sets of related lessons, including accompanying student materials, that are developed by the Institute for Learning are available at:

ShopIFL.com

Please note: The lesson is not meant to be a script to follow, but rather a set of questions that target specific mathematical ideas which teachers can discuss together in professional learning communities.

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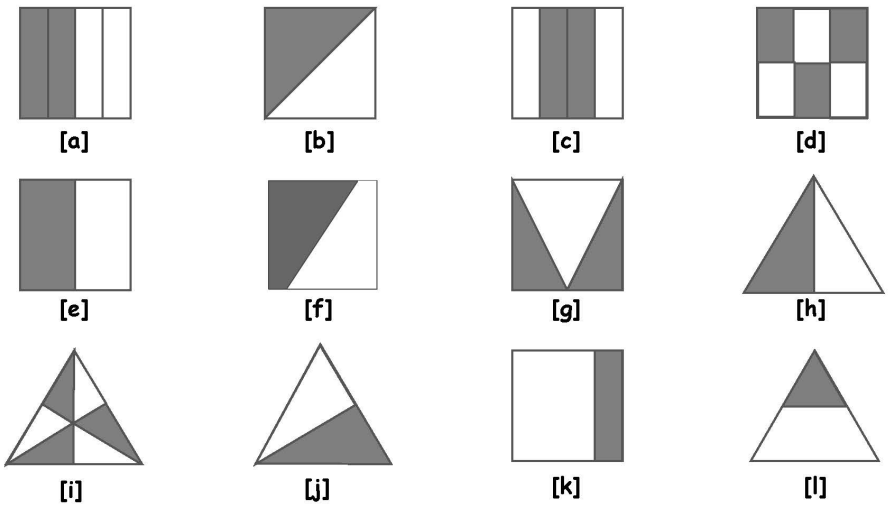
Half of the Whole Task

TASK

Name_____Date_____

Half of the Whole[1]

Identify all of the figures that have one half shaded. Be prepared to explain how you know that one half of the figure is shaded. Write a written description giving your reason why a figure is showing halves. If a figure does not show one half shaded explain why the figure is not showing halves.



1 Adapted from Watanabe, T. (1996, April). Ben's understanding of one-half. *Teaching Children Mathematics* (2): 460-464.

TASK

LESSON GUIDE

Half of the Whole

Rationale for Lesson: This lesson provides students with an opportunity to identify many different ways of representing halves. Students will identify $\frac{1}{2}$ as equal to $\frac{2}{4}$ and to $\frac{3}{6}$. The task also provides students with an opportunity to discuss discrete and continuous figures in order to determine that a figure represents half as long as $\frac{1}{2}$ of the area is shaded and $\frac{1}{2}$ is unshaded regardless of the location of the shaded area.

Task: Half of the Whole

Identify all of the figures that have one half shaded. Be prepared to explain how you know that one half of the figure is shaded. Write a written description giving your reason why a figure is showing halves. If a figure does not show one half shaded explain why the figure is not showing halves.

See student paper for complete task.

Common Core State Standards for Mathematical Practice[2]	MP1 Make sense of problems and persevere in solving them. MP3 Construct viable arguments and critique the reasoning of others. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.	
Common Core State Standards for Mathematical Content[2]	3.NF.A.1	Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
	3.NF.A.3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
	3.NF.A.3.A	Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
	3.NF.A.3.B	Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

2 National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO). (2014). Mathematics. *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math

LESSON GUIDE

Essential Understandings	• A fraction describes the division of a whole or unit (area/region, set, linear/measurement) into equal parts. • A fraction is relative to the size of the whole or unit. • If the numerator is half the quantity in the denominator then the fraction is equal to a half. • Rational numbers (fractions) have an infinite number of equivalent forms, and the forms are equivalent if the same portion of the set or area of the figure is represented or they represent the same point on the number line, regardless of if the pieces of the whole are adjacent or not.
Materials Needed	• Half of the Whole task sheet • Enlarged print-out of the figures • Scissors, tape, and glue stick

LESSON GUIDE

SET-UP PHASE

Look at the task. It has a lot of figures on the page. Circle all of the figures that show half of the figure shaded.

Can someone say back what you are supposed to do?

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Can't get started.	What are you trying to figure out? Do you see a figure that shows one-half shaded? How do you know that it shows half?	Can you figure a way to show that they are equal pieces? You can use scissors.
Circles only halves that are continuous.	If this is a cake, tell me what you know about the shaded and unshaded pieces.	Can you use the scissors to show how the pieces are equal?
Circles all of the correct halves.	Tell me what you know about halves.	Write an explanation for how Figures A, C, and E all show halves. How do the figures differ? How are they the same?

SHARE, DISCUSS, AND ANALYZE PHASE

LESSON GUIDE

EU: A fraction describes the division of a whole or unit (area/region, set, linear/ measurement) into equal parts.

- Which figures show halves? How do you know they show halves? Who agrees? Who disagrees?
- Is there another way of knowing that half is shaded? Can you show us how you know there is a fair share? *(I cut A in half and put the pieces on top of each other and they are the same size.)*
- Tell me about Figure C. How many people think it shows halves? How many people do NOT think it shows halves? Use your scissors to show me that you have halves.
- If halves are cut apart and put on top of each other and they are the same size or the same amount of area then there are two equal pieces or halves. **(Marking)**
- Someone looked at the set of 4 pieces and said, "I know that 2 is half of 4." What might this person be thinking? How did this person know that 2 was half of four by just looking at the total number of pieces? *(There are four pieces and 2 is half of four because there are 2 shaded and two unshaded pieces.)*
- Which of these tell about half of the figure? $\frac{3}{6}$? $\frac{4}{8}$? $\frac{5}{6}$? What if I have tenths, how many equal pieces are needed to show half? *(*$\frac{5}{10}$ *and* $\frac{5}{10}$ *)*
- Half of the total number of pieces is like doubles. So 3 shaded and 3 unshaded make 6 pieces altogether. 4 shaded and 4 unshaded makes 8 total pieces. **(Marking)**
- Can you name other names for half? Write down two names for $\frac{1}{2}$.

EU: Rational numbers (fractions) have an infinite number of equivalent forms, and the forms are equivalent if the same portion of the set or area of the figure is represented or they represent the same point on the number line, regardless of if the pieces of the whole are adjacent or not.

- Look at Figures A and C. Someone said A shows halves but they claimed that figure C does not show halves. What do you think? *(They are the same because each one shows four pieces and two are shaded and two are not shaded.)*
- What fraction can we write to describe Figure A? *(*$\frac{2}{4}$ *and* $\frac{1}{2}$ *)* Tell me about the meaning of the denominator in $\frac{2}{4}$. Tell me about the meaning of the numerator in $\frac{2}{4}$.
- How do Figures A and C compare with Figure E? *(All three show one-half shaded. A and C both have* $\frac{2}{4}$*, but Figure E shows* $\frac{1}{2}$*.)*
- Why can we write $\frac{1}{2}$ for Figures A, C, and E? *(2 is half of 4, so we can write* $\frac{1}{2}$*. All three figures have the same amount shaded.)*
- Both $\frac{1}{2}$ and $\frac{2}{4}$ describe one-half of a figure. These three figures have the same area shaded. **(Revoicing and Marking)**

LESSON GUIDE

EU: If the numerator is half the quantity in the denominator then the fraction is equal to a half.

- Talk about Figure D; I see more than two equal pieces. What fraction is shaded in Figure D? Turn and talk. *(Three-sixths shaded, record $\frac{3}{6}$.)*
- What does the three, or the numerator tell us?
- What does the denominator tell us in $\frac{3}{6}$?
- How do I know just by looking at the fraction that this is half? *(Three is half of all of the pieces, so $\frac{3}{6}$ is half of the total number of pieces.)* Is $\frac{100}{200}$ half? Is $\frac{9}{10}$ half?
- Give me another fraction that describes half of a figure? (Discuss names of fractions that show half. If students cannot name fractions for one-half, then name some for students and ask them to justify why the fraction describes half.)
- If you know the total number of pieces and the numerator tells you half of the total number of pieces, then you know the fraction is another name for one half. **(Marking)**
- Talk about Figure H, does it show halves? How do you know?
- How can both Figure D and Figure H show halves? They don't look like the same size.

EU: A fraction is relative to the size of the whole or unit.

- Both Figure J and Figure B show halves. How can both of these figures show half of the figure shaded? *(This is half of the square and this one is half of the triangle.)*
- How can both figures represent half of the figure shaded? The halves don't look the same. *(The halves are different. Half of Figure B is more than half of Figure J because the whole is bigger for Figure B.)*
- Figure A shows halves. We talked about this one. Does Figure H show halves? How do you know?
- How can both of the figures show halves? They don't look like the same size.
- Not all halves are equal. The size of the half depends on the whole. If you have a large whole and a small whole then half of a larger whole will be greater than half of a smaller whole. **(Marking)**

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