INSTITUTE for LEARNING

Grade 8

Calling Plans (Part 2)

UNIVERSITY OF PITTSBURGH

INSTITUTE for LEARNING

UNIVERSITY OF PITTSBURGH

This is a single lesson. Full sets of related lessons, including accompanying student materials, that are developed by the Institute for Learning are available at:

ShopIFL.com

Please note: The lesson is not meant to be a script to follow, but rather a set of questions that target specific mathematical ideas which teachers can discuss together in professional learning communities.

Table of Contents

TASK

Name____Date__________________

Calling Plans

Long-distance Company A charges a base rate of $5 per month, plus 4 cents per minute that you are on the phone. Long-distance Company B charges a base rate of only $2 per month, but they charge 10 cents per minute used.

Part 1

How much time per month would you have to talk on the phone before subscribing to Company A would save you money?[1]

1 Adapted from Achieve. (2002). Sample problems: Algebra. *Foundations for success: Mathematics expectations for the middle grades* (p. 149). Washington, DC: Author. Retrieved from http://www.achieve.org/files/FoundationSuccess.pdf

Part 2

Create a phone plan, Company C, that costs the same as Companies A and B at 50 minutes but has a lower monthly fee than either of the plans.

TASK

LESSON GUIDE

Calling Plans (Part 2)

Rationale for Lesson: In Part 2 of the Calling Plans Task, students create a phone plan that has a lower monthly fee than two other plans but costs the same as the other plans for a specified number of minutes. The task begins with two phone plans that have already been explored in an earlier task, *Calling Plans, Part 1*. As they explore possible phone plans, students consider the effects of the rate of change and base rate–both individually and in combination–on the behavior of plans that all cost the same amount for the specified number of minutes. In doing so, they deepen their knowledge of the impact that both slope and *y*-intercept have on the equation and graph of a line. They will also deepen their understanding of what it means to be the solution to an equation, as well as the solution to a system of equations.

Task: Calling Plans

Long-distance Company A charges a base rate of $5 per month, plus 4 cents per minute that you are on the phone. Long-distance Company B charges a base rate of only $2 per month, but they charge 10 cents per minute used.

Part 1

How much time per month would you have to talk on the phone before subscribing to Company A would save you money?[1]

Part 2

Create a phone plan, Company C, that costs the same as Companies A and B at 50 minutes but has a lower monthly fee than either of the plans.

Common Core State Standards for Mathematical Practice[2]	MP1 Make sense of problems and persevere in solving them. MP2 Reason abstractly and quantitatively. MP3 Construct viable arguments and critique the reasoning of others. MP4 Model with mathematics. MP5 Use appropriate tools strategically. MP6 Attend to precision. MP7 Look for and make use of structure. MP8 Look for and express regularity in repeated reasoning.	
Common Core State Standards for Mathematical Content[2]	8.F.B.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

2 National Governors Association Center for Best Practices (NGA Center) and Council of Chief State School Officers (CCSSO). (2014). Mathematics. *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math

LESSON GUIDE

	8.F.B.5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
	8.EE.C.8	Analyze and solve pairs of simultaneous linear equations. A. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. B. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.* C. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
Essential Understandings	• A linear function can be represented by an equation of the form $y = mx + b$ where m and b have a regular and predictable meaning in the context, table, graph, and equation. • Slope and intercept can be interpreted within the context of a real-world problem involving systems of equations. • The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs (x, y) that make both equations true statements or satisfy the equations simultaneously. • The solution to a system of two linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because the intersection point(s) make(s) both of the equations true statements or satisfies both of the equations in the system simultaneously.	
Materials Needed	• Student reproducible task sheet • Calculators, rulers, graph paper • Chart paper and markers	

LESSON GUIDE

SET-UP PHASE

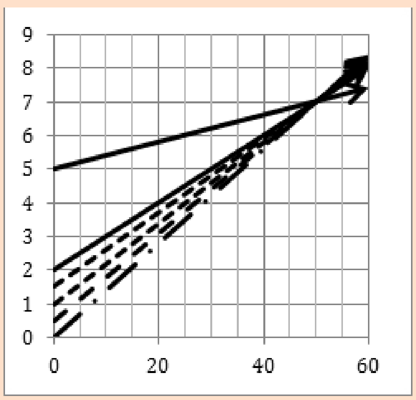
- Remind students of the issues they considered important in Calling Plans 1. Ask: *What did we say was important to keep in mind when you use your cell phone?* Get students to bring out ideas such as monthly cost and cost per minute. Attend to the ways that students describe these ideas in their own words so you can refer back to their words after they have read the task.
- Ask students to recall their work on *Calling Plans, Part 1*. Have student solutions from that task posted around the room and direct students' attention to them.
- Have students read the task individually, and then select a student to read it aloud as others follow along. Ask students to circle words that they do not understand, and make note of these words as you circulate around the room. Ask students who are familiar with the words to explain them in their own words. Ask others to add to these explanations. Provide further clarification if needed. Make sure to emphasize and repeat the words in context throughout the lesson.
- Then have a few students explain to the class what they know about companies A and B. They should state that Company A has a monthly fee of $5 and a per-minute charge of 4 cents, while Company B has a monthly charge of $2 and per minute charge of 10 cents. Ask students what the task is asking them to do and what they are trying to find when solving the problem. As students describe the task, make certain they indicate that the goal is to create a plan for Company C that will cost the same as Companies A and B at 50 minutes but has a lower monthly fee than Companies A and B. Also, stress that students will be expected to explain how and why they solved the problem a particular way and to explain their process in terms of the context of the problem.

EXPLORE PHASE

Possible Student Pathways	Assessing Questions	Advancing Questions
Can't get started.	What are you trying to figure out? What did we discover yesterday about when companies A and B have the same cost?	What are possible values for Company C's monthly fee? Explain how you know. How can you use one of those possible values to find an equation for Company C? What representations can you use to help you make decisions?
Guess-and-Check – starts with a given monthly fee OR starts with a given cost per minute but is unsuccessful in determining an equation for Company C.	What about the problem context suggests that you can choose $1.50 (or $1, 15 cents, 9 cents, etc.)? What do you mean by "it didn't work"? What needed to work?	If $1.50 is the monthly fee, where will that be on the graph? What will the line have to look like in order to satisfy the conditions of the problem? What else do you need to know in order to write the equation?

<table>
<tr><th>Possible Student Pathways</th><th>Assessing Questions</th><th>Advancing Questions</th></tr>
<tr><td>Guess-and-check – starts with a given monthly fee OR starts with a given cost per minute and finds an equation for Company C. Possible equations where y = total cost and x = number of minutes of talk:
• $y = .11x + 1.50$
• $y = .12x + 1$
• $y = .13x + .50$
• $y = .14x$ or $y = .14x + 0$</td><td>Why did you decide on $1.50 ($1, 15 cents, 14 cents, etc.)?

How did you find your equation?</td><td>If this is the equation of the Company C’s plan, describe the plan in words.

Are there any other plans that would also work? What representation can help you find those other plans?</td></tr>
<tr><td>Graphs or draws a sketch.

The solid lines are Company A and Company B. Any of the dotted lines represent possible plans for Company C.</td><td>How did you draw your graph?

What information about Company C does your graph tell you?

How does the graph convince you that your plan for Company C will work?</td><td>How can you use your graph to describe a plan for Company C?

What specific information about Company C’s plan do you need to get from the graph to create an equation describing the plan?</td></tr>
<tr><td>Finishes early.</td><td>How did you determine the equations you wrote for Company C?

How will each of you describe your solution strategies to the rest of the class, if called upon to do so?</td><td>How can you decide if you have all possible plans for Company C?

What patterns do you notice in the equations? Why are those patterns happening?</td></tr>
</table>

LESSON GUIDE

SHARE, DISCUSS, AND ANALYZE PHASE

EU: A linear function can be represented by an equation of the form $y = mx + b$ where m and b have a regular and predictable meaning in the context, table, graph, and equation.
EU: Slope and intercept can be interpreted within the context of a real-world problem involving systems of equations.

- Can someone remind me again what we were trying to figure out in this problem? *(We needed to find a phone plan that had a smaller monthly fee but would cost the same as Company A & B when 50 minutes were used.)*
- I noticed that Group E created an equation for the new plan. Can you come to the front and explain your plan for Company C? *(Company A had $5 monthly fee and Company B had a $2 monthly fee, so we picked $1 because it was smaller. From our work yesterday, we knew that the cost at 50 minutes was $7, so we tried to find the cost per minute to get to $7 at 50 minutes if the monthly fee was $1. When we divided $6 by 50 minutes we got $.12. So we got $y = 0.12x + 1$.)*
- What type of a function is modeled by this equation? *(A linear function.)* How do you know? *(It's in the form $y = mx + b$.) (It has a constant rate of change of .12.)* What do the m and b represent in an equation like this one that models a linear function? *(m is the slope and b is the y-intercept.)*
- Will someone who understands what Group E did explain why they divided 6 by 50? *(They chose a monthly fee of $1, so the cost of the 50 minutes would have to be $7 minus $1 which is $6. Then, they divided the $6 by 50 minutes to find that their Plan C would be 12 cents per minute.)* Someone else explain what the 6 and 50 mean in the problem context.
- Who can add on and explain to the class how Group E then decided on an equation to represent a plan for Company C? Also explain what x and y represent.
- What is Group E's phone plan for Company C in words? *(If you used Company C, you would pay $1 every month and 12 cents for every minute that you talked on your phone.)* How do we know that Group E's plan fits the criteria for our problem? *(The monthly fee of $1 is less than Company A and B, but the cost at 50 minutes is the same for all three companies.)*
- Group D had a different equation. Can you explain your equation to the class? *(We saw that Company B charged 10 cents for every minute on the phone, so we tried 11 cents. If you pay 11 cents per minute for 50 minutes, the cost will be $5.50. So we knew the monthly fee would be $1.50. Our equation is $y = .11x + 1.50$.)* What type of a function is modeled by this equation? How do you know?
- Who agrees with Group D and can restate their strategy in your own words?
- What information did both Groups D and E take into account as they tried to find an equation? *(They needed to make sure the plan cost $7 at 50 minutes.) (They needed to choose or find a monthly fee less than $2.) (They needed to choose or find a cost per minute.)*
- What is Group D's phone plan for Company C in words? *(If you used Company C, you would pay $1.50 every month and 11 cents for every minute that you talked on your phone.)*
- Group A, you started with a cost per minute and had problems finding a plan. Please explain what problems you ran into and how you resolved those problems. Others in this room ran into the same problems. *(We tried 9 cents per minute and could not make it work. If you pay 9 cents per minute for 50 minutes, the cost will be $4.50. That makes the monthly fee $2.50 and that's more than Company B.)*

- What information did your choice of 9 cents give you, and how did you use that information to resolve the problem? *(We tried 8 cents and that was worse, so we decided we needed to charge more than 9 cents. We tried 10 cents, but that was Company B, so then we went to 11 cents, like Group E.)*
- I saw several groups use a similar guess-and-check strategy, and learn from the so-called "wrong" choices they tried. It's important to realize that we can gain insight into how to solve problems by noting the information the mistakes we make give us. **(Marking)** Can someone explain how Group C used guess-and-check to get to the correct answer?
- **So, I'm hearing that a linear function can be represented by an equation in the form *y* = m*x* + b where m is the slope, or the constant rate of change, and b is the *y*-intercept. I'm also hearing that we can use what we know about the context of a problem to construct an equation for a linear function by determining what the possible values are for m and b. (Recapping the EU)** Can someone summarize the methods we've seen so far? Let's move on and look at some other methods I saw you use today.

EU: The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs (*x*, *y*) that make both equations true statements or satisfy the equations simultaneously.

EU: The solution to a system of two linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because the intersection point(s) make(s) both of the equations true statements or satisfies both of the equations in the system simultaneously.

- Group C please share your solution strategy. *[We sketched a graph for Company A and B, then looked for a line that went through (50, 7). We found many lines.]*
- Say more. Please explain to us how you decided on lines that meet the criteria stated in the problem. *[The starting point had to be below 2 and the line needed to pass through (50, 7).]*
- Who can name the mathematical term to describe the starting point on their graph? *(The y-intercept.)* Where did we see this earlier in the equations?
- (Student name), please explain why the line must start below 2 and include (50, 7). *[It must start below 2 because Company C must have a lower monthly fee than the other companies and the lowest monthly fee you can see on the graph is 2. Then (50, 7) is the point that makes the cost $7 for 50 minutes of phone usage for all three Companies.]*
- Let's just take a minute to explore the strategy used by Group C and several other groups. First, why graph the lines that work for Company A and Company B? What do we know about those lines? *(The line for Company A has all the solutions to the equation for Company A. Same for Company B. So we can try to draw another line on the graph for Company C.)*
- Can someone state that another way, and also state what the actual equations are that we are talking about? *(Company A's plan is $C = .04t + 5$ and Company B's Plan is $C = .10t + 2$. C is the total cost, and t is the talking time in minutes. Every point on this line is a solution to $C = .04t + 5$ and every point on the other is a solution to $C = .10t + 2$.)*
- If a point provides a solution to an equation, we can also say it satisfies the equation. Turn and talk. What does it mean to satisfy an equation? *(If "satisfy" means it is a solution, then it means the point makes the equation true.)*
- When we graph several equations on the same axes, like Group C and others did, remember that we call this a system of equations. What can we say about (50, 7)? *(They meet there.) (It's the point of intersection for all three plans.)*

- Can someone say more about the point of intersection, using the term "satisfy?" *(The point of intersection satisfies both equations.) (In fact, it satisfies the third equation, too.)*
- Why, then, did Group C draw several lines that pass through (50, 7)? Why was that a good strategy for solving this problem? **(Challenging)** *[The point (50, 7) must satisfy the equation for plan C. It has to be true for Company C, too. So any line that "works" must pass through it.]*
- How, then, can we use the graph of Company A and Company B to think about possible plans for Company C? *[We can draw a line that goes through (50, 7) and starts lower than 2. Maybe we choose a y-intercept like Group C did, a y-intercept of 1 or $1, and figure out the slope.]*
- Who can tell us a way to figure out the slope? *[We can use the points (0, 1) and (50, 7) to find the ratio of the (7 – 1) over (50 – 0).] [We could draw a slope triangle from (0,1) to (0, 7) and then over to (50, 7) and find the height up and the distance over.]*
- When we set up this ratio, what ratio do we end up with? *(6/50)* Where have we seen 6/50 before? Who can explain what it means in the context of this problem? *(That's exactly what group E did! The 6 is the number of dollars left over if the monthly fee is $1, and the 50 is the number of minutes that must cost $6.)*
- Let's go back to an earlier question, and use the graph to think about the answer. Are there other possible plans that also meet the criteria for Company C? How can we decide? *($y = .11x + 1.50$, $y = .12x + 1$, $y = .13x + .50$, $y = .14x$ OR $y = .14x + 0$)*
- Can someone else say what we know must be true graphically for any of the lines describing Company C with respect to the lines describing the plans for Companies A and B? Why?
- Are there any other points that will satisfy all three equations? *[No, lines can't bend, and these lines all meet at (50, 7), so they can't meet again; they can only meet once.]*
- No matter what plan we present for Company C, it must include the point (50, 7) so that all the plans will cost $7 for 50 minutes of phone use. We can see that graphically as a line that passes through the point (50, 7), and, in this case has a y-intercept less than 2. We can also see that algebraically because (50, 7) will make the equations for each of the three plans a true statement. Or, as we said earlier, (50, 7) will satisfy each of the equations. **(Recapping)** Can someone explain why (50, 7) must make all three of the equation true in this problem?
- Look back at the equations that we've said are possible for modeling a Plan C. Is there a pattern? Turn and talk. *(Every time you increase the rate per minute by 1 cent, the monthly fee decreases by 50 cents.)* How do we all feel about what she just said? What is causing this pattern to occur?

Application	The point (1, 5) is on the line representing $y = 2x + 3$. Find equations for at least 3 other linear functions that intersect $y = 2x + 3$ at (1, 5).
Summary	What can you say about the point of intersection of two lines that each represent a unique linear function?
Quick Write	Explain how you can use the slope and y-intercept of the line represented by the equation $y = 2x + 3$ to find other equations of linear functions that also pass through the point (1, 5).

Institute for Learning
Learning Research and Development Center

University of Pittsburgh
3939 O'Hara Street
LRDC Suite 310
Pittsburgh, PA 15260
T: 412-624-8319

www.instituteforlearning.org