

Sudoku Variations: Supporting Understanding across the Mathematics Curriculum

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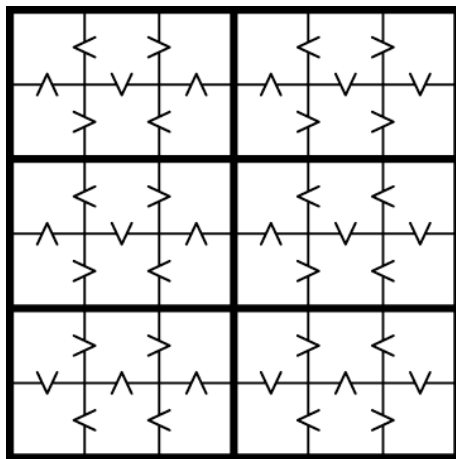
Miami University
Oxford, OH

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"GREATER THAN" SUDOKU

As with Sudoku, the numbers in the Greater Than Sudoku puzzle solutions appear once in each row, each column, and each outlined region. In addition, there are "greater than" symbols placed between adjacent squares in the outlined regions, indicating which number is greater than the other.

For example, in the solution to the 6 x 6 puzzle below, you can see that each number from 1 to 6 appears once in each row, column, and outlined region. You can also see that between any two adjacent squares in an outlined region, the "greater than" symbol points to the smaller of the two numbers.



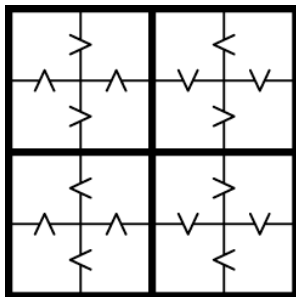
Greater Than Sudoku Example



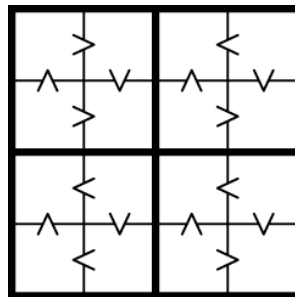
4 < 5 > 2	1 < 6 > 3
6 > 1 < 3	5 > 4 > 2
1 < 6 > 4	2 < 3 < 5
3 > 2 < 5	6 > 1 < 4
5 > 3 > 1	4 > 2 < 6
2 < 4 < 6	3 < 5 > 1

Greater Than Sudoku Example Solution

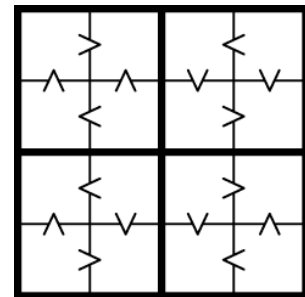
Use what you know about ordering of numbers and problem solving strategies to place the digits 1-4 in the following puzzles.



Greater Than Sudoku Puzzle 1



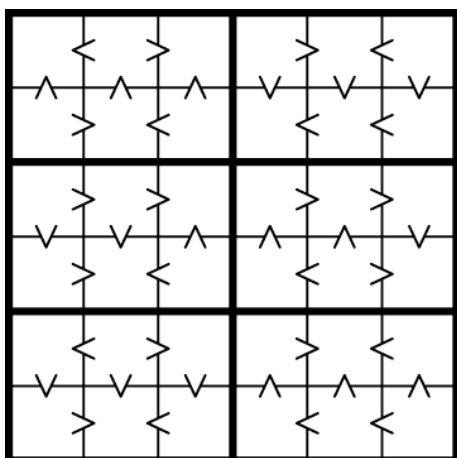
Greater Than Sudoku Puzzle 2



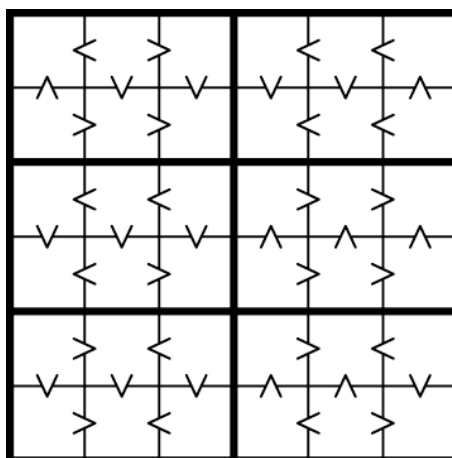
Greater Than Sudoku Puzzle 3

What strategies are the most helpful in solving these Greater Than Sudoku puzzles?

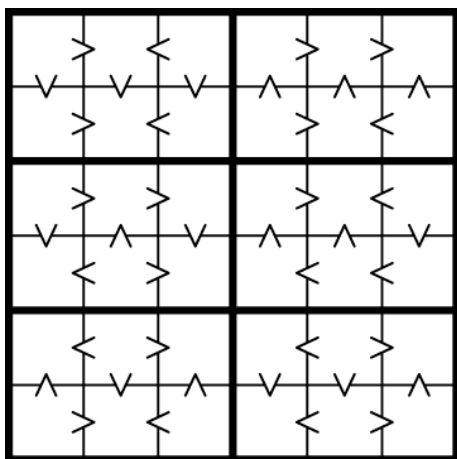
Solve the following Greater Than Sudoku puzzles



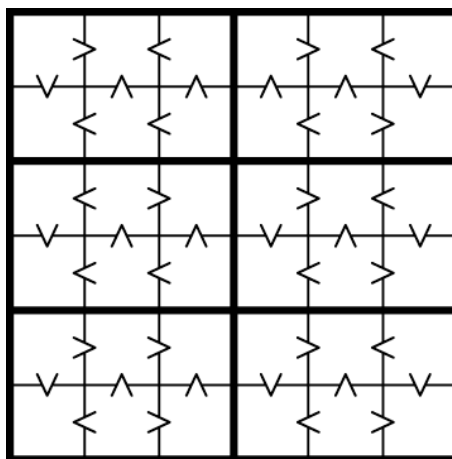
**Greater Than Sudoku
Puzzle 4 – Easy**



**Greater Than Sudoku
Puzzle 5 – Medium**

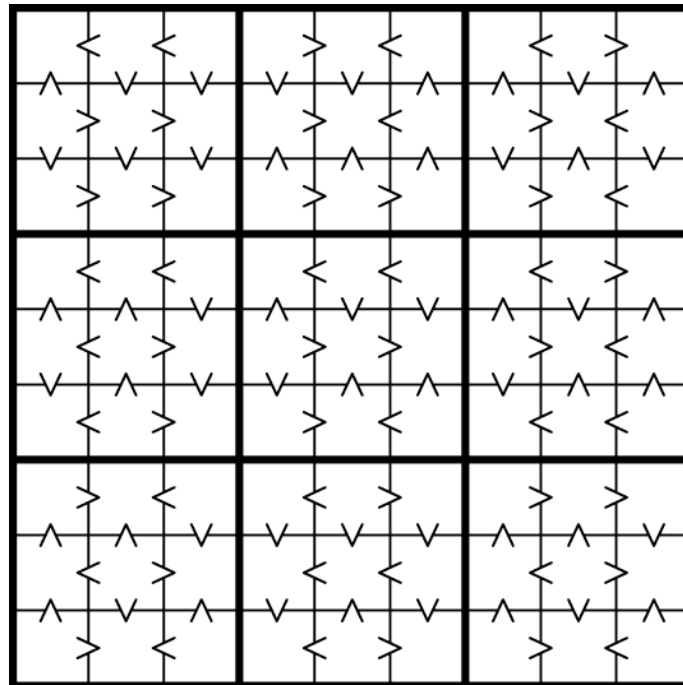


**Greater Than Sudoku
Puzzle 6 – Medium**



**Greater Than Sudoku
Puzzle 7 – Difficult**

Solve the following Greater Than Sudoku puzzle



**Greater Than Sudoku
Puzzle 8 – Difficult**
(from www.killersudokuonline.com)

Some Greater Than Sudoku Resources

- Killer Sudoku Online (<http://www.killersudokuonline.com>) carries a daily 9 x 9 Greater Than Sudoku showing all of the greater than symbols, a weekly 9 x 9 Greater Than Sudoku with some of the symbols removed, and a weekly 9 x 9 Greater Than Killer Sudoku that combines these two types. These can be printed out or solved online. Many of the previous puzzles are also archived on their website.
- Several 9 x 9 Greater Than Sudoku puzzles are available at Yoogi Games (<http://syndicate.yoogi.com/greatest-sudoku>). They also sell books of Sudoku puzzles including one of "Comparison Sudoku" (their name for Greater Than Sudoku).
- The blog "A Day in the Life" contains an entry where the writer describes his step-by-step procedure for solving a specific Greater Than Sudoku puzzle. This is available at <http://stuckinthecube.blogspot.com/2007/05/solving-greater-than-sudoku-puzzles.html>.
- A free mobile phone application of Greater Than Sudoku puzzles is available at <http://www.mobilerated.com/greater-than-sudoku-1701.html>.

Note: All of the above resources focus only on 9 x 9 Greater Than Sudoku puzzles. The 4 x 4 and 6 x 6 variations given here were created for this presentation. These were not difficult to design, but they required some verification for uniqueness and solvability.

GEOMETRY SUDOKU

As with Sudoku, the numbers in the Geometry Sudoku puzzle solutions appear once in each row and each column. There are no outlined regions. Instead, clues are given describing the shape that would be created if the uncircled numbers of that type were connected (consider connecting the centers of the squares in which these numbers are placed). The shapes that are given are the most specific shape name for those numbers. For example—a shape described as a parallelogram will not be a rectangle, square, or rhombus. If it were, then the more specific name would be used.

For example, in the 5 x 5 puzzle below, six circled numbers have been placed in the starting grid at the left. The remaining numbers must be placed so that they form the vertices of the shapes indicated. In the solution at the right, the numbers have been placed so that each number appears in each row and column, and so that the uncircled numbers of each type form the shapes that are indicated (see the three shape grids below).

				④
		①		
			④	
③		②		⑤

- 1 – Quadrilateral
- 2 – Parallelogram
- 3 – Rectangle
- 4 – Isosceles right triangle
- 5 – Rectangle

Geometry Sudoku Example

2	1	5	3	④
4	2	①	5	3
5	3	4	2	1
1	5	3	④	2
③	4	②	1	⑤

Geometry Sudoku Example Solution

2	1	5	3	④
4	2	①	5	3
5	3	4	2	1
1	5	3	④	2
③	4	②	1	⑤

- 2 – Parallelogram**
- 4 – Isosceles right triangle**

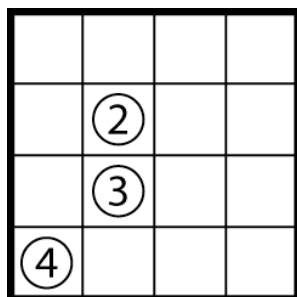
2	1	5	3	④
4	2	①	5	3
5	3	4	2	1
1	5	3	④	2
③	4	②	1	⑤

- 3 – Rectangle**
- 5 – Rectangle**

2	1	5	3	④
4	2	①	5	3
5	3	4	2	1
1	5	3	④	2
③	4	②	1	⑤

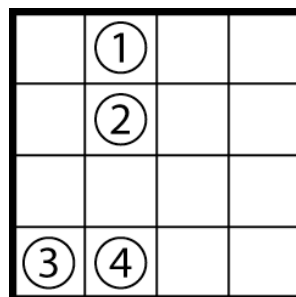
- 1 – Quadrilateral**

Solve the following Geometry Sudoku puzzles



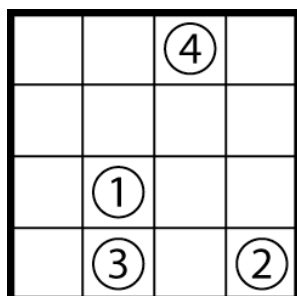
- 1 – Quadrilateral
- 2 – Isosceles triangle
- 3 – Isosceles triangle
- 4 – Isosceles triangle

Geometry Sudoku Puzzle 1



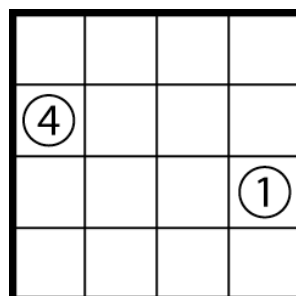
- 1 – Isosceles right triangle
- 2 – Isosceles triangle
- 3 – Isosceles triangle
- 4 – Isosceles right triangle

Geometry Sudoku Puzzle 2



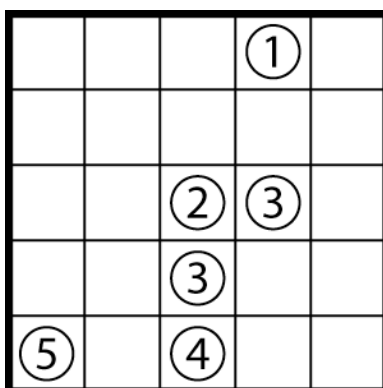
- 1 – Isosceles triangle
- 2 – Isosceles triangle
- 3 – Right triangle
- 4 – Isosceles right triangle

Geometry Sudoku Puzzle 3



- 1 – Isosceles right triangle
- 2 – Rhombus
- 3 – Square
- 4 – Isosceles right triangle

Geometry Sudoku Puzzle 4



- 1 – Quadrilateral
- 2 – Rectangle
- 3 – Isosceles triangle
- 4 – Parallelogram
- 5 – Rectangle

Geometry Sudoku Puzzle 5

Solve the following Geometry Sudoku puzzles

		③		⑤
⑤		④		
④		②		①

- 1 – Square
- 2 – Quadrilateral
- 3 – Parallelogram
- 4 – Isosceles triangle
- 5 – Isosceles triangle

Geometry Sudoku Puzzle 6

③				
	②			③
	④			①
	⑤			
				④

- 1 – Parallelogram
- 2 – Isosceles trapezoid
- 3 – Isosceles triangle
- 4 – Isosceles triangle
- 5 – Kite

Geometry Sudoku Puzzle 7

⑤				
③		①		②
		④		
④			②	

- 1 – Quadrilateral
- 2 – Right triangle
- 3 – Quadrilateral
- 4 – Isosceles right triangle
- 5 – Square

Geometry Sudoku Puzzle 8

Solve the following Geometry Sudoku puzzles

	②	⑤			⑥
⑥				③	
④				①	
	①		⑤		
					③
②			④		

- 1 – Parallelogram
- 2 – Square
- 3 – Quadrilateral
- 4 – Square
- 5 – Quadrilateral
- 6 – Rectangle

Geometry Sudoku Puzzle 9

					⑤
⑤	④				
	⑥	③		①	
		⑥	③		
	②				
②				④	①

- 1 – Parallelogram
- 2 – Rectangle
- 3 – Rectangle
- 4 – Quadrilateral
- 5 – Square
- 6 – Quadrilateral

Geometry Sudoku Puzzle 10

		⑥			⑤
⑤			②		
		③			④
			③	④	
			⑥		①
①	②				

- 1 – Square
- 2 – Quadrilateral
- 3 – Square
- 4 – Parallelogram
- 5 – Square
- 6 – Quadrilateral

Geometry Sudoku Puzzle 11

KILLER SUDOKU

As with Sudoku, the numbers in the Killer Sudoku puzzle solutions appear once in each row, each column, and each outlined region (when they are marked—smaller puzzles may not use these). In addition, groups of squares are surrounded by a dotted line and the sum of the numbers in these groups is indicated by a small number in the top left corner of the grouping. Finally, a number cannot be repeated in one of these groups.

For example, in the solution to the 6 x 6 puzzle below, you can see that each number from 1 to 6 appears once in each row, column, and outlined region. You can also see that groups of two, three, or four squares are surrounded by dotted lines and the sum of the numbers in these groups matches their indicated sum.

8	14				15
		8	10		
12				8	
10		7	8		
				8	
	18				

Killer Sudoku Example

⁸ 2	¹⁴ 4	3	1	6	¹⁵ 5
5	1	⁸ 6	¹⁰ 3	2	4
¹² 4	3	2	5	⁸ 1	6
¹⁰ 6	5	⁷ 1	⁸ 2	4	3
3	2	4	6	⁸ 5	1
1	¹⁸ 6	5	4	3	2

Killer Sudoku Example Solution

Solve the following Killer Sudoku puzzles, placing the digits 1-4 in each row and column.

6	9		
	8		
	9	5	3

Killer Sudoku Puzzle 1

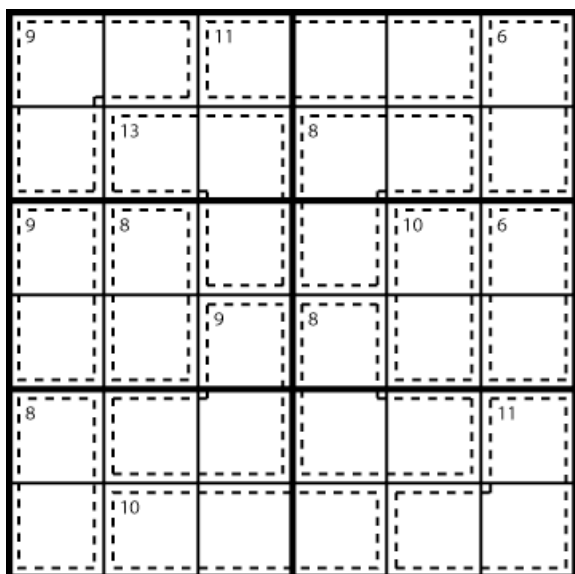
8		6	6
	7		
3			10

Killer Sudoku Puzzle 2

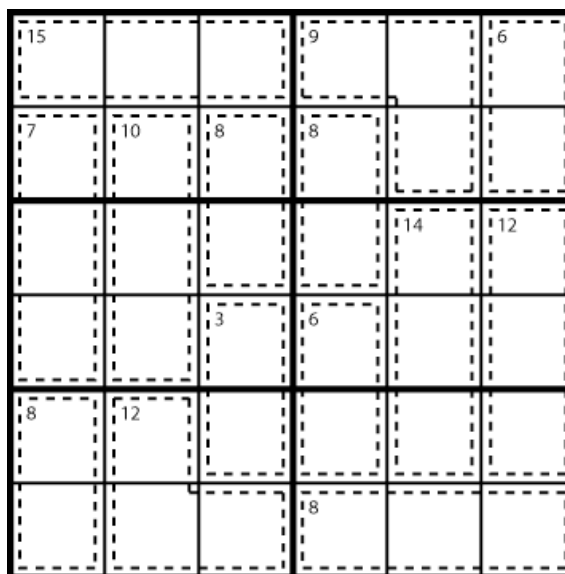
9	3		7
		8	
4			3
	6		

Killer Sudoku Puzzle 3

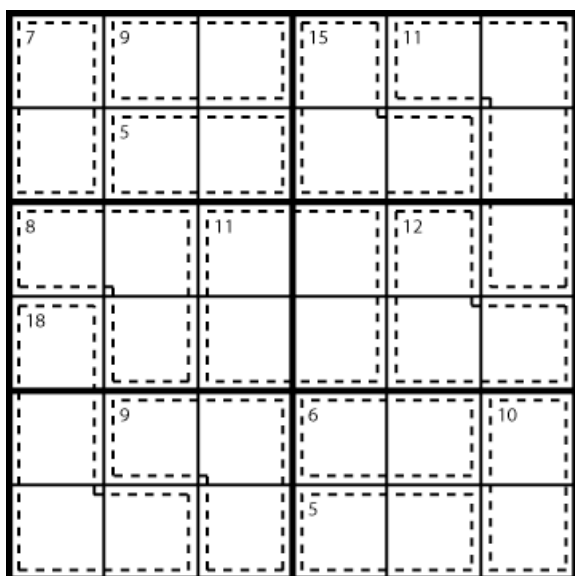
Solve the following Killer Sudoku puzzles (from www.sudoku.org.uk)



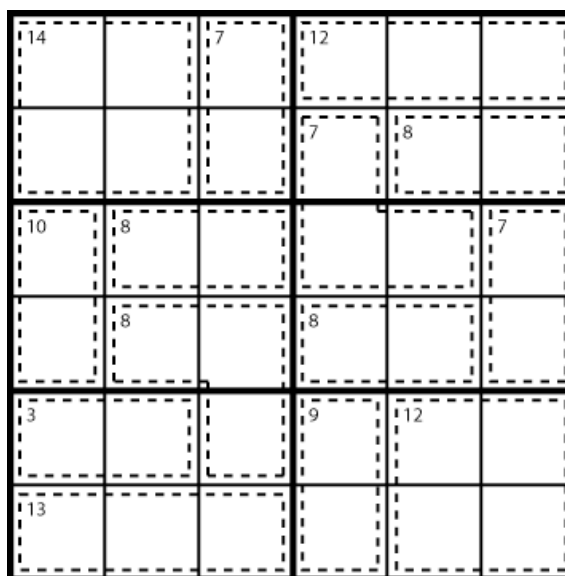
Killer Sudoku Puzzle 4



Killer Sudoku Puzzle 5



Killer Sudoku Puzzle 6



Killer Sudoku Puzzle 7

Solve the following Killer Sudoku puzzle

16	10		8	23	9	20	8	
	10						19	
4	24		6		14			
							7	15
10	5	26						
		22	15	21	6	18		5
17								
12			13		8	13		13
						8		

Killer Sudoku Puzzle 8

(from www.krazydad.com)

Some Killer Sudoku Resources

- Killer Sudoku Online (<http://www.killersudokuonline.com>) carries a daily 9 x 9 Killer Sudoku, a weekly 9 x 9 Killer Sudoku (very difficult), and a weekly 9 x 9 Greater Than Killer Sudoku that combines these two types. These can be printed out or solved online. Many of the previous puzzles are also archived on their website.
- www.sudoku.org.uk maintains a website featuring several different Sudoku variations, including 6 x 6 Killer Sudoku puzzles, (<http://www.sudoku.org.uk/DailyMinis.asp>) and 9 x 9 Killer Sudoku puzzles (<http://www.sudoku.org.uk/killersudoku.asp>).
- A free Killer Sudoku shareware game is available from Yoogi Games (<http://www.yoogi.com/killer-sudoku>).
- Jim Bumgardner maintains the blog "KrazyDad." Here he publishes a number of interesting variations on logic puzzles including hundreds of 9 x 9 Killer Sudoku puzzles (<http://www.krazydad.com/killersudoku>).

KENKEN

As with Sudoku, the numbers in the KenKen puzzle solutions appear once in each row and each column. In addition, individual squares or groups of squares are surrounded by a border. At the top of each grouping, there appears a small number and usually an operation symbol (addition, subtraction, multiplication, or division). That small number represents the result when all of the numbers in the group are combined using the indicated operation.

For example, in the 4 x 4 puzzle below, you can see that each number from 1 to 4 appears once in each row and column.

3-		6×	
5+	2÷	3-	
			9+
4+			

KenKen Example

3- 4	1	6× 3	2
5+ 3	2÷ 2	3- 4	1
2	4	1	9+ 3
4+ 1	3	2	4

KenKen Example Solution

- In the KenKen example above, the first two squares in the top row are grouped and are labeled with a 3 and a subtraction sign (3-). This means that when one number in this group is subtracted from the other, they have a difference of 3.
- To the right of that group, there is a set of three squares grouped together and labeled with a 6 and a multiplication sign (6×). This means that the numbers in these three squares have a product of 6.
- On the left side of the grid above, there are two squares grouped together and labeled with a 5 and an addition sign (5+). This means that the numbers in this group have a sum of 5.
- To the right of that group are two squares labeled with a 2 and a division sign (2÷). This means that when one number in this group is divided by the other number, they have a quotient of 2.

Finally, a number can appear more than once in a group—as long as it does not appear in the same row or column as another occurrence of that number.

Solve the following KenKen puzzles (from www.kenken.com)

1-		2×	
2-	2÷	4	
		18×	
3+			4

KenKen Puzzle 1

7+	6×		3+
	1	5+	
2÷	12×		3
		2-	

KenKen Puzzle 2

1-		4	2×
7+			
2÷		10+	
4	2÷		

KenKen Puzzle 3

1-	2÷		7+
	3-		
2÷		3	2÷
12×			

KenKen Puzzle 4

3-		1-	
4+	24×		2÷
		4	
1-		3-	

KenKen Puzzle 5

1-	5+	3	2÷
		2÷	
3-	6+		12×

KenKen Puzzle 6

Solve the following KenKen puzzles (from www.kenken.com)

12+			8×	
10+	8+	25×		2-
		2	12+	
1	2÷			

KenKen Puzzle 7

1-		10+		
5×		5+	60×	
1-				5+
	10×		2÷	
7+				1

KenKen Puzzle 8

16+			2÷	
4×		60×		9+
15×	48×	1		1-

KenKen Puzzle 9

2÷	6×	10+	15×	
				6+
8+		9+		
	80×	7+	1	2-

KenKen Puzzle 10

2÷	5+		36×		5
	12×			4-	
6+		2÷		3	4-
3÷		6	1-		
30×	1-		4×		12×
	5-		3		

KenKen Puzzle 11

3	5-	11+		2÷	9+
8+		5×	3		
				6	2÷
3÷		72×	3-		
20×				2×	
1-		2	1-		

KenKen Puzzle 12

Solve the following KenKen puzzle

3645×		15+			2÷		9×	
1-			1680×					2÷
	672×		2-		7+		26+	
		50×	18×			36×		
210×				5-				288×
	1	15+			16+			
	2-	24×		1	5-		10+	
2÷		24×	2÷	4-	8-	17+		1-

KenKen Puzzle 13
(from www.kenken.com)

Some KenKen Resources

- KenKen.com features six new KenKen puzzles each day, from 4 x 4 to 9 x 9. They can be played online or printed out. KenKen.com also supplies daily KenKen puzzles to other websites like the New York Times. KenKen is a registered trademark name owned by Nextoy. Other versions of KenKen-type puzzles appear under the names Kendoku and Mathdoku.
- KenKen puzzle books can now be found in bookstores and online.

SOLUTIONS

Greater Than Sudoku

2 > 1	3 < 4
^ ^	v v
4 > 3	2 > 1
1 < 2	4 > 3
^ ^	v v
3 < 4	1 < 2

Puzzle 1

3 > 2	1 < 4
^ v	^ v
4 > 1	3 > 2
1 < 4	2 < 3
^ v	^ v
2 < 3	4 > 1

Puzzle 2

2 > 1	3 < 4
^ ^	v v
3 < 4	2 > 1
1 < 3	4 > 2
^ v	v ^
4 > 2	1 < 3

Puzzle 3

1 < 3 > 2	6 > 4 < 5
^ ^ ^	v v v
5 > 4 < 6	1 < 2 < 3
6 > 5 > 1	4 > 3 > 2
v v ^	^ ^ v
4 > 2 < 3	5 < 6 > 1
3 < 6 > 5	2 > 1 < 4
v v v	^ ^ ^
2 > 1 < 4	3 < 5 < 6

Puzzle 4

1 < 6 > 3	2 < 4 < 5
^ v v	v v ^
5 > 4 > 2	1 < 3 < 6
4 < 5 < 6	3 > 2 > 1
v v v	^ ^ ^
2 < 3 > 1	6 > 5 > 4
6 > 2 < 5	4 > 1 < 3
v v v	^ ^ v
3 > 1 < 4	5 < 6 > 2

Puzzle 5

6 > 1 < 4	3 > 2 < 5
v ^ ^	^ ^ v
2 < 3 < 5	4 < 6 > 1
3 < 4 > 2	5 > 1 < 6
v ^ ^	v ^ v
1 < 5 < 6	2 < 4 > 3
5 > 2 > 1	6 > 3 < 4
v ^ ^	v ^ v
4 < 6 > 3	1 < 5 > 2

Puzzle 6

4 > 3 < 6	5 > 2 > 1
v v v	^ ^ ^
2 > 1 < 5	6 > 3 < 4
5 > 4 > 3	2 > 1 < 6
v ^ v	^ ^ v
1 < 6 > 2	3 < 4 < 5
3 < 5 > 1	4 < 6 > 2
^ v ^	v v ^
6 > 2 < 4	1 < 5 > 3

Puzzle 7

4 < 6 < 9	7 > 2 < 3	1 < 8 > 5
^ v v	v v ^	^ v ^
8 > 5 > 3	6 > 1 < 4	7 > 2 < 9
v v v	^ ^ ^	v ^ v
7 > 2 > 1	9 > 8 > 5	4 > 3 < 6
3 < 4 < 8	1 < 7 < 9	5 < 6 > 2
^ ^ v	^ v v	^ v ^
5 < 7 > 6	8 > 3 > 2	9 > 1 < 4
v ^ v	v ^ ^	v ^ ^
1 < 9 > 2	5 > 4 < 6	3 < 7 < 8
2 > 1 < 7	4 < 9 > 8	6 > 5 > 3
^ ^ v	v v v	^ ^ v
6 < 8 > 4	3 < 5 < 7	2 < 9 > 1
v v ^	v v ^	v v ^
9 > 3 < 5	2 < 6 > 1	8 > 4 < 7

Puzzle 8

SOLUTIONS

Geometry Sudoku

1	4	2	3
3	②	1	4
2	③	4	1
④	1	3	2

Puzzle 1

2	①	3	4
4	②	1	3
1	3	4	2
③	④	2	1

Puzzle 2

3	2	④	1
1	4	2	3
2	①	3	4
4	③	1	②

Puzzle 3

1	3	4	2
④	2	1	3
3	4	2	①
2	1	3	4

Puzzle 4

3	2	5	①	4
2	5	1	4	3
1	4	②	③	5
4	1	③	5	2
⑤	3	④	2	1

Puzzle 5

2	1	③	4	⑤
⑤	3	④	1	2
1	4	5	2	3
3	2	1	5	4
④	5	②	3	①

Puzzle 6

③	1	2	4	5
4	②	5	1	③
2	④	3	5	①
1	⑤	4	3	2
5	3	1	2	④

Puzzle 7

⑤	1	2	4	3
③	4	①	5	②
2	5	3	1	4
1	2	④	3	5
④	3	5	②	1

Puzzle 8

1	②	⑤	3	4	⑥
⑥	4	1	2	③	5
④	5	3	6	①	2
3	①	2	⑤	6	4
5	6	4	1	2	③
②	3	6	④	5	1

Puzzle 9

6	1	4	2	3	⑤
⑤	④	2	1	6	3
4	⑥	③	5	①	2
1	5	⑥	③	2	4
3	②	1	4	5	6
②	3	5	6	④	①

Puzzle 10

4	3	⑥	1	2	⑤
⑤	1	4	②	6	3
2	6	③	5	1	④
6	5	1	③	④	2
3	4	2	⑥	5	①
①	②	5	4	3	6

Puzzle 11

SOLUTIONS

Killer Sudoku

1	4	2	3
2	1	3	4
3	2	4	1
4	3	1	2

Puzzle 1

3	1	2	4
4	3	1	2
2	4	3	1
1	2	4	3

Puzzle 2

4	1	2	3
2	3	1	4
1	4	3	2
3	2	4	1

Puzzle 3

1	5	2	6	3	4
3	4	6	1	5	2
5	6	3	2	4	1
4	2	1	3	6	5
2	3	5	4	1	6
6	1	4	5	2	3

Puzzle 4

6	5	4	3	2	1
2	1	3	6	4	5
1	6	5	2	3	4
4	3	2	1	5	6
3	4	1	5	6	2
5	2	6	4	1	3

Puzzle 5

5	6	3	4	2	1
2	1	4	6	5	3
1	4	2	3	6	5
6	3	5	1	4	2
3	2	6	5	1	4
4	5	1	2	3	6

Puzzle 6

5	3	1	6	2	4
2	4	6	1	3	5
6	5	3	2	4	1
4	1	2	3	5	6
1	2	5	4	6	3
3	6	4	5	1	2

Puzzle 7

9	6	4	7	5	3	8	2	1
7	8	2	1	4	6	3	9	5
1	5	3	2	8	9	7	4	6
3	7	9	4	6	5	2	1	8
2	4	5	3	1	8	9	6	7
8	1	6	9	7	2	4	5	3
5	9	7	6	3	4	1	8	2
4	3	8	5	2	1	6	7	9
6	2	1	8	9	7	5	3	4

Puzzle 8

SOLUTIONS

KenKen

1-	4	3	2×	1	2
2-	3	2	2+	4	1
	1	4	18×	2	3
3+	2	1	3	4	4

Puzzle 1

7+	4	2	3	3+	1
	3	1	5+	4	2
2÷	2	12×	4	1	3
	1	3	2-	2	4

Puzzle 2

1-	2	3	4	2×	1
7+	3	4	1	2	
2÷	1	2	10+	3	4
4	4	2÷	1	2	3

Puzzle 3

1-	3	1	2	7+	4
	2	3-	4	1	3
2÷	4	2	3	3	2÷
12×	1	3	4	2	

Puzzle 4

3-	4	1	1-	2	3
4+	1	24×	4	3	2÷
	3	2	4	4	1
1-	2	3	3-	1	4

Puzzle 5

1-	2	5+	4	3	2÷
	3	1	2÷	4	2
3-	1	3	6+	2	12×
	4	2	1	3	

Puzzle 6

12+	4	5	3	8×	2	1
10+	3	1	5	4	2	
	2	3	1	5	4	
	5	4	2	12+	1	3
1	1	2	4	3	5	

Puzzle 7

1-	2	3	10+	4	1	5
5×	1	5	5+	2	3	4
1-	4	1	3	5	2	
	5	10×	2	1	2÷	4
7+	3	4	5	2	1	

Puzzle 8

16+	4	5	3	2÷	1	2
4×	2	4	60×	5	3	9+
	1	2	4	5	3	
15×	5	3	1	2	4	
	3	1	2	4	5	

Puzzle 9

2÷	1	2	6×	10+	15×	3	5
	2	3	1	5	6+	4	
8+	3	1	5	4	2		
	5	4	2	1	3		
	4	5	3	2	1		

Puzzle 10

2÷	4	5+	2	3	36×	1	6	5
	2	3	4	6	5	1		
6+	1	5	2	4	3	4-	6	
3÷	3	1	6	5	4	2		
30×	6	4	5	2	1	3		
	5	6	1	3	2	4		

Puzzle 11

3	3	5-	1	11+	6	5	2	9+
8+	2	6	1	3	4	5		
	4	2	5	1	6	6	2÷	3
3÷	1	3	4	2	5	6		
20×	5	4	3	6	1	2		
1-	6	5	2	2	4	3	1	

Puzzle 12

3645×	9	5	7	2	6	8	4	1	3
1-	4	9	1	7	8	5	6	3	2
672×	3	7	9	6	4	2	5	8	1
	8	4	5	1	3	6	9	2	7
210×	6	3	2	5	7	4	1	9	8
1	7	1	6	9	2	3	8	5	4
	5	6	8	3	1	7	2	4	9
2÷	2	8	3	4	9	1	7	6	5
	1	2	4	8	5	9	3	7	6

Puzzle 13