

# Quality Common Mathematics Unit Assessments

*Design* is a funny word. Some people think design means how it looks.  
But of course, if you dig deeper, it's really how it works.

—Steve Jobs

**H**ow do you decide if the unit-by-unit common mathematics assessment instruments you design are high quality? This is a question every teacher and leader of mathematics should ask. Tim Kanold describes his experience at Stevenson HSD 125 when he first asked this question of the middle school mathematics teachers from one of the Stevenson feeder districts.

The mathematics assessment work Tim describes created an early beta model for a test evaluation tool he and his fellow teachers could use to evaluate the quality of their unit-by-unit mathematics assessments. Figure 2.1 (page 14) provides a much deeper

and robust during-the-unit or end-of-unit assessment instrument evaluation tool your collaborative team can use to evaluate quality and build new and revised unit assessment instruments.

Your collaborative team should rate and evaluate the quality of its most recent end-of-unit or chapter assessment instruments (tests and quizzes) using the tools in figure 2.1 and figure 2.2 (page 15). How does your team's end-of-unit assessment score? Do you rate your current tests a twelve, sixteen, or twenty-two? How close does your assessment instrument come to scoring a twenty-seven or higher out of the thirty-two points possible in the test-quality protocol?

## *Personal Story* TIMOTHY KANOLD

During my fourth year as the director of mathematics at Stevenson, it was clear to me that we were not very assessment literate. By this I mean we had very little knowledge about each other's assessment routines and practices. I remember collecting all the mathematics tests being given to our grades 6–8 feeder district middle school students during the months of October and November, and feeling a certain amount of dismay at the wide range of rigor, the lack of direction for how the assessments were organized, and our lack of clarity about the nature of the mathematics tasks being chosen for the exam.

In some instances, the questions we were asking sixth-grade students were more rigorous than the questions we were asking eighth-grade students. We had little or no shared direction for this most important aspect of our professional life. We needed to do a deep dive into the research and the expectations of high-quality mathematics assessments. It began a journey for all of our mathematics teachers to not only become assessment literate, but to use our mathematics assessments as a way to sustain student effort and learning at the right level of rigor across the grades.

High-Quality Assessment Criteria	Description of Level 1	Requirements of the Indicator Are Not Present	Limited Requirements of the Indicator Are Present	Substantially Meets the Requirements of the Indicator	Fully Achieves the Requirements of the Indicator	Description of Level 4
1. Identification and emphasis on essential learning standards (student-friendly language)	Essential learning standards are unclear, absent from the assessment instrument, or both. Some of the mathematical tasks (questions) may not align to the essential learning standards of the unit. The organization of assessment tasks is not clear.	1	2	3	4	Essential learning standards are clear, included on the assessment, and connected to the assessment tasks (questions).
2. Balance of higher- and lower-level-cognitive-demand mathematical tasks	Emphasis is on procedural knowledge with minimal higher-level-cognitive-demand mathematical tasks for demonstration of understanding.	1	2	3	4	Test is rigor balanced with higher-level and lower-level-cognitive-demand mathematical tasks present and aligned to the essential learning standards.
3. Variety of assessment-task formats and use of technology	Assessment contains only one type of questioning strategy—selected response or constructed response. There is little to no modeling of mathematics or use of tools. Use of technology (such as calculators) is not clear.	1	2	3	4	Assessment includes a blend of assessment types and modeling tasks or use of tools. Use of technology (such as calculators) is clear.
4. Appropriate and clear scoring rubric (points assigned or proficiency scale)	Scoring rubric is not evident or is inappropriate for the assessment tasks.	1	2	3	4	Scoring rubric is clearly stated and appropriate for each mathematical task.
5. Clarity of directions	Directions are missing or unclear. Directions are confusing for students.	1	2	3	4	Directions are appropriate and clear.
6. Academic language	Wording is vague or misleading. Academic language (vocabulary and notation) are not precise, causing a struggle for student understanding and access.	1	2	3	4	Academic language (vocabulary and notation) in tasks is direct, fair, accessible, and clearly understood by students. Teachers expect students to attend to precision in response.
7. Visual presentation	Assessment instrument is sloppy, disorganized, difficult to read, and offers no room for student work.	1	2	3	4	Assessment is neat, organized, easy to read, and well-spaced, with room for student work. There is also room for teacher feedback.
8. Time allotment	Few students can complete the assessment in the time allowed.	1	2	3	4	Students can successfully complete the assessment in the time allowed.

**Figure 2.1: Team discussion tool—Assessment instrument quality evaluation rubric.**

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<p><b>Directions:</b> Examine your most recent common end-of-unit assessment, and evaluate its quality against the following eight criteria in figure 2.1 (page 14). Write your responses to each question in the following spaces.</p>
<p>1. Are the essential learning standards written on the assessment?  <b>Discuss:</b> What do our students think about learning mathematics? Do they think learning mathematics is about doing a bunch of random mathematics problems? Or, can they explain the essential learning standards in student-friendly / <i>can...</i> statements for each group of questions? Can they solve any mathematical tasks that might reflect a demonstration of learning that standard? Are our students able to use the essential learning standards and tasks to determine what they have learned and what they have not learned yet?</p> <p>Note: In order for students to respond to the end-of-unit assessment feedback when the teacher passes it back, this is a necessary assessment feature.</p>
<p>2. Is there an appropriate balance of higher- and lower-level-cognitive-demand mathematical tasks on the assessment?  <b>Discuss:</b> What percentage of all tasks or problems on the assessment instrument is of lower-level cognitive demand? What percentage is of higher-level cognitive demand? Is there an appropriate balance? Is balancing rigor a major focus of our work?</p> <p>Note: Use figure 2.4 (page 23) as a tool to determine the rigor. This will help you to better understand the level of cognitive demand. Also, see the appendix (page 111) for more advice on this criterion. As a good rule of thumb, the rigor-balance ratio should be about 30/70 (lower- to higher-level cognitive demand) on the assessment as appropriate to the standards on the assessment.</p>
<p>3. Is there variety of assessment formats and a clear use of technology?  <b>Discuss:</b> Does our assessment use a blend of assessment formats or types? If we use multiple choice, do we include questions with multiple possible answers? Do we include tasks that allow for technology as a tool, such as graphing calculators? Do we provide tasks that assess appropriate use of tools or modeling?</p> <p>Note: Your end-of-unit assessments should not be of either extreme—all multiple-choice or all constructed-response questions.</p>
<p>4. Are scoring rubrics clear and appropriate?  <b>Discuss:</b> Are the scoring rubrics to be used for every task clearly stated on the assessment? Do our scoring rubrics (total points or proficiency scale) make sense based on the complexity of reasoning for the task? Are the scoring points or scale assigned to each task appropriate and agreed upon by each teacher on our team? Is there clear understanding about the student work necessary to receive full credit for each assessment task or question? Is it clear to each team member how partial credit will be assigned?</p>

**Figure 2.2: Team discussion tool—High-quality assessment evaluation.**

continued →

<p>5. Are the directions clear?  <b>Discuss:</b> What does clarity mean to each member of our team? Are any of the directions we provide for the different assessment tasks confusing to the student? Why?</p> <p>Note: The verbs (action words) you use in the directions for each set of tasks or problems are very important to notice when discussing clarity. Also, be sure that in the directions you clearly state the student work you expect to see and will grade using points or a scale.</p>	<p>6. Is the academic language precise and accessible?  <b>Discuss:</b> Are the vocabulary and notation for each task we use on our common assessment clear, accessible, and direct for students? Do we attend to the precision of language used during the unit, and do students understand the language we use on the assessment?</p> <p>Note: Does the assessment instrument include the proper language supports for all students?</p>	<p>7. Does the visual presentation provide space for student work?  <b>Discuss:</b> Do our students have plenty of space to write out solution pathways, show their work, and explain their thinking for each task on the assessment instrument?</p> <p>Note: This criterion often is one of the reasons not to use the written tests that come with your textbook series. You can use questions from the test bank aligned to your instruction, but space mathematics tasks and assessment questions as needed to allow plenty of room for students to demonstrate their understanding.</p>	<p>8. Do we allot enough time for students to complete the assessment?  <b>Discuss:</b> Can our students complete this assessment in the time allowed? What will be our procedure if they cannot complete the assessment within the allotted time so all students receive equitable opportunities for demonstrating learning?</p> <p>Note: Each teacher on the team should complete a full solution key for the assessment he or she expects of students. For upper-level students, it works well to use a time ratio of three to one (or four to one) for student to teacher completion time to estimate how long it will take students to complete an assessment. For elementary students, it may take much longer to complete the assessment. All teachers should use the agreed-on time allotment.</p>
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You should expect to eventually write common assessment tests that would score fours in all eight categories of the assessment evaluation.

For deeper clarification and explanation on each of the evaluation criteria in the assessment tool, use figure 2.2 (page 15) to reflect on how your team rates and scores on your common assessment instruments.

### TEACHER *Reflection*



Based on the criteria from figures 2.1 and 2.2, summarize the result of your evaluation of an assessment you created and used with your students.

What action can you immediately take to improve your assessments?

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The first four design criteria of the assessment instrument evaluation tool (identification and emphasis on essential learning standards, balance of higher- and lower-level cognitive demands, variety of assessment-task formats and use of technology, and appropriate scoring rubric) are perhaps the most important, and yet, also the most limiting aspect of many mathematics unit assessments—both during and at the end of a unit. They are vital to making your assessments “work” for students, as Steve Jobs indicates in the opening quote for this chapter.

These first four mathematics assessment design criteria can often create places of *great inequity* in your mathematics assessment process and professional work. For that reason, we provide extended details for each of these four criteria to help you and your colleagues’ growth and development in designing your common mathematics assessments.



### TEAM RECOMMENDATION

#### Design High-Quality Common Unit Assessments

Use figures 2.1 and 2.2 to determine the current strengths and areas to improve on your team common assessments.

- Commit to creating your common unit assessments before the unit begins.
- Respond to the prompt: What is your team plan to improve your common unit assessments this year?

### Identification and Emphasis on Essential Learning Standards

Your team can organize assessments in many ways, such as the following.

- Format (multiple choice first and constructed response second)
- Difficulty (from easier tasks to more difficult tasks)
- Order (place the tasks in the order they were identified for the assessment)
- Essential learning standard

The essential learning standards should be the *organizational driver for your assessment design* because the essential learning standards sit at the top of the instructional framework “to signify that setting goals is the starting point for all decision making” (Smith, Steele, & Raith, 2017, p. 195).

Your team should list an essential learning standard for the unit, followed by the mathematics tasks or questions you believe represent evidence of student learning and proficiency for that standard. There should be no more than three to six such standards on any end-of-unit mathematics assessment and perhaps one to two on a unit quiz or mid-unit assessment.

There are many names for the types of standards your team will use, such as *power standards*, *priority*

*standards, promise standards, and so on.* When your team works within a PLC culture and for the purposes in this book, we suggest using the term *essential learning standards* for the unit. These are the *big idea* standards you expect your students to learn.

### TEACHER Reflection

Review the criteria from figures 2.1 and 2.2. *How should you organize your assessments?* This is a good first question for your team to ask. Describe your current practice.

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Rick DuFour and Bob Marzano (2011) ask you to think of the essential learning standards as part of your team's guaranteed and viable curriculum for each unit. A *guaranteed curriculum* is one that promises the community, staff, and students that a student in school will learn specific content and processes regardless of the teacher he or she receives. A *viable curriculum* refers to ensuring there is adequate time for all students to learn the guaranteed curriculum during the school year.

Clear team agreement on the essential learning standards for the unit is a priority. It is the starting point for answering PLC critical question 1 (DuFour et al., 2016): What do we want all students to know and be able to do? This is the launching point for more equitable learning experiences for every student in your grade level or course.

The three to six essential learning standards also inform your team's intervention design and response to critical questions 3 and 4, discussed in part 2 of this book (page 65).

Ultimately, the reason for designing your common assessments around the organizational driver of the essential learning standards is to identify where to target future emphasis for students, including:

1. Mathematics tasks or questions, such as the proper distribution and ensured standards alignment of those mathematical tasks placed on the common assessment
2. Team-designed student intervention and extension activities
3. Instructional decisions during and after a unit (We discuss this more thoroughly in *Mathematics Instruction and Tasks in a PLC at Work* in this series.)

Your collaborative team may want to use or create a proficiency map that identifies, by unit, essential learning standards with which students should be proficient by the end of each unit. A quick overview of unit pacing can help your team make connections between standards and what students should learn, which in turn informs quality assessment and a planned team response to student learning. (Visit [go.SolutionTree.com/MathematicsatWork](http://go.SolutionTree.com/MathematicsatWork) for examples of several K–5 unit-by-unit, grade-level proficiency maps.)

### TEACHER Reflection

Look closely at your students' current assessments. How many essential learning standards are you listing for the assessments? Too many? Too few? If you do not list the standards, can you identify how many the tests assess?

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Be sure your mathematics assessments, unit tests, and quizzes do not list too many essential learning standards. In order to design high-quality mathematics assessments your team and students can use for formative purposes, focus only on the big idea and most essential learning standards. Also, be sure to write the essential learning standards in student friendly-language using *I can* statements. This helps students verbalize their part in the assessment, learning, and reflection process.

However, the *big idea* standards generally do not provide enough detail for your daily lessons. You most likely use or have heard of many terms for your lesson focus, such as *learning objective*, *learning target*, *daily objective*, *daily learning standard*, *lesson objective*, and so on. These more detailed and specific standards for the daily lesson help you and your students unwrap the essential learning standards (the big ideas for the unit) into daily skills and concepts to use for the assessment tasks (questions) on the test. For the purposes of our PLC work, we reference these specific parts of standards as *learning targets*.

Your daily learning targets help support the tasks selected for your common assessments but are too specific for test organization purposes. The essential learning standards in each unit comprise sections of the assessment and allow for student reflection and continued learning. (See pages 20–21 for specific student reflection examples.)

To help you compare and contrast the difference between essential learning standards and daily learning targets, figure 2.3 (page 20) provides a sample that illustrates a grade 4 fractions unit. The essential learning standards it lists in the left column present a more formal representation of each essential learning standard, similar to how it might appear in your state standards.

The middle column shows essential learning standards as a teacher would write them on the assessment or test in student-friendly *I can* language. The right column unwraps each standard into daily learning targets for your lesson design and planning using combinations of verbs and noun phrases in the original formal standard.

Using the sample grade 4 fractions unit, your team could create shorter common assessments for each of the essential learning standards throughout the unit. You and your students could then use the results to proactively receive feedback for additional instruction and help *before* the end-of-unit assessment. (Visit [go.SolutionTree.com/MathematicsatWork](http://go.SolutionTree.com/MathematicsatWork) to find similar models for first grade, seventh grade, and high school.)

## TEACHER *Reflection*

How does your team create and develop its understanding of the essential learning standards for the unit?

How do you break down the essential learning standards for assessment purposes into daily learning targets for instruction purposes?

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When you work with your colleagues to determine the essential learning standards for common unit assessments, you ensure equitable learning expectations and experiences for students in classrooms across your team.



## TEAM RECOMMENDATION

### Determine Essential Learning Standards for Student Assessment and Reflection

- Identify the formal learning standards students must be proficient with by the end of the unit.
- Group the standards as needed to create three to six essential learning standards for the unit. Write these using student-friendly language and begin each with *I can* statements.
- As a team, make sense of the essential learning standards and discuss what students must know and be able to do to demonstrate proficiency.

<p align="center"><b>Formal Unit Standards (Generic state standard language)</b></p>	<p align="center"><b>Essential Learning Standards for Assessment and Reflection (Uses student-friendly language)</b></p>	<p align="center"><b>Daily Learning Targets (Explains what students should know and be able to do; unwrapped standards)</b></p>
<p>1. Explain why a fraction <math>\frac{a}{b}</math> is equivalent to a fraction <math>\frac{(n \times a)}{(n \times b)}</math> by using visual fraction models, with attention to how the number and size of the parts differ, even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</p> <p>2. Compare two fractions with different numerators and different denominators, for instance, by creating common denominators or numerators or by comparing to a benchmark fraction such as <math>\frac{1}{2}</math>. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols <math>&gt;</math>, <math>=</math>, or <math>&lt;</math>, and justify the conclusions.</p>	<p>I can explain why fractions are equivalent and create equivalent fractions.</p> <p>I can compare two fractions and explain my thinking.</p>	<ul style="list-style-type: none"> <li>• Explain why a fraction <math>\frac{a}{b}</math> is equivalent to a fraction <math>\frac{(n \times a)}{(n \times b)}</math> by: <ul style="list-style-type: none"> <li>▶ Using visual fraction models</li> <li>▶ Telling how the number and size of the parts differ even though the fractions are the same size</li> </ul> </li> <li>• Generate equivalent fractions.</li> <li>• Compare two fractions with different numerators and different denominators by: <ul style="list-style-type: none"> <li>▶ Creating common numerators</li> <li>▶ Creating common denominators</li> <li>▶ Comparing to a benchmark fraction</li> </ul> </li> <li>• Record fraction comparisons using <math>&lt;</math>, <math>=</math>, or <math>&gt;</math>.</li> <li>• Justify fraction comparisons (for instance, using visual models).</li> <li>• Recognize that comparisons are valid only when the two fractions refer to the same whole.</li> </ul>
<p>3. Understand a fraction <math>\frac{a}{b}</math> with <math>a &gt; 1</math> as a sum of fractions <math>\frac{1}{b}</math>.</p> <p>a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p> <p>b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.</p> <p>c. Add and subtract mixed numbers with like denominators (for example, by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction).</p>	<p>I can add and subtract fractions and show my thinking.</p>	<ul style="list-style-type: none"> <li>• Compose a fraction using unit fractions.</li> <li>• Add fractions by joining parts of the same whole.</li> <li>• Subtract fractions by separating parts of the same whole.</li> <li>• Decompose a fraction more than one way into a sum of fractions with like denominators.</li> <li>• Add mixed numbers with like denominators using: <ul style="list-style-type: none"> <li>▶ Equivalent fractions</li> <li>▶ Properties of operations</li> <li>▶ Relationship between addition and subtraction</li> </ul> </li> <li>• Subtract mixed numbers with like denominators using: <ul style="list-style-type: none"> <li>▶ Equivalent fractions</li> <li>▶ Properties of operations</li> <li>▶ Relationship between addition and subtraction</li> </ul> </li> </ul>

<b>Formal Unit Standards</b> <b>(Generic state standard language)</b>	<b>Essential Learning Standards for Assessment and Reflection</b> <b>(Uses student-friendly language)</b>	<b>Daily Learning Targets</b> <b>(Explains what students should know and be able to do; unwrapped standards)</b>
<p>4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</p> <p>a. Understand a fraction <math>\frac{a}{b}</math> as a multiple of <math>\frac{1}{b}</math>. For example, use a visual fraction model to represent <math>\frac{5}{4}</math> as the product <math>5 \times (\frac{1}{4})</math>, recording the conclusion by the equation <math>\frac{5}{4} = 5 \times (\frac{1}{4})</math>.</p> <p>b. Understand a multiple of <math>\frac{a}{b}</math> as a multiple of <math>\frac{1}{b}</math>, and use this understanding to multiply a fraction by a whole number (for example, use a visual fraction model to express <math>3 \times (\frac{2}{5})</math> as <math>6 \times (\frac{1}{5})</math>, recognizing this product as <math>\frac{6}{5}</math>. (In general, <math>n \times (\frac{a}{b}) = \frac{n \times a}{b}</math>.)</p>	<p>I can multiply a fraction by a whole number and explain my thinking.</p>	<ul style="list-style-type: none"> <li>• Multiply a fraction by a whole number.</li> <li>• Show that a fraction <math>\frac{a}{b}</math> is a multiple of <math>\frac{1}{b}</math> and write an equation to match.</li> <li>• Multiply a fraction by a whole number using the idea that <math>\frac{a}{b}</math> is a multiple of <math>\frac{1}{b}</math>. <ul style="list-style-type: none"> <li>▶ Use a visual model.</li> <li>▶ Write expressions.</li> </ul> </li> </ul>
<p>3d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators (e.g., by using visual fraction models and equations to represent the problem).</p> <p>4c. Solve word problems involving multiplication of a fraction by a whole number (for example, by using visual fraction models and equations to represent the problem). For example, if each person at a party will eat <math>\frac{3}{8}</math> of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will the party need? Between what two whole numbers does your answer lie?</p>	<p>I can solve word problems involving fractions.</p>	<ul style="list-style-type: none"> <li>• Solve word problems involving addition and subtraction of fractions with like denominators referring to the same whole using: <ul style="list-style-type: none"> <li>▶ Visual fraction models</li> <li>▶ Equations</li> </ul> </li> <li>• Solve word problems involving multiplication of a fraction by a whole number. <ul style="list-style-type: none"> <li>▶ Use a visual model.</li> <li>▶ Write equations.</li> </ul> </li> </ul>

**Figure 2.3: Sample grade 4 fractions unit—Essential learning standards.**

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