INTRODUCTION

When the National Council of Teachers of Mathematics (NCTM) Educational Materials Committee asked us to write a book for beginning elementary math teachers, we were excited and overwhelmed. We were excited to share teaching practices honed through years of experience as classroom teachers, coaches, supervisors, and staff developers—practices we developed by building on others' work and research. Yet this was a daunting challenge: How could we convey the complexities of teaching math without overwhelming new teachers, or make information about math teaching accessible without oversimplifying?

We first thought about goals for students: What is important for students to learn? What does a successful math student look like? To succeed in today's information age, students must interpret, generate, and represent data, and they need to be fluent with numbers and operations to do mental calculations. They also need deep, connected mathematical knowledge to access and transfer to new and complex problems. NCTM (1989, 2000) presents a framework of Process Standards through which students must engage to learn math with understanding. The Common Core State Standards for Mathematics, CCSSM (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), incorporates and expands on the NCTM Process Standards in their Standards for Mathematical Practice. Both documents highlight the goals of developing proficiency in problem solving and procedural fluency with conceptual understanding.

NCTM Process Standards

- problem solving
- reasoning and proof
- representation
- communication
- connections

CCSSM Standards for Mathematical Practice

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

The guiding principles and essential practices we share with you in this book align with both the NCTM Process Standards and the CCSSM Standards for Mathematical Practice.

Successful mathematics students also demonstrate the dispositions of effective problem solvers: curiosity, flexibility, persistence, risk taking, and reflection. Such students are curious about patterns and relationships, question why numbers and operations behave the way they do, explore the underlying mathematical structure of problems, and wonder what conjectures they can make. They are flexible as they manipulate numbers in problems to make them easier to solve or as they consider

other ways to solve problems. As they grapple with problems, they persist in understanding them, solving them, proving their solutions correct, and seeking to make new connections and generate new problems. They take risks in inventing strategies or considering new strategies or ideas. As they problem solve, successful math students continually reflect on and monitor their thinking and work, asking themselves, "Does this make sense?"

To support math learning, classrooms must be mathematical communities where students work together to solve problems, raise questions, invent procedures, explore patterns and relationships, formulate proofs, and justify their solutions and ideas through classroom discourse. Together with students, teachers create a safe, supportive environment where important mathematical ideas emerge, are debated, and are investigated.

This book presents both the challenges and the opportunities inherent in developing mathematical thinkers. This book will help you transform the challenges into opportunities for rich learning. You might already be asking yourself these questions:

- How can I differentiate my teaching to meet the diverse needs of my students?
- What assessments best advance student learning?
- How can students understand math if I do not show them different strategies and tell them about the underlying mathematical structures and properties?
- What advice do I give parents about how to support their children's math learning?

These questions and more are what beginning math teachers should ask themselves and their colleagues as they revise and refine their practices. This book suggests ways to address such questions as you support your students' development of mathematical ideas.

This is not a book of math activities. Instead, it focuses on how students learn math and on pedagogy. Vignettes, all based on real classroom discussions, illustrate teaching practices to support math learning. Chapters about the lesson cycle come from conversations teachers had when they planned, enacted, and reflected on a lesson. These vignettes model effective mathematics teaching and learning.

Start by taking small steps to improve your teaching, gradually working toward skillful practice. As you read each chapter, try one or two ideas. With more experience, you can return to the chapters to clarify your thinking, delve more deeply, and refine your teaching by trying out more complex ideas. As you revisit this book, we hope that it will support you to make deeper, more meaningful connections between your teaching experience and our model of math practice.

The book comprises four sections. Section I (Mathematics Teaching and Learning) envisions what mathematics learning and teaching look like and the fundamental underpinnings of best practices. Chapter 1 consists of vignettes from a grade 2 classroom followed by important aspects of teaching and learning. Students engage in problem solving with peers, and their teacher facilitates a class discussion in which students analyze different solutions. Chapter 2 describes how children learn math with understanding, and presents guiding principles and brain-engaging practices to maximize math learning. Understanding how students learn—by developing neural connections—can help you appreciate the vitality of the practices that this book, NCTM, and CCSSM recommend. In later chapters you will find "essential practices" that are aligned with the guiding principles and brain engaging practices, but are more specific to the topic of each chapter.

Section II (Laying the Groundwork) focuses on work you can do either before the school year starts or within the first few months to make successful math learning more likely. Chapter 3 identifies important first steps, such as developing relationships with colleagues and establishing support networks in your school. We also explain how you can learn to use your curriculum materials and how to establish productive relationships with families. Chapter 4 offers ideas about classroom arrangements (and sample floor plans) to maximize math learning through class discussions, student access to materials, and group problem solving. Chapter 5 discusses emotional safety and its influence on complex thinking. Learn to establish a supportive math community where students help each other do their best thinking, work, and problem solving. Chapter 6 describes management routines that support math learning, presenting a metacognitive problem-solving routine vital to learning. Chapter 7 describes computational fluency, a key learning goal. The chapter also explains instructional routines to build computational fluency and shows how to engage students in generalizing about underlying mathematical ideas and properties.

Section III (The Lesson Cycle) illustrates through vignettes how two fourth-grade teachers plan, enact, and reflect on a lesson. Chapter 8 describes tasks most likely to yield significant learning and explains how to make curriculum tasks more mathematically engaging. Chapter 9 shows how these teachers adapt a well-known framework to plan a division lesson and how they document their planning to guide enactment and reflection. Chapter 10 illustrates how purposeful planning enabled easier, more effective decisions during teaching. One teacher introduces the lesson, and we see how she and her colleague interact with students as they solve problems. The teacher engages students in sharing their strategies and thinking in a class discussion at the end of the lesson. Chapter 11 shows how the teacher and her colleague reflect on the lesson together and document their reflections. They look for evidence of student learning and analyze student work to identify next instructional steps for students and ways to improve the lesson and the teaching.

Section IV (Essential Elements of Effective Mathematics Teaching) covers classroom discourse, assessment, differentiation, and homework—including background on why each element is important, suggestions to get started, and ways to reflect and improve. Chapter 12 describes productive discourse in classrooms and suggests questions you can ask to elicit student thinking. Learn to foster broad participation in rich discussions, how to make the math thinking visual, and how to engage students in math discussions. Chapter 13 explains how effective assessment starts with setting and communicating clear goals and expectations for learning. We explore assessment, giving students descriptive feedback, and engaging students in assessing their own work and thinking. Chapter 14 introduces differentiation as a necessary practice to ensure classroom equity. We describe how to differentiate instruction, emphasizing ways to engage English language learners and struggling students in developing mathematical thinking. Methods of teaching English language learners and struggling learners are fundamentally similar to ways of effectively teaching all students to learn mathematics with understanding. Chapter 15 explains the purposes and types of homework. We suggest how to ensure that homework is purposeful in supporting student learning and offer ideas to differentiate homework.

more4U

Supporting chapters dealing with technology and family engagement are available at this book's More4U website (look on the title page for your access code).

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Just as students need time to develop mathematical understanding, teachers need time to develop expertise in teaching. Becoming an effective teacher is a process, not an event. As with any worthwhile pursuit, you will feel challenged, inspired, and sometimes confused. This is to be expected and even embraced—this is what learning is about. Draw on the same dispositions you want your students to exhibit: curiosity, flexibility, persistence, risk taking, and reflection. We hope that you find this book a supportive and reassuring companion that you can return to throughout your journey.