

Preface

Teaching is a complex endeavor. The knowledge base behind mathematics teaching includes the knowledge of mathematics, of connections among mathematical ideas, of students, of students' learning, of school culture, and much more. The process of teaching involves creating a learning community, challenging students to make sense of mathematical ideas, and supporting students' developing understanding. The teaching process involves a myriad of decisions, of which just a few are "When do we tell?" "When do we ask more questions?" and "When do we merely encourage more thinking?" Consequently, it is not surprising that learning to teach well is a career-long endeavor! With roots in preservice education, it is sustained throughout one's career in an ongoing process of learning what students understand, how they understand it, and what learning activities most effectively support meaningful and useful understanding. This yearbook is organized around three aspects of teaching: foundations for teaching, the enactment of teaching, and the support of teaching nurtured in preservice education and strengthened throughout a teacher's career.

Part 1 provides perspectives on a few of the many underlying foundations. Carol Malloy begins by addressing an overarching concern across the entire teaching spectrum: equity—the concept of significant mathematics being learned meaningfully by all students. She shares research and conceptual frameworks that ultimately converge into a vision of excellent teaching, effective for all students. Jennifer M. Bay-Williams, Elizabeth M. Skipper, and Susan K. Eddins argue for the value of understanding entire trajectories of mathematical ideas, from primary through high school, when designing instruction at any level. Curriculum coherence and understanding where students have been and where they are going are essential for effective teaching and learning. Within the frame of curriculum and materials, Fran Arbaugh and Catherine A. Brown ask, "What Makes a Mathematical Task Worthwhile?" If we know that worthwhile tasks are crucial learning tools, then what are their characteristics? Having criteria by which to recognize or create such tasks is essential in teachers' work. Arbaugh and Brown offer suggestions. M. Kathleen Heid, Rose Mary Zbiek, and Glendon W. Blume note that when technology plays a role in instruction, teachers' mathematical foundations may need to be deepened. These authors make specific suggestions for the study of functions. Finally, Carne (Barnett) Clarke and Alma Ramirez high-

light the difficulties inherent in students' use of language: students' construction of meanings and their use of language are tightly interwoven. Clarke and Ramirez identify potential pitfalls and suggest pathways around them.

Part 2 addresses the enactment of teaching. Along with colleagues, Barbara and David Clarke have done extensive research on primary school students' learning. Where learning was exceptionally strong, they and fellow researchers looked closely at the children's teachers. As a result they have been able to identify several characteristics associated with effective teaching. Also at the primary school level, Aki Murata, Naoyuki Otani, Nobuaki Hattori, and Karen C. Fuson provide a detailed picture of a Japanese classroom taught in the United States. Through this example they illustrate how teachers communicate their valuing of all students' responses while at the same time moving students toward specific instructional goals. Margaret Smith, in a similar vein, draws from the video data of the Third International Mathematics and Science Study (TIMSS) and contrasts elements of teaching algebra lessons typical of the United States and Japan. Debra I. Johanning and Teri Keusch provide images from a specific middle grades fractions lesson that suggest how classroom learning communities can be built. Moving beyond classrooms, Phyllis Whitin suggests extending learning communities into homes. She gives specific suggestions and guidelines for involving family members in thinking together with children about intriguing mathematical ideas.

Although problem solving has been a centerpiece of mathematics instruction for many decades, the role of problem posing has gotten less attention. David J. Whitin helps remedy this situation by illustrating cases of problem posing at many instructional levels and by highlighting the instructional benefits of encouraging students to pose, as well as solve, problems. Joan Cohen Jones's article closes Part 2 by revisiting the issue of equity with an array of ideas drawn from a variety of classrooms and sources.

Teachers think regularly about supporting students' learning. Less often do teachers or the public think about supporting teachers' learning. Part 3 addresses this crucial element in building effective teaching systems: the career-long support of teaching. The first three articles address teachers' content knowledge. Charlene E. Beckmann, Pamela J. Wells, John Gabrosek, Esther M. H. Billings, Edward F. Aboufadel, Phyllis Curtiss, William Dickinson, David Austin, and Alverna Champion represent a team of mathematicians and mathematics educators who have worked together to strengthen mathematics courses taken by many undergraduates including future teachers. Their approach has involved adapting and extending *Standards*-based grades K–12 curricular activities to the college classroom. They provide examples from several courses where the Reasoning strand is highlighted. At the secondary school level, Irene Bloom shares how challenging problems advance prospective teachers' mathematical understanding as they make

connections within and across mathematics strands. Similarly, Kathleen Cramer engages in-service elementary school teachers through an array of problem-solving activities focused on big ideas. Her work reveals factors that reduce anxiety and build confidence in doing mathematics.

The next several articles concentrate on knowledge where teaching strategies and mathematical content knowledge intersect. Theresa J. Grant and Kate Kline have done a remarkable job of building trust among in-service elementary school teachers learning to use reform curricular materials. Participating teachers willingly allow themselves to be videotaped and to have colleagues observe and analyze their lessons. Together, colleagues learn to deal sensitively and thoughtfully with many vital decisions in teaching. Lynn C. Hart, Deborah Najee-ullah, and Karen Schultz, too, have built trusting relationships with teachers, in part by opening their own work to others through modelling a number of strategies including critical self-reflections. As a result, teachers are more willing to analyze objectively their own teaching and that of colleagues. P. Mark Taylor's research indicates that when teachers work together within schools, features of administrative support and the culture of the school may support or hinder their collaboration. Related to issues of school culture, the article by Patricia A. Jaberg, Cheryl A. Lubinski, and Sigrid Aeschleman highlights aspects of the principal's role in giving support for teachers' professional development.

Thinking carefully about assessment as another avenue for professional development is illustrated in the article by Sandra Wilcox and Elizabeth M. Jones. A sample problem from the Balanced Assessment project is shared with multiple implications for teachers' reflection and for students' activities. Professional development related to technology poses its own challenges. Karen Hollebrands and Rose Mary Zbiek furnish a road map for the stages of professional growth related to the integration of technology into teaching. Finally, Janet Warfield and Cheryl Lubinski delve into the realm of supporting teachers as they begin to assume leadership roles with colleagues. Their work poses a crucial question, "If we believe students learn well by struggling with challenges, then how do we extend that philosophy to our work with new teacher leaders?"

Throughout, authors have shared beliefs, insights, experiences, and findings that inform, intrigue, and expand our perspectives of mathematics teaching. As I look back at the articles and the Teaching Principle of the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM] 2000), the original impetus for this yearbook, I hear many of the messages of that Principle echoed in the authors' voices.

"To be effective, teachers must know and understand deeply the mathematics they are teaching" (NCTM 2000, p. 17) is the foundation for the articles by Beckmann and colleagues; Bloom; Cramer; and Heid, Zbiek, and Blume.

“Effective mathematics teaching requires a serious commitment to the development of students’ understanding of mathematics.... Teaching mathematics well involves creating, enriching, maintaining, and adapting instruction to move toward mathematical goals, capture and sustain interest, and engage students in building mathematical understanding” (NCTM 2000, p. 18). For more details, see the articles by Barbara and David Clarke; Murata and colleagues; Phyllis Whitin; and Bay-Williams, Skipper, and Eddins.

“Effective teaching requires a challenging and supportive classroom learning environment.... Effective teaching conveys a belief that each student can and is expected to understand mathematics and that each will be supported in his or her efforts to accomplish this goal” (NCTM 2000, p. 18). This statement is the heart behind the articles by Malloy and by Joan Cohen Jones.

“Are students’ discussion and collaboration encouraged? Are students expected to justify their thinking? If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others’ arguments, then creating an environment that fosters these kinds of activities is essential” (NCTM 2000, p. 18). Scenes with these features are found in the articles by Johanning and by Smith.

“In effective teaching, worthwhile mathematical tasks are used to introduce important mathematical ideas and to engage and challenge students intellectually. Well-chosen tasks can pique students’ curiosity and draw them into mathematics” (NCTM 2000, p. 18). Arbaugh and Brown elaborate on what constitutes a worthwhile mathematical task.

“Effective teaching involves observing students, listening carefully to their ideas and explanations, and using the information to make instructional decisions” (NCTM 2000, p. 19). These and related issues of communication are discussed by Clarke and Ramirez.

“Opportunities to reflect on and refine instructional practice—during class and outside class, alone and with others—are crucial in the vision of school mathematics outlined in *Principles and Standards*. To improve their mathematics instruction, teachers must be able to analyze what they and their students are doing and consider how those actions are affecting students’ learning” (NCTM 2000, p. 19). Reflection is a major component in the work of Grant and Kline and that of Hart, Najee-ullah, and Schultz.

“Collaborating with colleagues regularly to observe, analyze, and discuss teaching and students’ thinking ... is a powerful, yet neglected, form of professional development in American schools (Stigler and Hiebert 1999)” (NCTM 2000, p. 19). Hollebrands and Zbiek, Warfield and Lubinski, and Taylor address collaboration.

“Using a variety of strategies, teachers should monitor students’ capacity and inclination to analyze situations, frame and solve problems, and make sense of mathematical concepts and procedures. They can use this information to assess their students’ progress and to appraise how well the mathe-

mathematical tasks, student discourse, and classroom environment are interacting to foster students' learning" (NCTM 2000, p. 19). See David Whitin for having students "frame" problems and Wilcox for the uses of assessment to help make instructional decisions.

"The work and time of teachers must be structured to allow and support professional development that will benefit them and their students" (NCTM 2000, p. 19). The principal's role, one aspect of structuring the work of teachers, is discussed by Jaberg, Lubinski, and Aeschleman.

Accompanying this yearbook is a companion professional development handbook. The handbook offers teachers and teacher educators guidance in bringing to life issues and ideas from this yearbook in teacher education activities.

The work of creating this book was succored and guided by the thoughtful and enthusiastic work of a hard-working and caring panel:

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Edna Vasquez, *Southfield High School, Southfield, Michigan*

George Bright, the general editor for the three NCTM yearbooks published in 2002–2004, was a special source of insight and encouragement.

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Rheta N. Rubenstein
Sixty-sixth Yearbook Editor

REFERENCES

- National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.
- Stigler, James W., and James Hiebert. *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*. New York: Free Press, 1999. Quoted in National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics* (Reston, Va.: NCTM, 2000), p. 19.