

Isn't MATH

THE UNIVERSITY OF ARIZONA



CIRCLES AND ELLIPSES are everywhere, even on the logo of Grand Central Terminal in New York City. In Vanderbilt Hall, formerly the Waiting Room and now used for cultural and commercial events, five majestic gold chandeliers of seemingly concentric circles hang from the ceiling. A photograph taken directly beneath one of these chandeliers makes the circles appear concentric, but the circles actually lie in parallel planes, with the largest circle closest to the ceiling and the smallest closest to the floor (see inset below). The three innermost circles have light bulbs that are equally spaced. The number of bulbs in each of these rings follows an arithmetic sequence: 8, 24, and 40. However, the fourth ring has 48 bulbs instead of 56, resulting in bulbs that are slightly more widely spaced.



CHANDELIER, VANDERBILT HALL
INSET: CHANDELIER, VANDERBILT HALL, VIEW FROM BELOW



LOWER LEVEL WINDOWS, GRAND CENTRAL



WHISPERING GALLERY, GRAND CENTRAL

- 1 Assume that the circles are coplanar. Let the circumference of each of the three innermost circles be measured in "light bulb units," or lbu. Thus, the smallest circle has a circumference of 8 lbu.
- (a) In terms of lbu, **FIND** the radii of the three innermost circles.
- (b) Assume that the distance between all the circles is the same but that the number of bulbs in the outermost ring is 48 instead of 56. **WHAT** is the distance, in terms of lbu, along the arc between two consecutive bulbs on this circle?

- 2 As shown in the photograph above of the Vanderbilt Hall chandelier, the concentric circles are not coplanar but lie in parallel planes. Let the distance between the

parallel planes be 4 lbu.

- (a) **SHOW** that if the chandelier were a solid, it would be a frustum of a cone.
- (b) In terms of lbu, what is the volume of the frustum?

- 3 Let the circumference of the smallest horizontal ring of lights be 16 lbu and that of the largest horizontal ring of lights be 44 lbu. Let the distance between each horizontal ring of lights be 3 lbu. **WHAT** would be the circumferences of the other rings if the vertical curves were elliptical?

- 4 The lighting design on the food court ceiling (see photograph, right) has circles within circles. Within two of these large concentric circles, smaller circles

alternate with rectangles. Within two other concentric circles, some circles are nearly tangent to one another.

If we are given two concentric circles, **CAN** the ring bounded by the circles always be filled with a chain of tangent smaller circles? **FIND** an expression for the ratio of the radii of the two concentric circles so that the tangent small circles fit. See figure 1 to help find the conditions under which the chain is possible.

- 5 On either side of the entrance to the Oyster Bar and Restaurant, on a lower level of the terminal, are windows that, taken as a whole, seem to be in the form of an ellipse (see photograph, left:

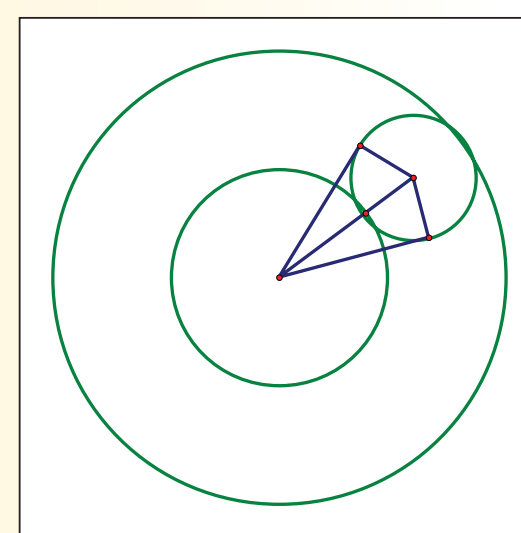
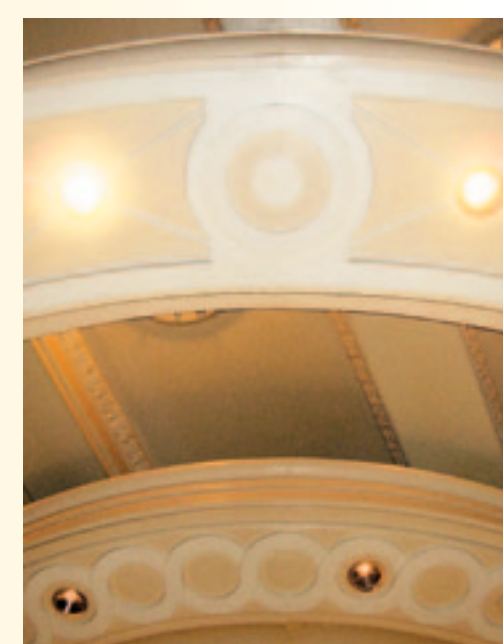


Fig. 1 Use the diagram to determine the ratio of the radii of the two circles.

Lower Level Windows, Grand Central). By superimposing an appropriate grid and making measurements along the vertical bars of the windows, **WRITE** an equation for the ellipse or show that the curve is not an ellipse.

- 6 Outside the entrance to the Oyster Bar and Restaurant is the Whispering Gallery (photograph, above left), which has low ceramic arches. Positioned at diagonally opposite corners of this gallery, two people—facing the wall—can hear each other as they whisper. Both the curvature of the ceiling and the nature of the acoustic tiles account for this phenomenon.

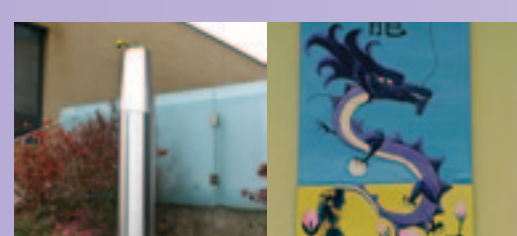


Food Court Ceiling, Grand Central

Assume that the curvature of the gallery is elliptical. **WHERE** do the two people need to stand to hear each other whispering?



M A T H



I S



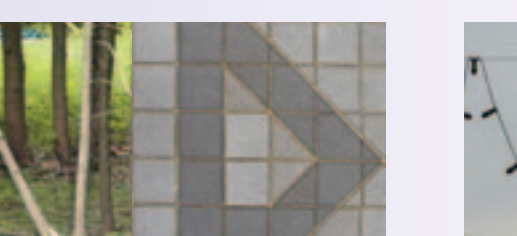
A L L



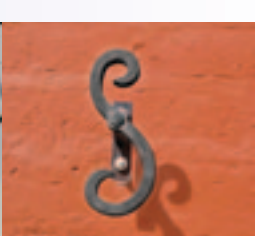
A R O



U N D



U S



S