

# Introduction

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It's likely that when you were in elementary school learning math, your teachers emphasized knowing the standard procedures necessary to get the right answer. Today mathematics educators are concerned with creating opportunities for students to learn mathematics with understanding. Why do teachers concern themselves with "understanding"? It is not enough to know how to do something. The real power of mathematics is achieved by knowing both how and why. For example, when you were in elementary school, you probably learned how to divide decimals using a set of procedures (called an algorithm) to perform the division. Knowing how to follow those procedures to correctly divide decimals was an important skill to develop. In real-life terms, it has likely contributed to the way you quickly decide which size container or brand to purchase at the grocery store. And that's just one small instance of how solving problems and deriving correct answers is important in everyday living. Being able to explain why the algorithm works is just as important. When students learn to explain why certain procedures work, they are better prepared to apply what they know when they face new mathematical situations. And this ability, enabled by understanding, can carry over to confronting the ever-changing conditions of life, too.

## Research and Good Mathematics Teaching

Alan Schoenfeld, a professor at the University of California, Berkeley, has spent decades conducting research on students' mathematical thinking. He has found that students' ability to solve problems is jeopardized when their opportunities for learning focus solely on memorizing procedures. But you might be wondering, "Is it reasonable to expect my child to explain why algorithms work?" or perhaps, "Isn't it the teacher's job to explain why things work?" Yes, it is the teacher's responsibility to support students in understanding why things work, and teachers regularly explain and clarify ideas

to do this. However, research indicates that students learn mathematics better when given opportunities to talk about and explain mathematical ideas. By “better” we mean children retain their knowledge instead of forgetting it right after taking a test. They can apply what they know in other mathematical situations, develop critical thinking skills, and be motivated to learn more. Focusing on understanding is simply the right thing to do for students.

Current methods for teaching mathematics are informed by the huge body of research on students' mathematical thinking that has accumulated over the last fifty years. Researchers have investigated many aspects of learning mathematics, including the evolution of counting strategies, operations with numbers, fraction and decimal ideas, reasoning with data, and so on. One result of this work has been the development of standards for teaching mathematics, such as the *Common Core State Standards for Mathematics*, published in 2010, and the National Council of Teachers of Mathematics's *Principles and Standards for School Mathematics*, published in 2000. It is research on student thinking that shapes these standards, which outline what mathematical teaching and content is appropriate for different grade levels. A more recent development guided by research is a series of papers, called progressions documents, which describe how student ideas on a particular topic, such as multiplication, develop over time (<http://ime.math.arizona.edu/progressions/>). In “Taking a Closer Look at Connections” (below), we continue this conversation by looking at how students' ideas about numbers develop and become more sophisticated during the elementary school years, illustrating this point with examples of student work from various grades.

Another large body of research examines how different teaching methods and practices affect students' opportunities for learning, including the following:

- ◆ The kinds of questions that teachers ask
- ◆ The use of problems that involve real-world situations
- ◆ Different ways to structure collaborative groups
- ◆ How to manage classroom discussions
- ◆ Effective use of hands-on materials such as base-ten blocks and other tools like calculators

What researchers have discovered is that good teachers regularly pose challenging yet accessible problems that can be solved in several ways. They do not assume that students necessarily understand all that is presented in class (either by the teacher or other students), even when most students seem to be following along. Rather, good teachers make it a habit to ask students questions, encouraging them to share their ideas and ask their own questions. Thus, good teaching is largely about creating a classroom environment (or culture) where students are comfortable sharing their ideas and asking questions, and where students know that learning to understand mathematics requires wrestling with tough questions. Good teachers know that learning mathematics is more

than “getting the right answer” to a problem; it is about the process of making sense of problems and seeing one’s ideas fit with the ideas of others. The best teachers know how to create a classroom environment that makes it possible for students to do so.

## The Four Principles to Understanding Children’s Mathematical Worlds

To help your children be successful math learners, there are four principles to embrace and accept as you take this journey into and through their mathematical worlds:

1. *Children do impressive mathematics. Adults, parents and teachers, must realize it is essential to understand the different strategies children might use to reach a solution. By doing so, adults can grasp how children think about and approach mathematics, and be in a better position to support children’s learning.* Children can reason about ideas, invent methods for solving problems, explain their thoughts, and so on. They also can use pictures and other models to think about and to represent many ideas. Although they may not solve problems or reason about ideas the way adults do, they have insightful ways to engage in mathematics. Children not only develop well-constructed ideas but also build new connections among those ideas all the time. Children are capable of vividly and accurately explaining their thinking about concepts and giving mathematical reasons for why they chose the solution approaches they did to solve a problem. Even if their answers are incorrect, the solution might be a reasonable estimate and, more critically, it might show their understanding of important underlying mathematical concepts.
2. *Parents must be aware of the expectations for students in the 21st-century mathematics classroom.* Ask most parents what they want for their children in school, and the answer may be some variation of “Do well” or “Get good grades.” But what does a child actually have to know and do in order to achieve such goals? Because a highly technological and constantly evolving culture like ours demands flexibility as an attitude and a skill, stating rules by rote and mindlessly following procedures to solve problems will not be adequate for future success in life—or figuring out the latest electronic game. The modern math classroom requires children to be mathematically flexible enough to think through problems to find a solution. And to be flexible in math, children must understand the procedures that they use and the concepts behind those procedures. A familiar example may help. You were most likely taught to solve problems such as  $1\frac{1}{2} \div \frac{3}{4}$  by inverting the divisor— $\frac{4}{3}$ —and then multiplying the two fractions to arrive at  $\frac{12}{6}$ , or 2. You probably have no difficulty solving this type of problem correctly. However, what does the answer of 2 tell you? It is larger than either of the

two numbers you started with. How can that be? In today's classroom, children are expected to explain why this procedure works and what the numbers in the problem mean. While there are some very sophisticated mathematical ideas behind this procedure, such as understanding how multiplication and division are related, children can explain it in their own terms if they've seen and experienced such explanations in their classrooms. Part of the process of helping your children is to observe how they solve problems and note what they seem to understand.

3. *The mathematics classroom children experience today operates very differently from the one their parents remember.* For instance, when you visit your child's classroom, you notice that the students are engaged in activities unlike those you participated in during math class. Students often work together as teams to complete assignments. The teacher does not tell students how to solve problems; instead he or she asks students to solve problems with several classmates, to agree on a solution, and to be prepared to present their work to the whole class when it comes together for a discussion. For these exchanges, the teacher expects students to explain their solution methods and to give reasons why those methods make sense.

Whole-group discussions are significant in today's mathematics teaching and learning environment. These guided conversations give students a chance to hear about other classmates' thinking, to make connections among the various ideas, to refine them, and so on. This is just one of many arrangements and instructional strategies teachers use to manage discussions and to plan for and meet the learning objectives they have for their students. So while your children's math experiences may be very different from your own, they are designed to allow them to solve and reason about problems and to connect new ideas with those they already know. By understanding the new ways in which your children engage in mathematical learning, you will be better prepared to participate in that learning, from helping with homework to playing math games.

4. *Understanding mathematics requires children to make connections among mathematical ideas.* Children develop understanding of mathematics by recognizing how ideas are related to one another. For example, the way a child understands subtraction often involves thinking about addition because every subtraction problem can be stated in terms of addition; for instance, if you want to determine what you get when you subtract 4 from 9, you can think about what number added to 4 gives you 9. Your child is also encouraged to make connections among different representations—a crucial part of understanding mathematics. To solve problems involving decimals, for example, children must be able to represent decimals using base-ten

blocks, rectangular diagrams, and a number line as well as use formal decimal notation. Knowing how to move flexibly among these different representations is a key indicator of a student's understanding of place value, fractions, and number sense. Homework and math games are natural opportunities to talk about different mathematical ideas and how they are connected to each other.



## TAKING A CLOSER LOOK at Connections

Young children think about the same concepts that older children do, albeit those ideas become more refined as they progress through the elementary grades. For instance, children might explore ideas related to multiplication at various grade levels using different information to determine a solution. Consider the following problem:

*If there are 7 children in our room, how many eyes are in the room?*

Younger children might count each of their friends, pointing to each friend twice, to count the number of eyes. A first grader might draw the following solution to the problem:



Fig. 0.1. A possible first-grade solution for “How many eyes are in our room?”

Hank, a second grader, not only skip-counted by twos to find the number of eyes but provided an equation documenting his solution:



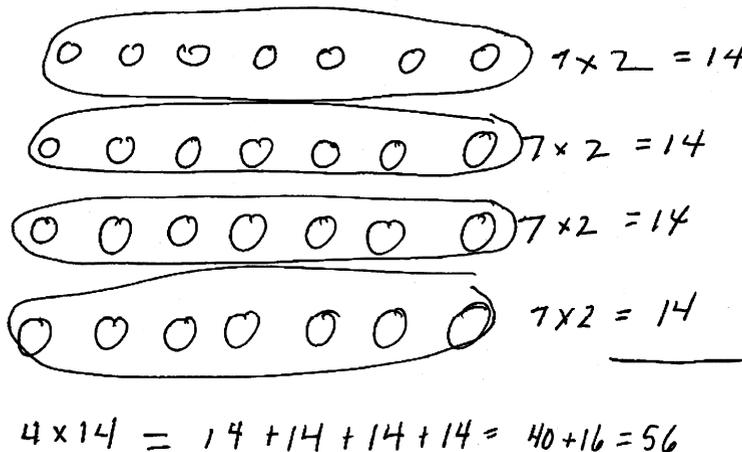
Fig. 0.2. A second grader's solution for “How many eyes are in our room?”

A third grader might explain that each child has 2 eyes and there are 7 people, so 7 groups of 2 eyes is 14 eyes and  $2 + 2 + 2 + 2 + 2 + 2 + 2 = 14$ .

Older children may use this information to solve a more challenging problem:

*How many eyes are in the room if there are 28 children?*

One child reasoned that there are 14 eyes in each row of 7 children because  $7 \times 2 = 14$ , and inasmuch as there are 4 rows of 14 eyes ( $14 + 14 + 14 + 14$ ), there are 4 groups  $\times$  14 eyes, equaling 56 eyes.



**Fig. 0.3.** A possible fourth-grade solution for the number of eyes of 28 people

In our examples, the younger children counted or added to find the total number of eyes. Hank, the second grader, made connections between addition and multiplication and began to explore multiplication. And finally in the third and fourth grades, children seemed to understand how they could think interchangeably about addition and multiplication to solve the problem.

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Remarkably, children across the grades consider many of the same relationships, but those ideas become increasingly advanced as they continue to make connections among those ideas. You will notice in the following chapters how similar concepts surface from time to time across the elementary grades and how these ideas become increasingly more challenging. It is vital for children's continuing success in math that they make sense of concepts early on so they can continue to build on those ideas to develop an understanding of the mathematics they are learning.

## Things to Remember

- ◆ The power of mathematics is knowing *why* as well as *how* a procedure works.
- ◆ Children do impressive mathematics. By understanding the different strategies they might use to reach a solution, adults can better grasp how children think about and approach mathematics.
- ◆ Parents must be aware of the expectations for students in today's mathematics classroom.
- ◆ The mathematics classroom children experience now operates very differently from the one their parents remember.
- ◆ Understanding mathematics requires children to make connections among mathematical ideas.
- ◆ Before reading on, think about the four principles. Do they make sense to you? Can you see how they work together to give you a window into your child's mathematical world? Do you recognize how they can be used to guide your own actions as you support your child's math learning?
- ◆ Think about how you make sense of and solve problems or make decisions at work and at home. Do you talk about the issue with other people to get ideas? Do you think about a similar situation that you have successfully resolved? Do you make a list or a diagram of various options? Noticing how you solve real-life problems can help you connect to the problem-solving strategies your children use to learn math.