CHAPTER 1

DEVELOPING DESIGN LITERACY TO SUPPORT A CULTURE OF MATHEMATICAL MODELING

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Mathematical models and modeling are one of the pillars of the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and many other college- and career-ready standards. The mathematical modeling cycle incorporates iteration, problem definition, product development, and testing and refining, and it is closely related to the design cycles used in engineering design and software development (Cirillo et al., 2016). Modeling requires a heavy emphasis on approximation and the analysis of a work in progress (prototype), also known as rough draft thinking (Jansen, 2020). Prototyping is the process of generating a test case or sample design to test an idea. Design can be seen as encompassing not only mathematical modeling but also prototyping as a means to generate solutions to meet the specifications of a problem or situation (Kolko, 2015).

Design literacy is connected to both the creation and understanding of artifacts and images in a broad sense and is not limited to only graphic design (Heller, 2004). The term *mathematical design literacy* can be explained as the connection between mathematical design thinking (Simon, 2020) and mathematical literacy (Organisation for Economic Co-operation and Development, 2022). *Mathematical design literacy* refers to an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve design problems that address challenges humans face. To get a better sense of how we can develop a culture that supports mathematical design thinking and modeling in the classroom, we can examine the ways design literacy encourages and helps students develop innovation and creativity. First, however, we need to understand the classroom as a workplace and build in opportunities for innovation and creativity.

In this chapter, we present three tasks intended to support the development of mathematical design literacy as students create and understand artifacts and phenomena that they have experienced firsthand and outside of the classroom.

TEACHER NOTE

Kolko (2015) identifies five principles that informed not only how we built our tasks but the classroom environment in which the tasks are assigned, completed, and assessed:

1. Create models to examine complex problems.
2. Focus on users' experiences, especially their emotional ones.
3. Tolerate failure.
4. Exhibit thoughtful restraint.
5. Use prototypes to explore potential solutions.

OVERVIEW OF THE TASKS

We include three tasks that may be completed as a progression or assigned individually. The first task, Burning Candles, offers a good introduction to prototyping and iteration (cycles of testing and revision). This task is built on an explicit problem, as students work to design and animate candles using specific linear functions and coordinate geometry. Although the functions are specified, students apply design thinking as they identify the geometries of the candle and flames, and models emerge looking quite different from one another based on student-led decision-making.

The second task, Logo Design, is a graphic design project in which students create works of art using transformed geometric objects and color. The dynamic geometry environment supports iteration and prototyping because students receive instantaneous feedback on their designs at each step. This task gives students experience with having regular failures when the mathematics does not match their intended artistic vision, but failure is not interpreted as the end result; instead, it is viewed as a necessary and useful part of the creative process.

The third task blends the first two, encompassing both the graphic design of Logo Design and the dynamic animation of Burning Candles. Relying on students' lived experiences with kaleidoscopes, the Kaleidoscope task provides an opportunity for students to explore a more complex phenomenon with opportunities for decision-making along the way. During this task, students will likely want to incorporate a wider range of geometric motions and objects, which encourages problem posing and problem solving en route to developing a final dynamic sketch (Harper & Cox, 2017). Thoughtful restraint is also important for this task, as students must choose which aspects of a kaleidoscope are most important for them to model.

There are two important principles on which we build a classroom culture that supports the development of design literacy and mathematical modeling. First, we provide adequate tools so that students can experience mathematics as something to be designed. Digital geometry environments (DGEs) provide an interface that allows students to engage with and mathematize real-world situations. Such technology can help students represent and model natural phenomena while the mathematical concepts are an explicit focal point.

Second, we make space for students to be human and present in the classroom by focusing on student relationships with mathematics, including considering students' emotions. Asking students to engage in mathematical design requires room within the classroom and school cultures for risk-taking and ambiguity. As students work through problems that emerge during the design process, their ambition and creativity may be hampered by feelings of risk, as many are conditioned to value a polished solution over the actual process of building a solution. Ambiguity about strategy and even purpose may make students uncomfortable in an academic setting, where we are usually evaluated on the production of solutions. The fear that a solution will not be found (and thus an assignment not completed) might impair our willingness to ask and pursue ambitious mathematical questions, create ambitious models for data or natural phenomena, or move beyond the boundaries of what we already know into an area that could offer profound insights. To help students overcome their fears, we must foster a culture tolerant of intellectual risk-taking (Jansen et al., 2016/2017) and tolerate failure.

These principles—to provide access to both adequate tools and a focus on students' relationships with mathematics—are important considerations in light of the other design principles, particularly because of the academic setting. Although it is important to create viable designs, we must also acknowledge that in the classroom context, the process of designing is as important, if not more important, than the final product. If our goal is for students to be able to move with dexterity or fluency within a solution space, then we must provide them with room to maneuver.

TASK 1.1: Burning Candles

Part 1: Using digital geometry environments, build a model of a candle melting at a constant rate of 12 mm every 20 seconds. Design or modify your model so the candle is controlled by a slider called "time" that can be animated. When the slider is animated, your candle should "melt" and report the amount that has burned.

Part 2: Using what you learned in the previous part, build a model of two different candles, P and Q, that are lit at the same time and burning at different, but constant, rates. For instance, when Candle P has burned 16 mm, Candle Q has burned 10 mm.

Part 3: Using what you have learned, model two candles *of different heights* that are lit at the same time and finish burning at the same time.

Digital geometry environments (DGEs) provide an atmosphere in which students can build mathematical representations of complex phenomena. Teachers can support mathematical design by emphasizing mathematics as an action and not a product. Students move back and forth between their experiential knowledge of the selected phenomenon and the growing mathematical representation in the DGE. The mathematics needed to create a valid representation within DGE varies, but it generally requires knowledge of geometric concepts to construct the candles and the functions to control the behavior of those geometric objects, as well as a willingness to engage in the modeling cycle of formulation, computation, interpretation, and validation. DGE is particularly useful in this context because it enables students to work with more precision in creating geometric objects and observe their dynamic behavior when linked to functions. This task supports every stage of the modeling cycle, but it particularly supports the work of validating the resulting model. More mathematics will be unpacked following an explanation of the task.

Learning Objectives

Students will be able to

- use geometric objects and linear functions to create an animation that depicts a melting candle;
- work fluidly with rates of change, including both increasing and decreasing linear functions; and
- interpret expressions for functions in terms of the candle's height and burn time.

STANDARDS ADDRESSED BY THIS TASK

Common Core State Standards for Mathematical Practice

- **Practice 2:** Reason abstractly and quantitatively.
- **Practice 4:** Model with mathematics.
- **Practice 5:** Use appropriate tools strategically.

Common Core State Standards for Mathematics Content Standards

- **HSG-MG.A:** Apply geometric concepts in modeling situations.
- **HSG-MG.A.3:** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
- **HSF-BF.A:** Build a function that models a relationship between two quantities.

> **HSF-LE.B.5:** Interpret expressions for functions in terms of the situation they model.

Source. National Governors Association Center for Best Practices (NGA Center) & Council of Chief State School Officers (CCSSO). (2010). *Common core state standards for mathematics*. https://learning.ccsso.org/wp-content/uploads/2022/11/ADA-Compliant-Math-Standards.pdf

Situating the Task

Burning Candles is a task that lives at the intersection of geometry and algebra and could be easily incorporated into a unit on linear functions or coordinate geometry. The task is broken into three sequential parts that can be incorporated into separate class periods or combined as needed. Each part of the task requires 20 to 40 minutes of student exploration and appropriate time to debrief each task as a class. Students will need to be familiar with DGE to complete the task. Specifically, they will need to be able to create and manipulate objects such as points, segments, and shapes and link those objects to a slider. They should also be able to create algebraic expressions that model the burning action of the candles. Students unfamiliar with DGE will need an additional day prior to beginning the task to familiarize themselves with the environment. In the student work shown in Figure 1.1, students used GeoGebra for their project; Desmos Geometry and Geometer's Sketchpad® are suitable alternatives. Specific skills needed to complete this task include graphing points with coordinates that are represented algebraically, connecting points with segments, and animating created objects.

To begin this task, students should have a preliminary understanding of linear functions and proportional reasoning, although that knowledge might be developing and could deepen by doing this task. For example, students will move between contextual and symbolic representations of linear relationships using geometric objects as a means to support a feedback cycle. That cycle will help students become more precise and understand the relationship between the graphical slope and y-intercept and the contextual meaning behind each.

Teachers must be familiar with the chosen DGE so they can help familiarize their students with the DGE and support students when they have questions about how to do something. Teachers do not need to wait until they are experts, however, as the task offers potential for learning alongside their students as they troubleshoot and create.

Implementing the Task

Launch

To build contextual knowledge, you may want to begin by inviting students to share their experiences with candles. Candles are important in many cultures and can be integral parts of life events and traditions. Students who have not seen a candle burn may benefit from a demonstration. A thin birthday candle might be the most appropriate choice because it burns quickly, demonstrating the phenomenon more effectively. You may also activate prior mathematical knowledge of quantity and measurement by asking students how they might measure the progress of a burning candle. Students may have ideas that the answer involves the volume of wax burned, the changing height of the candle, or even the height of the flame.

To provide clarity about the task and prepare students for exploration, you may introduce the tasks separately or as a cluster for students to explore. If you choose to introduce them one at a time, you will not need to do much of a launch for Parts 2 and 3 beyond sharing the scenarios.

Explore

For help anticipating student responses, see the section on student ways of thinking. As students work on the task, you can monitor students' actual responses to the task. Having students work in pairs or small groups encourages them to talk out loud, creating more observable thinking and work.

While students work on the tasks, it is important to encourage prototyping and iteration. Try asking the following questions:

- What shape might you start with?
- What happens when you drag your slider?
- How might you modify the sketch based on what happened?

It is also important to tolerate failure. You could say, "That definitely didn't work. What's next?" or "Why do you think it did that?" Focus on the students' experience as well with questions such as these:

- Did that surprise you?"
- How did that feel when the candle did that?
- I sense that you're frustrated. What is the problem you're trying to solve?

Summarize

As you move around the classroom, select and sequence work to be presented during the lesson summary. In this section, we provide some guidance for how to select interesting and useful work and how to sequence it during your summary.

There are two main reasons to debrief each part separately. First, to improve design literacy, it is beneficial to help students be more conscious of the iterative nature of their work and to draw attention to what did not work along the way.

TEACHER NOTE

The following questions might spur students to share their ideas:

- As you designed your candles, what challenges did you overcome?
- Was anyone surprised by something that did *not* work?
- Could anyone talk about a moment when you tested something and got an idea about what to try next?

Second, if there is wide variation in the ways students are constructing their candles in Part 1, it would be useful for them to share their work before beginning Part 2. Similarly, if there is variation in student success, it might be helpful for students to see how their classmates are linking the slider to the geometric construction.

TEACHER NOTE

Asking the following questions can help you jump-start discussion:

- Once you created your slider, how did you learn to connect it to the candle?
- What geometric objects did you use to model the candle? Did you customize it in any way?
- How did you verify that your candles were melting at the right rate?
- Did anyone's candle grow instead of melt? If so, why do you think that happened?
- What changed in your model when you wanted your candle to burn faster or slower?
- What else could we use these tools and this mathematics to model (e.g., the motion of an analog clock, runners in a race, a basketball shot)?

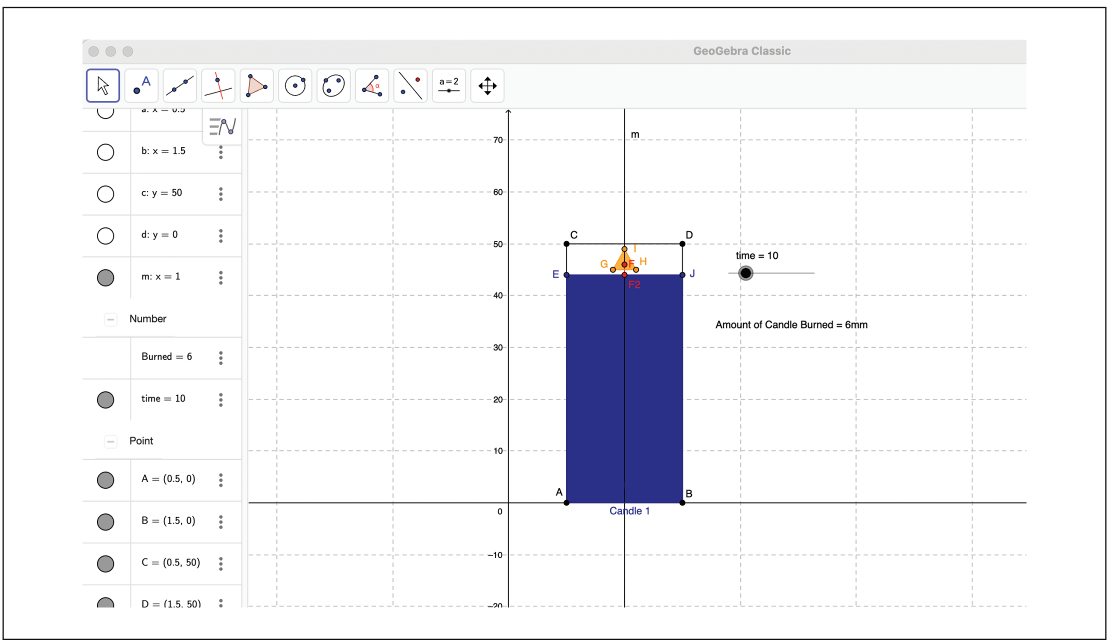
It is important to make connections between the work demonstrated by students: How do different groups solve the same design challenges? In fact, it is essential to connect the work back to the aspects of design literacy; the challenge is to match a well-known phenomenon. The mathematics of the task may be more familiar to you and your students than the design aspects of the work. Although it may be tempting to focus on just the mathematics in the discussion, you and your students can gain a lot by discussing a work in progress, failures, and iteration, as addressing these topics can strengthen students' design literacy. This task will help students become more comfortable sharing rough draft thinking in the future and more open to a growth mindset.

Similarly, you may feel that the mathematics is more easily covered in the Explore and Summarize phases than the design work. Students may need to stop working on the task before they feel like they have a completed design if they have chosen to pursue more ambitious features such as a flickering flame or 3D shapes, even if they have solved the central mathematical problem. Honor students' creativity and ambition in the classroom and find ways to encourage them to move beyond the constraints of the task and work on problems they identify.

Students' Ways of Thinking: Opportunities for Assessment and Differentiation

As the first milestone, students often begin work on Part 1 of this task by creating a rudimentary rectangle, cylinder, or even trapezoid by placing a shape or plotting points on the coordinate plane along with a slider. The shape can be any height and width. The second milestone is that students will need to algebraically link the y-coordinates of vertices C and D to the slider. In Figure 1.1, the student's end creation is a candle that is 50 mm tall, with point C having coordinates (0.5, 50 - .6 * *time*) and point D having coordinates (1.5, 50 - .6 * *time*). As the slider is moved to the right, the rectangle grows shorter and shorter.

Figure 1.1
A Sample Burning Candle



Once the melting mechanism has been constructed, students reach a third milestone and often feel empowered to expand their model to include a flame; some students who feel ambitious may attempt to create a flickering motion. The final milestone reached is that students find a way to represent the amount of the candle that has burned and is no longer visible. In Figure 1.1, the student visualizes this with a frame (rectangle *ABCD*) that does not change as the blue rectangle melts. Additionally, this student has added dynamic text to their sketch to report the amount of candle burned. Each of these four milestones provide opportunities to differentiate learning for students.

In Parts 2 and 3 of the task, students can use what they learned from Part 1 and build by adding additional candles to their sketch. It is a major milestone for small groups to realize that multiple candles can (and should) be linked to the same slider. The cognitive struggle that leads to this milestone is important, and teachers should resist the urge to overscaffold here and instead encourage iteration after failure. Try asking, "What did you learn?" and "What will you try next?"

Extending Students' Thinking

To extend the task for motivated students or those who are comfortable exploring your DGE unassisted, ask them to consider a scenario in which the candles are lit at different times. Emphasize that the candles should still be linked to the same slider, then have students use conditional thinking to show or hide objects within the GeoGebra Object Properties menu. There may be other strategies worth exploring in other DGEs as well, which you may want to explore alongside your students as a co-learner and cocreator. This exploration will build on the design literacy developed through the task without extending the mathematics further than the original assignment.

TASK 1.2: Logo Design

This task will allow your creative side to shine! Consider what your logo will represent. If you are not sure where to start, consider making a new logo for your favorite brand of shoes, clothes, fast food, or something else you love. Bring the brand to life with color and design!

Your logo needs to

- represent a company, activity, program, brand, or product; and
- use at least two transformations.