1 Introduction

Purpose of This Guide

Your first question when looking at NCTM's Curriculum Focal Points might be "How can I use NCTM's Focal Points with the local and state curriculum I am expected to teach?" The intent of this guide is to help instructional leaders and classroom teachers build focus into the curriculum that they are expected to teach through connecting related ideas and prioritizing topics of emphasis at each grade level. NCTM's Curriculum Focal Points documents are not intended to be a national curriculum but have been developed to help bring more consistency to mathematics curricula across the country. Collectively, they constitute a framework of how curriculum might be organized at each grade level, prekindergarten through grade 8. They are also intended to help bring about discussion within and across states and school districts about the important mathematical ideas to be taught at each grade level. Because of the current variation among states' curricula, the Curriculum Focal Points are not likely to match up perfectly with any state curriculum. This volume, a guide to the Focal Points for grade 2, explores the mathematics that is emphasized in a focused curriculum. Major aspects of the kindergarten Focal Points are summarized here to provide background for grade 2. See *Focus in Kindergarten* (NCTM 2010) for more details about the kindergarten Focal Points.

Purpose of Curriculum Focal Points

The mathematics curriculum in the United States has often been characterized as a "mile wide and an inch deep." Many topics are studied each year—often reviewing much that was covered in previous years—and little depth is added each time the topic is addressed. In contrast, higher performing countries tend to select a few fundamental topics each year and develop them in greater depth. In addition, because education has always been locally controlled in the United States, learning expectations can significantly differ by state and local school systems.

In the 1980s, the National Council of Teachers of Mathematics (NCTM) began the process of bringing about change to school mathematics programs, particularly with the first document to outline standards in mathematics, titled *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). This document provided major direction to states and school districts in developing their curricula. NCTM's *Principles and Standards for School Mathematics* (2000) further elaborated the ideas of the 1989 Standards, outlining learning expectations in the grade bands of pre-K–2, 3–5, 6–8, and 9–12. *Principles and Standards* also highlighted six principles, which included the Curriculum Principle, to offer guidance for developing mathematics programs. The Curriculum Principle emphasized the need to link with, and build on, mathematical ideas as students progress through the grades, deepening their mathematical knowledge over time.

A curriculum is more than a collection of activities: It must be coherent, focused on important mathematics, and well articulated across the grades.

—The Curriculum Principle, *Principles and Standards for School Mathematics*

NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (2006) is the next step in helping states and local districts refocus their curricula. It provides an example of a focused and coherent curriculum in prekindergarten through grade 8 by identifying the most important mathematical topics or "Focal Points" at each grade level. The Focal Points are not discrete topics to be taught and checked off, but rather a cluster of related knowledge, skills, and concepts. By organizing and prioritizing curriculum and instruction in grades pre-K–8 around Focal Points at each grade level, teachers can foster more cumulative learning of mathematics by students, and students' work in the later grades will build on and deepen what they learned in the earlier grades. Organizing mathematics content in this way will help ensure a solid mathematical foundation for high school mathematics and beyond.

Impact of Focal Points on Curriculum, Instruction, and Assessment

Significant improvement can be made in the areas of curriculum, instruction, and assessment by identifying Focal Points at each grade level. At the curriculum level, Focal Points will allow for more rigorous and in-depth study of important mathematics at each grade level. This rigor will translate to a more meaningful curriculum that students can understand and apply, thereby ensuring students' learning and increasing students' achievement. At the instructional level, Focal Points will allow teachers to more fully know the core topics they are responsible for teaching. Professional development can also be tailored to deepen teachers' knowledge of these Focal Points and connect these ideas in meaningful ways. Assessments can be designed that truly measure students' mastery of core topics rather than survey a broad range of disparate topics, thus allowing for closer monitoring of students' development. At the classroom assessment level, having a smaller number of essential topics will help teachers determine what their students have learned and provide sufficient time to ensure that these topics have been learned deeply enough to use and build on in subsequent years. If state assessments are more focused as well, more detailed information can be gathered for districts and schools on areas for improvement.

Using This Guide in Study Groups or Learning Communities

Many teachers tell us that they did not have an opportunity in school to build sufficient understanding of many topics that they now teach. Therefore our discussion of the grade 1 Focal Points is detailed enough for teachers to begin building such understanding. We suggest that teachers form study groups (such as those in lesson study, mathematics circles, or other learning communities) to read and discuss parts of this volume, to work together to build a deeper understanding of the Focal Points topics, and to plan how to develop

such understanding with students by adapting as needed their present grade 1 teaching and learning materials. A helpful approach for other teacher working groups has been to share students' insights and questions and to look at students' work to understand different ways that students are solving problems, to address errors, and to help move students forward in a learning path that fosters both understanding and fluency. Because teachers' lives are busy and demanding, they are better served by concentrating on small chunks of this volume at a time and working through them deeply rather than trying to do too much and getting discouraged. Teachers' learning, like students' learning, is a continuing process, but one that can be very rewarding.

Focal Points and the Common Core State Standards for Mathematics

The Common Core State Standards for Mathematics, released June 2, 2010, were the next step after the Focal Points. The Common Core State Standards (Common Core State Standards Initiative 2010) were the result of an intensive interactive writing and feedback process that included authors of the Focal Points document. These standards specify the focal point areas at each grade level along with additional standards to meet all math domains in kindergarten through grade 8. There is very strong agreement between the Curriculum Focal Points for kindergarten, grade 1, and grade 2 and the Common Core State Standards for these grades. The language differs in some places, and the Common Core State Standards are longer and more specific in some cases. But these are minor variations in a very coherent picture across these two documents.

One minor difference is that the Common Core State Standard for kindergarten on relating counting and cardinal numbers (K.CC.4) is in the prekindergarten rather than in the kindergarten Number Core table in NCTM's *Focus in Prekindergarten* and *Focus in Kindergarten* books. This is because prekindergarten children frequently understand this relationship (see the National Research Council report on early mathematical learning *Mathematics Learning in Early Childhood: Paths toward Excellence and Equity* (Cross, Woods, & Schweingruber, eds, 2009). This is a crucial relationship, and it is necessary in the Common Core State Standards for kindergarten so that students who have not learned this relationship earlier can do so. See the *Focus in Prekindergarten* or *Focus in Kindergarten* books for discussion of this standard and activities that can support the standard.

A second minor difference is that the Common Core State Standards are explicit about fluency with single-digit addition and subtraction for kindergarten and grade 1 (these are in Table 2.1):

K.OA. 5. Fluently add and subtract within 5.

1.OA.6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

The Focal Points are consistent with these standards, but these numbers were implicit rather than explicit in the Focal Points.

Table 1.2 at the end of this introduction contains all of the Grade 2 Common Core State Standards and the Grade 2 Curriculum Focal Points and the Connections to the Focal Points. These have all been rearranged to fit within the four critical areas in the introduction to the Common Core State Standards. The original Grade 2 Curriculum Focal Points are on the page preceding Table 1.2. The Common Core State Standards are longer and more specific about some concepts. A few standards are implicit in the Focal Points or go somewhat beyond them. These are all smaller standards that do not have major emphasis in grade 2; so they were not focal points but might be minor standards in a full grade-level standards document. These standards are on integrating understanding of lengths on rulers and on number line diagrams (2.MD 5 and 6), working with time and money (2.MD 7 and 8), and relating repeated measurements and a line plot (2.MD.9).

Because of the minor differences in language between the Common Core State Standards and the Curriculum Focal Points, the language of the math standards has been used for grade 2 in Tables 2.1 and 3.1. These tables also contain some extra explanatory language because the standards are so short.

The Common Core Sate Standards for Mathematical Practice are closely related to the NCTM Process Standards. These both are discussed in the final part of the book. We want to begin and end with a view of classrooms in which grade-appropriate mathematical reasoning is at the center of individual and classroom mathematical activity.

Bringing Focus into the Classroom: Classrooms That Build Understanding and Fluency

Students cannot build understanding in a classroom in which the teacher does all the talking and explaining. Pedagogical principles for classrooms that do help students build understanding are outlined in *Principles and Standards for School Mathematics* (NCTM 2000) and the National Research Council reports *Adding It Up* (Kilpatrick, Swafford, and Findell 2001) and *How Students Learn: Mathematics in the Classroom* (Donovan and Bransford 2005). A learning-path pedagogical perspective that coordinates the principles from these three sources is outlined in Fuson and Murata (2007). This approach also integrates understanding and fluency. In such an approach, teachers create a nurturing, meaning-making "math talk" community in which students discuss their mathematical thinking and help one another clarify their own thinking, understand and overcome errors, and describe the method they use to solve a problem. Teachers and students assist everyone's learning by coaching one another during such math talk and during problem solving if needed. Teachers and students model, structure and clarify, instruct or explain, question, and give feedback.

Using mathematical drawings

The use of mathematical drawings during problem solving and explaining of mathematical thinking helps listeners understand the thinking and the explanation of the speaker. The use of mathematical drawings during homework and classwork helps the teacher understand students' thinking and thus provides continual assessment to guide instruction as the teacher addresses issues that arise in such drawings (e.g., errors or interesting mathematical thinking). Mathematical drawings do not show situational details of the real object; such drawings should be done in art class, not in mathematics class. Mathematical drawings focus on the mathematically important features and relationships, such as the quantity and operations, and can use small circles or other simple shapes to show single-digit and multidigit quantities. These representations can evolve into schematic numerical drawings that show relations or operations. Throughout this volume, we use mathematical drawings that can be produced and understood by students.

Learning phases

The learning-path pedagogical perspective that integrates understanding and fluency has four phases for each new topic area. The phases begin by building understanding and then move to emphasizing fluency. For each new mathematics topic, teachers—

a) begin by eliciting students' thinking;

b) teach research-based mathematically desirable and accessible methods that reflect the standard algorithmic approach, discuss and repair errors, and ensure that standard approaches are discussed and related to methods that students understand;

c) help students achieve fluency with a general method while continuing to build relationships and understanding; and

d) continue cumulative practice occasionally all year so that students remember what they have learned.

Moving to mathematically desirable methods

Eliciting students' thinking when beginning each new topic is important so that the teacher can build on that thinking and modify and extend it as needed. The teacher needs to emphasize sense making by all participants through all four of the phases above. Although some students will develop fairly advanced methods, allowing too much time for students to "invent" methods can leave less-advanced students doing a primitive method that is slow and perhaps error prone for an extended period. Mathematically desirable methods that are generalizable to larger numbers and that use important mathematical aspects of the quantities involved (for example, hundreds, tens, and ones) need to be introduced if they have not arisen from other students or from the instructional program. These methods should also be accessible to students and build on their ways of thinking. We discuss such methods for the grade 1 Focal Points.

These methods enable everyone to use a method that they can understand and explain but that is also mathematically desirable.

The standard algorithmic approach

The Focal Points specify topics for which students should achieve fluency with the standard algorithm. By this phrase, mathematicians mean the *standard algorithmic approach* that involves certain basic steps and not the specific ways in which numerals are written to show these steps. So, for example, multidigit addition and subtraction involve two major steps: adding or subtracting like units (hundreds and hundreds, tens and tens, and ones and ones) and composing/decomposing a unit when needed (composing a new ten or new hundred from ten ones or ten tens when adding and decomposing a hundred or a ten to make ten tens or ten ones when subtracting). These concepts are discussed and exemplified in the section on multidigit adding and subtracting. Simpler and more complex ways to write this same standard algorithmic approach for addition and subtraction are presented later. Each way has disadvantages and advantages, and students can identify and discuss these.

Adding It Up (Kilpatrick, Swafford, and Findell 2001) clarified that in fact no such thing as *the* standard algorithm exists. Many different algorithms (systematic methods of repeated steps for carrying out a computation) have been used over time in the United States, and many different algorithms are used presently in other countries. Students from other countries may bring such written methods into a classroom in the United States. Students from the United States will bring the current common methods from home. All such methods need to be discussed and related to mathematical drawings or other quantities so that all methods can be understood. A student should be allowed to use any method that is mathematically desirable and that the student can explain. Mathematically desirable methods use the standard algorithmic approach and therefore meet any state goal that requires use of the standard algorithm (this phrase is just another way to say the standard algorithmic approach). Some mathematics programs suggest that students not use the standard algorithm because it often involves a complex way of writing steps, but this method will come from some homes and does need to be included in the class discussion. This view emphasizes that the steps and the meanings underlying the algorithm are the important features, and understanding these—and why they work—is a major focus of the work with the algorithm.

Conceptual prerequisites

Helping all students move rapidly to a mathematically desirable and accessible method requires that they have the conceptual prerequisites for such methods. The teacher may need to build in these prerequisites in advance before introducing the topic. We summarize important prerequisites for the grade 1 Focal Points here. See *Focus in Kindergarten* (NCTM 2010) for more details about these prerequisites that need to be developed in kindergarten.