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Improving Mathematics for All Students

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The relevance of mathematics education to engineers, physical scientists, and mathematicians has long been obvious. In the last twenty-five years, there has been a vast increase in the number of fields to which mathematics has been applied, accelerated since 1950 by the availability of electronic computers. Testimonies to mathematics are easily found in economics, biology, and business.[1]

> The use of mathematical language…is already desirable and will soon become inevitable. Without its help the further growth of business with its attendant complexity of organization will be retarded and perhaps halted. In the science of management, as in other sciences, mathematics has become a "condition of progress."[2]

There is no sign that this trend will diminish.

Our society has not yet adapted to today's many new roles for mathematics. Expertise—indeed, even minimal competence—in mathematics is still considered the province of a select and somewhat unusual few. For too many people, the requirement that they be mathematically literate is another threatening aspect of an increasingly threatening world. The extent to which this is true reflects the extent to which mathematics education for everyone must improve.

Inadequacies of the Present Situation

Current results from mathematics education are distressing. In spite of increased use of mathematics in everyday life, many students consider mathematics their least liked subject. Furthermore, large numbers of students terminate eight or nine years of school mathematics training with what amounts to a remedial course in computational arithmetic. They never reach even the first conceptual stage in mathematics—the use of algebraic symbols in problem solving.

Even students who take mathematics beyond arithmetic often have little competence with the mathematics studied. The content of textbooks written for prospective elementary school teachers indicates how little they are expected to have remembered from their days in school. Students who currently take four years of high school mathematics know more mathematics than students in former years. Yet even they (and many of their teachers) are often ignorant about the nature of mathematics and mathematical thinking.

Most educated nonmathematicians have respect for mathematics but little friendliness toward or sense of control over it. Many find it difficult to understand how anyone can enjoy mathematics, perhaps because the learning of mathematics is made such a serious, no-compromise business. It is so easy to identify mathematical mistakes that students are "nailed'' for every lapse. Hence, for all but the best students school mathematics experience is associated with much failure and few rewards.

To summarize, the present situation for large numbers of students seems to be characterized by (1) dislike of the subject, (2) lack of opportunities to become acquainted with any more than trivial foundations of the subject, (3) limited competence with even the meager amount of mathematics learned, (4) ignorance about the nature of mathematicians' work, (5) fear or puzzlement when understanding of mathematics is expected, and (6) memories of failure whenever the subject of mathematics is broached. It is no wonder that most people consider mathematics to be relevant to others (perhaps) but irrelevant to themselves (certainly).

What about the Reforms?

During the sixties, mathematics education was the object of considerable study and reworking by individuals and groups interested in its reform, but treatment of the problems noted above was not a primary goal of most such groups. Reform efforts generally succeeded in achieving limited but necessary objectives. At the secondary level, reform efforts were frankly aimed at "college capable" youngsters and courses were patterned to fit into existing school settings, resulting in greatly improved college preparatory courses. At the elementary school level, more varied, richer content was provided by adding work in algebra, geometry, and number theory to the computational arithmetic that had dominated the scene. All in all, much mathematical theory was added to the curriculum, including work with logic, sets, relations, other bases, and nonmetric geometry; to some extent, the curriculum was reorganized to emphasize these as fundamental ideas.

Efforts were not aimed solely at content. "Learning by discovery" was taken up by many groups. Programmed materials were popular in the early stages. More recently, computer-assisted instruction, use of calculating machines, and mathematics laboratories have become popular. Throughout these years, there has been increased use of all types of teaching aids.

Substantial work was done to change mathematics learning from memorizing large numbers of unrelated rules to understanding the nature of mathematics as a system developed from a reasonably small number of postulates. Accordingly, the amount of theory has been increased, with a resulting expansion in the degree of conceptualization required of students. Much of mathematics is theory, much of it conceptual, and such content belongs in the curriculum.

However, while mathematics curriculum is now better organized and structured, mathematical *experience* for many students is no more meaningful or enjoyable than it was in the past. To large numbers of students, the study of mathematics seems as irrelevant today as it seemed prior to reforms of the past decade. This feeling is compounded by students' fears, feelings of incompetence, and general ignorance of the nature of mathematics and mathematical applications.

Attempts have been made to create curriculum materials that interest students, focus on the nature of mathematics, and give strong play to mathematical applications. Today such materials do exist. Generally, however, these attempts have been confined to units or at most one year of a student's experience. The problem of relevance as it relates to the entire mathematics curriculum has not been given enough consideration.

Finally, one should note that mathematics learning may be even more susceptible to the influence of poor or uninformed teaching than most subjects, since failure to understand an important concept at any school level often has later harmful consequences. If so, reform may be impossible where elementary school teachers are poorly informed in mathematics.

Making Mathematics Teaching More Relevant

It is impossible to give a description of "relevant school mathematics" which would be endorsed by all members of the mathematical community. However, the authors feel that most mathematics educators would concur with the following suggestions for changes. While one cannot speak accurately about the content aspects of needed reforms in the mathematics curriculum without at the same time considering methodology, for clarity each will be discussed individually.

Content Aspects

1. *Greater emphasis on the uses of numbers.*—Arithmetic has always been and continues to be the mainstay of school mathematics. Yet understanding and control over the main uses of numbers often do not emerge from this training. The child is familiar with counting but not with indexing and ordering, two other important uses of integers. He should be introduced early to the various sorts of coordinate systems which are all around him. And, perhaps most important, he needs continued practice in selecting appropriate computational maneuvers for given situations. In today's mechanized world, recognizing the arithmetic processes to use in a given situation may be as important as ability to calculate an answer accurately.

2. *Continual tie-ins between mathematics and its applications, with particular focus on the nature of mathematical applications.*—The principal link between mathematics and its uses is the building and exploitation of mathematical models, which have given mathematics its extraordinary relevance to new fields in recent years. The essential idea is as follows: (a) Since any situation taken in its entirety is extremely complicated, one makes assumptions and decisions that result in a simpler representation of the situation. (b) One next fits mathematical tools to this representation. (c) One then works through the mathematics abstractly, without reference to the actual situation. (d) Finally, one tries out his answers in the original problem. Everyone should understand this process for his own use and to comprehend an increasingly technical and bewildering world. This idea needs to be emphasized and reemphasized throughout the child's school years.

3. *Emphasis on mathematics as a means of examining common features of dissimilar systems or situations.*—Much of mathematics deals with properties and structures common to apparently unrelated mathematical phenomena. For example, multiplication and addition are two totally different operations, yet share many structural properties. Similarly, the power of mathematics in real-world applications stems in part from the fact that the same mathematics may be used, in totally unrelated situations. For instance, division is used to find out what percentage x is of y; it is also the process by which one calculates the length of one side of a rectangle given its area and an adjacent side. The same formula can be a mathematical model for many different phenomena—the same concept can be useful in a variety of situations. The abstract ideas of functions, relations, and transformations also unify diverse phenomena, serving as useful tools for studying mathematics and its applications and also for conceptualizing nonmathematical situations.

4. *Focus on the nondefinite realms of mathematics: estimation, approximation, comparison, arbitrariness of definitions and postulates, etc.*—Much school instruction foolishly and compulsively demands the unique answer. Yet only rarely are exact answers required in applications of mathematics encountered by most people. More frequently, some reasonable estimate is needed along with a sense of what is an appropriate level of precision in a given situation. Many useful comparisons and estimations are made in terms of "order of magnitude"; indeed, an order-of-magnitude change in a situation sometimes leads to revolutions in thinking and procedure, not merely qualitative change.[3] Few people can consider alternatives and consequences when proposals involve millions or billions of dollars. Where this prevents thoughtful citizen consideration of public policy, there is cause for concern.

5. *Use of mathematical ideas to analyze nonmathematical arguments and situations; decision*

making.—The above example is one instance of the relationship between mathematics and reality which needs increased attention—the use of mathematics to analyze and gain information about non-mathematical situations. Studying these situations not only heightens student interest, it also serves as a foundation for more technical studies. It is a feature of mathematics that logical reasoning can lead to contradictory results when individuals begin with different assumptions. If Senator A believes in a monolithic communist design lo take over the world while Senator B believes that there are many forms of communism and that communist countries may differ with each other, then Senators A and B are going to arrive at different foreign-policy conclusions. Yet both senators may be reasoning in an entirely logical, rational manner. This analysis of decision making is mathematical in nature; it can and should be a primary focus of mathematics teaching.[4]

Methodological Aspects

1. *Individualization to accommodate student interests; student-generated problems.*—There is more opportunity for students to become actively involved in all aspects of mathematics when the content emphasizes decision making, applications, estimation, and approximation, as suggested above. This increased involvement would hopefully lead to greater student interest in standard and teacher-selected content, with concurrent increases in students' desires to use mathematics in their own spheres of interest. Mathematics has traditionally been taught as a closed discipline, "closed" in the sense that curriculum is determined from the outside and not adapted to students. Nothing would more effectively increase the relevance of mathematics for students than occasionally to "open" the mathematics course to serve their interests.

2. *Emphasis on activities which are not "pass-fail" or "right-wrong" and on problems which have many interpretations and answers.*—Research has long indicated that success is a powerful motivator. But in mathematics education, activities have often been organized so that failure is common and success is not. Emphasis on analyzing issues and applications, as suggested above, lends itself to projects, essay questions, many-faceted question-and-answer sessions, and individualization. In these activities, specific content and standard skills are needed and hopefully learned. For instance, an algebra class might be asked, "Given today's rate of population growth, approximately when will the United States have a population of one billion?" In a geometry class, "Why is it necessary to have undefined terms in a mathematical system?" In general mathematics or arithmetic, "What percentage of students in the school have their own phones?" (This question requires consideration of sampling and confidence intervals, ideas rarely seen in the curriculum even though sampling pervades applications.)

3. *Increased motivation through content not strictly mathematical.*—Throughout this article, extramathematical applications have been stressed. Such applications occasionally should be used as motivation before the relevant mathematics is studied. Historically, much of mathematics has arisen out of the need to solve particular nonmathematical problems, a thought sequence to which students should be introduced. Frequently, mathematics developed for its own sake later becomes precisely what is needed as a mathematical model to explain a real-world phenomenon. Such was the case with "imaginary numbers," now indispensable in electrical circuitry and many other applications, and with non-Euclidian geometry, which now serves as a model in relativity theory. Interactions between mathematics and its uses need to be illustrated in practice throughout a youngster's mathematics schooling.

4. *Greater use of outside-school resources and activities and teaching aids of all kinds.*—Mathematics could be brought closer to the student by: use of a mathematics laboratory with concrete

materials, calculators, or a computer; classes in which mathematics and science students work together; the survey or analysis of data for schools; and visits to local industries or from industry representatives. Mathematics laboratories deserve particular mention. We probably spend 10 percent of class time in testing. Should we not spend at least that amount of time providing experimental bases for mathematical insights? Some classes might work in a laboratory in ways similar to those used in biology, chemistry, or physics laboratories. There are as many uses for a calculator in mathematics as for a microscope in biology.

The authors teach in a secondary school and also have responsibility for observing other teachers. There is an appalling lack of any types of aids (other than the blackboard) in most of the classes observed in spite of the abundance of such materials on the market. Mathematics teachers and educational technologists have a joint responsibility to improve this situation. One possibility might be an exhibit building developed cooperatively by manufacturers of educational media to serve as a showplace for materials and developments in educational technology. This would serve industry and education in much the way the Merchandise Mart serves the home-furnishings industry.

5. *More use of games and recreational aspects of mathematics*.—Although many mathematics teachers love mathematical games, they seldom devote class time to such games. There is presently a wealth of material available which motivates the study of standard topics through games. Furthermore, there is much that some branches of mathematics have in common with games—reliance on objects (players), with particular rules and particular meanings for terms.

6. *Greater attention to problem-solving processes and procedures, including the nature of problem generation*.—Finally, greater attention should be given to the nature and process of problem solving, in contrast to the present, almost exclusive emphasis on answers. The student can and should have opportunities to create his own problems for mathematical solutions and to modify and temper them so as to ease discussion of them, to estimate a solution, refine the estimation, arrive at a possible solution, check the solution, and revise the problem or solution if necessary. This is the type of activity which life demands and which mathematics is well equipped to provide.

All the suggestions in this paper are directed toward first considering the ways mathematics is treated by mathematicians and those who apply mathematics and then adapting these findings to the classroom. The fact is that mathematics is potentially relevant to everyone. Our task is to develop the emphases and methodologies that will make mathematics both palatable and accessible to virtually all individuals.

Endnotes

1. William J. Baumol, "Mathematics in Economic Analysis," in *The Spirit and the Uses of the Mathematical Sciences* (New York: McGraw-Hill Book Co., 1969), p. 247; and Robert Kozen, "On Mathematics and Biology," ibid., p. 204.
2. Albert Battersby, *Mathematics in Management* (Baltimore: Pelican Books, 1966), p. 11.
3. R. W. Hamming, "Intellectual Implications of the Computer Revolution," *American Mathematical Monthly* 70 (January 1963): 4–11.
4. Like most suggestions in this article, this idea is by no means novel (see Harold Fawcett, *The Nature of Proof*, Thirteenth Yearbook of the National Council of Teachers of Mathematics [New York: Bureau of Publications, Teachers College, Columbia University, 1938]).