Differentiating the Common Core State Standards for Gifted and Advanced Students

All differentiation is based on an understanding of the characteristics of gifted and high-potential students *and* the content standards within a domain. The new Common Core State Standards provide an opportunity for the field of gifted education to examine its practices and align them more fully to the 2010 NAGC Pre-K–Grade 12 Gifted Programming Standards for curriculum, instruction, and assessment. For example, similar to the NAGC Gifted Programming Standards, the CCSS emphasize problem solving (see Evidence-Based Practices 3.4.1–3.4.4, NAGC, 2010, p. 10, and Standards for Mathematical Practice 1, NGA & CCSSO, 2010a, p. 6). Because the Gifted Programming Standards in curriculum require educators to engage in two major tasks in curriculum planning—alignment to standards in the content areas and the development of a scope and sequence—using the CCSS is a natural point of departure. The effort must occur in vertical planning teams within districts and states in order to increase the likelihood of consistency and coherence in the process. There are three major strategies that may be employed to accomplish the task for gifted education:

- *Provide pathways with appropriate pacing of the CCSS for gifted learners.* Some of the CCSS address higher level skills and concepts that should receive focus throughout the years of schooling, such as a major emphasis on reasoning and sense-making. However, there are also discrete skills that may be clustered across grade levels and compressed around higher level skills and concepts for more efficient mastery by gifted students. Teachers might use preassessments in determining which students require more accelerated pacing. For example, within the CCSS domain of Measurement and Data, some students in first grade might be estimating lengths in standard units, while others might be solving problems involving measurement and estimation of liquid, volumes, and/or masses of objects (grade 3) or converting like measurement units within a given measurement system (grade 5).
- *Provide examples of differentiated task demands to address specific standards.* Standards like problem solving in mathematics lend themselves to differentiated interpretation through demonstrating what a typical learner on grade level might be able to do at a given stage of development versus what a gifted learner might be able to do. The differentiated examples should show greater complexity and creativity using a more advanced curriculum base. In mathematics, whereas typical learners might solve multistep word problems using a variety of models and strategies throughout grades K–12, gifted learners might pose and solve new, related problems of their own at an earlier stage of development. Other degrees of differentiation may take place by adding complexity to the task and using enrichment techniques that address student needs and district demographics, such as using mathematical equations and modeling to solve community problems.
- *Create interdisciplinary product demands to elevate learning for gifted students and to efficiently address multiple standards at once.* Because English language arts and mathematics

standards can be grouped together in application, much of the project work that gifted educators might already use could be revised to connect to the new CCSS and to show how multiple standards could be addressed across content areas. For example, research projects could be designed that address the research standard in English language arts and the data representation standard in mathematics by (a) delineating a product demand for research on an issue, (b) asking researchable questions that require quantitative approaches, (c) using multiple sources to answer them, (d) collecting data, (e) interpreting data (e.g., by creating a scatterplot and deciding if there is a line of best fit and describing the related variables), and then (f) representing findings in tables, graphs, and other visual displays that are explained in text and presented to an audience with implications for a plan of action. Such a project might be possible for the gifted learner at an earlier grade than for a typical learner.

To differentiate the Common Core State Standards for Mathematics, educators need to be aware of the eight Standards for Mathematical Practice and additional standards that should be considered for promising mathematics students before differentiating the curriculum. This section will address these needs and provide specific examples of differentiation that examine learning progressions in operations and algebraic thinking, fractions and the number system, geometry, and statistics and probability.

The Common Core State Standards for Mathematical Practice

When considering the implications of the CCSS for the development of mathematical talent, it is important to take into account the eight Standards for Mathematical Practice that educators should seek to develop in their students in addition to the individual mathematics content standards. These Standards for Mathematical Practice are an integral part of the Common Core State Standards for Mathematics and are described in detail (NGA & CCSSO, 2010a, pp. 6–8). These build on the NCTM (2000) process standards of problem solving, reasoning and proof, communication, representation, and connections, and include the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001): adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations, and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy). The eight Standards for Mathematical Practice for all students from kindergarten through college and careers are:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

It is important that students actively engage in these practices daily in their mathematics classes. Students need ongoing opportunities to experience the joy of investigating rich concepts in depth and applying mathematical reasoning and justification to a variety of scientific, engineering, and other problems.

In order to support mathematically advanced students and to develop students who have the expertise, perseverance, creativity, and willingness to take risks and recover from failure, which is necessary for them to become mathematics innovators, we propose that a ninth Standard for Mathematical Practice be added for the development of promising mathematics students—a standard on mathematical creativity and innovation: *Solve problems in novel ways and pose new mathematical questions of interest to investigate.*

The characteristics of the new proposed standard would be that students are encouraged and supported in taking risks, embracing challenge, solving problems in a variety of ways, posing new mathematical questions of interest to investigate, and being passionate about mathematical investigations.

Developing Innovative and Creative Mathematicians

To aid in the development of passionate, innovative, and creative mathematicians, teachers might use a heuristic such as the one shown in Figure 2. Using this heuristic, students may start at any point on the diagram and proceed in any order. One possible order might be:

- Relate the problem to other problems that you have solved. How is this similar to other mathematical ideas that you have seen? How is it different?
- Investigate the problem. Think deeply and ask questions.
- Evaluate your findings. Did you answer the question? Does the answer make sense?
- Communicate your results. How can you best let others know what you have discovered?
- Create new questions to explore. What else would you like to find out about this topic? Start a new investigation.

To assist students in their creation of new mathematical insights, some suggested questions for creative mathematical investigations include the following (Sheffield, 2006):

- *Who?* Who has another solution? Who has another method? Who agrees or disagrees?
- *What or what if?* What patterns do I see in these data? What generalizations might I make from the patterns? What proof do I have? What are the chances? What is

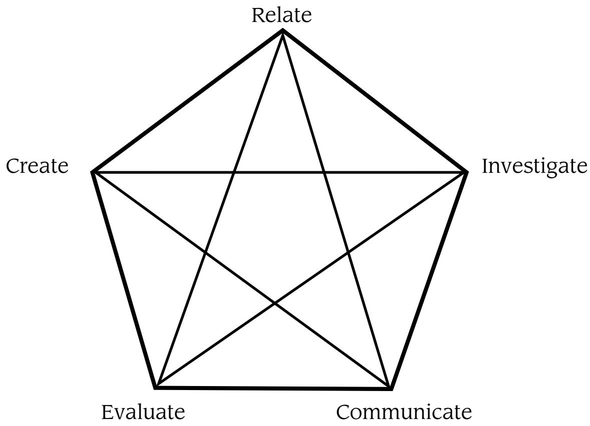

Figure 2. Heuristic for innovative and creative mathematicians. From *Extending the Challenge in Mathematics: Developing Mathematical Promise in K–8 Students* (p. 15), by L. J. Sheffield, 2003, Thousand Oaks, CA: Corwin Press. Copyright 2003 by Corwin Press. Reprinted with permission.

the best answer, best method of solution, or best strategy to begin with? What if I change one or more parts of the problem? What new problems might I create?

- *When?* When does this work? When does this not work?
- *Where?* Where did that come from? Where should I start? Where might I go next? Where might I find additional information?
- *Why or why not?* Why does that work? If it does not work, why does it not work?
- *How?* How is this like other mathematical problems or patterns that I have seen? How does it differ? How does this relate to real-life situations or models? How many solutions are possible? How many ways might I to represent, simulate, model, or visualize these ideas? How many ways might I sort, organize, and present this information?

Even our best mathematics students are often not encouraged to be creative. Educators need to support them as they move from questions with one right answer to those that require reasoning and justification and to problems and explorations that have several solutions or related problems that will deepen and extend the concepts being learned. Educators need to remember that the real learning frequently begins after the original problem has been solved.

If educators wish for students to develop a deeper understanding of concepts and to become creative and investigative mathematicians, they should use criteria for assessment that encourage depth and creativity, such as those noted by Sheffield (2000):

- *Depth of understanding:* the extent to which core concepts are understood, explored, and developed.
- *Fluency:* the number of different correct answers, methods of solution, or new questions formulated.
- *Flexibility:* the number of different categories of answers, methods, or questions.
- *Originality:* solutions, methods, or questions that are unique and show insight.
- *Elaboration or elegance:* clarity and quality of expression of thinking, including charts, graphs, drawings, models, and words.
- *Generalizations:* patterns that are noted, hypothesized, and verified for larger categories.
- *Extensions:* related questions that are asked and explored, especially those involving why and what if.

The instructional pace is also a critical consideration in the education of gifted students in mathematics. Advanced learners may demonstrate rapid or early mastery of some of the mathematics standards, especially those involving skill at computation and mastery of algorithms, requiring accelerative opportunities at key stages of development. Appropriate pacing for these students, including accelerated courses, means that students have the time and opportunity to delve deeply and creatively into topics, projects, and problems of interest. Therefore, it's important that advanced learners receive their instruction from well-prepared teachers who are knowledgeable regarding mathematics and strategies to use with advanced learners.

Teachers of the gifted should be mindful of the importance of providing problem-finding and problem-solving skills and strategies to stimulate mathematical and spatial reasoning and to work with a wide range of mathematical topics, such as number theory, geometry, and discrete mathematics. Early exposure to topics such as probability, statistics, and logic also are viable approaches to be used to support applied and cross-curricular skills, including conducting meaningful research in science and engineering. Extracurricular opportunities such as math clubs, circles, competitions, mentors, and online experiences should also be readily available without additional cost to students.

In encouraging these high levels of mathematical creativity and giftedness (Chapin, O'Connor, & Anderson, 2009), teachers should realize that the role of students is to:

- think, reason, make sense, and go deeper;
- talk to a partner and generate new ideas;
- repeat and rephrase what others have said and explain why they agree or disagree;
- make generalizations and justify conclusions;
- add on new ideas, new methods of solution, new questions, and original problems and related solutions;
- record solutions, reasoning, and questions;
- pose new mathematical questions of interest to investigate; and
- create innovative mathematical problems and solutions.

The role of the teacher is to:

- ask questions that encourage mathematical creativity, reasoning, and sense-making;
- elicit, engage, and challenge each student's thinking;
- listen carefully to students' ideas;

- ask students to clarify, justify, connect, and extend their ideas;
- assist students in attaching mathematical notation and language to their ideas;
- reflect on student understanding, differentiate instruction, and encourage participation; and
- guide students to resources, including those online, in print, and in person, such as mentors, apprenticeships, competitions, clubs, math circles, and other extracurricular opportunities.

Specific Examples for Differentiating Mathematics

The Standards for Mathematical Content appear in a variety of domains depending on the grade level. These are:

- Counting and Cardinality (K)
- Operations and Algebraic Thinking (K–5)
- Number and Operations in Base Ten (K–5)
- Measurement and Data (K–5)
- Geometry (K–HS)
- Number and Operations–Fractions (3–5)
- Ratios and Proportional Relationships (6–7)
- The Number System (6–8)
- Expressions and Equations (6–8)
- Statistics and Probability (6–HS)
- Functions (8–HS)
- Number and Quantity (HS)
- Algebra (HS)
- Modeling (HS)

The following pages offer examples of activities that use the eight Standards for Mathematical Practice and support the implementation of the Common Core State Standards for Mathematics. The advanced activities also make use of the proposed ninth Standard for Mathematical Practice on mathematical creativity and innovation. (Options for mathematical creativity are italicized.)

The sample activities were designed to give exemplars in a variety of areas including number and operations, algebraic thinking, geometry, and data and statistics. Sample activities are given for primary, intermediate, middle, and high school standards. Each activity begins with a selected task, gives a variety of questions for both typical and advanced learners, and describes suggestions for implementation that include ideas for different types of formative and summative assessment. Note that sometimes the initial problem is the same for both typical and advanced learners, and questions and formative assessments are used to differentiate and develop mathematical creativity and giftedness.

Formative assessment in these activities includes the use of pretests, differentiation of tasks and questions to assess during the problem-solving process, observation and analysis of student work, portfolios, and authentic cross-disciplinary tasks and research.