Framework

The conceptual structure for statistics education is provided in the two-dimensional framework model shown in Table 1. One dimension is defined by the statistical problem-solving process components that can be used to advance statistical literacy. The second dimension is composed of three developmental levels.

Statistical Problem-Solving Process

The purpose of the statistical problem-solving process (see Figure 3) is to collect and analyze data to answer statistical investigative questions.

This investigative process involves four components, each of which involve exploring and addressing variability:

I. Formulate Statistical Investigative Questions

II. Collect/Consider the Data

III. Analyze the Data

IV. Interpret the Results

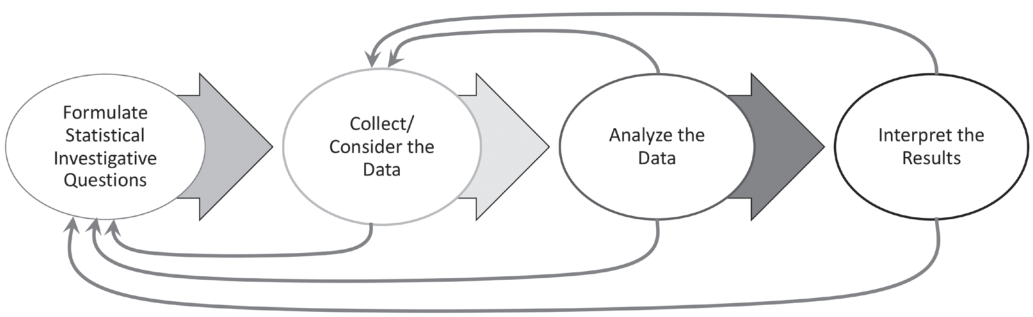

Figure 3: Statistical problem-solving process

I. Formulate Statistical Investigative Questions

Anticipating Variability – Beginning the Process
Formulating statistical investigative questions that anticipate variability leads to productive investigations. For example, the following are all statistical investigative questions that anticipate variability and could lead to a rich data collection process and subsequent analysis of the data:

- *How fast will my plant grow?*
- *Do plants exposed to more sunlight grow faster?*
- *How does sunlight affect the growth of a plant?*

In contrast, the question *How tall is the plant?* is answered with a single height; it is therefore not a statistical investigative question. *How tall is the plant* is a question we ask to collect data. Many other data collection questions could be asked to help collect the necessary data to answer the statistical investigative question: *Do plants exposed to more sunlight grow faster?* The fact that there will be differing heights for the different exposures of sunlight implies that we anticipate an answer based on measurements of plant heights that vary.

While statistical investigative questions begin worthwhile studies, the use of questioning is prominent throughout all four components of the statistical problem-solving process. Such uses of questioning will be illustrated throughout the examples at the different levels.

In addition to anticipating variability, there are other features of a statistical investigative question that are important. The variable(s) of interest must be clear; the group or population that the question is focused on must be clear; the intent of the question should be clear – is the question requiring a description of the data, is the question comparing a variable across two or more groups, is the question looking at an association between two variables; the question should be about the whole group (anticipating variability) and not about an individual (giving a deterministic answer); the question should be answered through data collection (primary data) or with the data in hand (secondary data); and the question should be purposeful.

II. Collect/Consider Data

Acknowledging Variability—Designing for Differences

Data collection designs must acknowledge variability in data. Some study methods are used to reduce and detect variability in data, such as Statistical Process Control and random sampling. Others are used to induce variability to test treatments, such as Design of Experiments. In the latter approach, experimental designs are chosen to acknowledge the differences between groups subjected to different treatments. Random assignment to the groups is intended to reduce differences between the groups due to factors that are not manipulated or controlled in the experiment. In all designs, a main statistical focus is to look for, account for, and explain variability.

After the data are available—whether they were collected first-hand or acquired from another source—they need to be interrogated. For example, questions about how the variables differ by type, the possible outcomes of each of the variables, and how the data were collected are necessary to clarify whether the data are useful for answering the statistical investigative question. The data collection design impacts the scope of generalizability and the possible limitations in analysis and interpretation.

III. Analyze the Data

Accounting of Variability—Using Distributions

When we analyze data, we seek to understand its variability. Reasoning about distributions is key to accounting for and describing variability at all developmental levels. Graphical displays and numerical summaries are used to explore, describe, and compare variability in distributions.

For example, the batting averages of the American League baseball teams and the batting averages of the National League baseball teams for a particular year can be displayed in two comparative dotplots and boxplots. These graphs show the variability of each league's distribution of batting averages. We can take into account variability by describing the overlap and the separation of the distributions of the two leagues.

Another example of taking variability into account is the margin of error in public opinion polling. When the results of an election poll state that "42% of those polled support a particular candidate with a margin of error of +/- 3 percentage points at the 95% confidence level," the focus of the margin of error is to account for sampling variability.

IV. Interpret the Results

Allowing for Variability—Looking beyond the Data

Statistical interpretations are made in the presence of variability and must take variability into account. For example, we should interpret the result of an election poll as an estimate that may vary from sample to sample of voters being polled. When interpreting the results of a randomized comparative medical experiment, we must remember there are two important sources of variability: randomization to treatment group, and variability from individual to individual. When we generalize the results and look beyond the study data collected, we must take into account these sources of variability.

Three Developmental Levels: A, B, and C

Experienced statisticians understand the role of variability in the statistical problem-solving process. When they formulate their first question, they anticipate the data collection, the nature of the analysis, and the possible interpretations—all of which involve possible sources of variability. In the end, mature practitioners reflect upon all aspects of data collection and analysis as well as the question itself when interpreting results. Likewise, they link data collection and analysis to each other as well as to the other components in the statistical problem-solving process.

Beginning students cannot be expected to make all of these linkages. They require years of experience and training to develop more mature reasoning. Much like mathematics education, statistics education should be viewed as a developmental process.

As in GAISE I, to meet the goals of statistical literacy, this report provides a framework for statistical education within Pre-K–12 settings over three Levels, A, B, and C. Students at very young ages innately have notions of variability and probability. Related research is summarized in *Statistics in Early Childhood and Primary Education* (Leavy, Meletiou-Mavrotheris & Paparistodemou, 2018). Level A capitalizes on these understandings by more formally introducing students to the statistical problem-solving process. Level B continues to build the statistical toolbox. By the time students reach Level C, the student can be provided with ambitious learning goals towards the development of statistical literacy in Pre-K–12 education. Level C sets lofty goals for students finishing Pre-K–12 education in today's data driven society. This sets the stage for students to mature past these levels to further develop more complex statistical investigative questions and analysis techniques while working with ever-evolving data types.

Although these three levels may parallel grade levels, they are based on development in statistical literacy, not age. There is <u>no</u> attempt to tie these levels to specific grade levels. Thus, a middle school student who has had no prior experience with statistics will need to begin with Level A concepts and activities before moving to Level B. This prerequisite holds for a secondary student as well. If a student has not had Level A and B experiences prior to high school, then it is not appropriate for that student to begin with Level C expectations. Investigations and scenarios are more teacher-driven at Level A but become more student-driven at Levels B and C.

The Framework Table

As was structured in the framework table from GAISE I, each of the four stages or process components is described as it develops across levels. It is understood that work at Level B assumes and develops further the concepts from Level A; likewise, Level C assumes and uses concepts from the lower levels. The essentials from GAISE I are similar in GAISE II but enhanced with more specifics and some additional essentials to account for the evolution of the statistical field since GAISE I.

Reading down a column will describe a complete problem investigation for a particular level.

Table 1: The Framework

Process Component	Level A	Level B	Level C
I. Formulate Statistical Investigative Questions	Understand when a statistical investigation is appropriate Pose statistical investigative questions of interest to students where the context is such that students can collect or have access to all required data Pose summary (or descriptive) statistical investigative questions about one variable regarding small, well-defined groups (e.g., subset of a classroom, classroom, school, town) and extend these to include comparison and association statistical investigative questions between variables Experience different types of questions in statistics: those used to frame an investigation, those used to collect data, and those used to guide analysis and interpretation	Recognize that statistical investigative questions can be used to articulate research topics and that multiple statistical investigative questions can be asked about any research topic Understand that statistical investigative questions take into account context as well as variability present in data Pose summary, comparative, and association statistical investigative questions about a broader population using samples taken from the population Pose statistical investigative questions that require looking at a variable over time Understand that there are different types of questions in statistics: those used to frame an investigation, those used to collect data, and those used to guide analysis and interpretation Pose statistical investigative questions for data collected from online sources and websites, smartphones, fitness devices, sensors, and other modern devices	Formulate multivariable statistical investigative questions and determine how data can be collected and analyzed to provide an answer Pose summary, comparative, and association statistical investigative questions for surveys, observational studies, and experiments using primary or secondary data Pose inferential statistical investigative questions regarding causality and prediction

Table 1: The Framework

Process Component	Level A	Level B	Level C
II. Collect Data/ Consider Data	Understand that data are information; recognize that to answer a statistical investigative question, a person may collect data themselves specifically for that purpose, or a person may use data that have been collected by other people for another purpose Understand how to collect and record information from the group of interest using surveys and measurements collected from observations and simple experiments Understand that a variable measures the same characteristic on several individuals or objects and results in data values that may fluctuate Understand that within a data set there can be different types of variables (e.g., categorical or quantitative) Interrogate the data set to understand the context of the variables as they may relate to statistical investigative questions Understand that data are not always pristine but may contain errors, have missing values, etc., and that decisions have to be made about how to account for these issues	Understand that data are information collected and recorded with a purpose and can be organized and stored in a variety of structures (e.g., spreadsheets) Understand that a sample can be used to answer statistical investigative questions about a population. Recognize the limitations and scope of the data collected by describing the group or population from which the data are collected Understand that data can be used to make comparisons between different groups at one point in time and the same group over time Recognize that data can be collected using surveys and measurements, and develop a critical attitude in analyzing data collection methods Understand that quantitative variables may be either discrete or continuous Understand how to interrogate the data to determine how the data were collected, from whom they were collected, what types of variables are in the data, how the variables were measured (including units used), and the possible outcomes for the variables Understand that data can be collected (primary data) or existing data can be obtained from other sources (secondary data) Understand how random assignment in comparative experiments is used to control for characteristics that might affect responses	Word as: Apply an appropriate data collection plan when collecting primary data or selecting secondary data for the statistical investigative question of interest. Distinguish between surveys, observational studies, and experiments Understand what constitutes good practice in designing a sample survey, an experiment, and an observational study Understand the role of random selection in sample surveys and the effect of sample size on the variability of estimates Understand the role of random assignment in experiments and its implications for cause-and-effect interpretations Understand the issues of bias and confounding variables in observational studies and their implications for interpretation Understand practices for handling data that enhance reproducibility and ensure ethical use, including descriptions of alterations, and an understanding of when data may contain sensitive information Understand how concerns about privacy and human subjects may affect the collection and distribution of data Understand that in some circumstances, the data collected or considered may not generalize to the desired population, or this data may be the entire population

Table 1: The Framework

Process Component	Level A	Level B	Level C
III. Analyze the Data	Understand that the distribution of a categorical variable or quantitative variable describes the number of times a particular outcome occurs Represent the variability of categorical variables or quantitative variables using appropriate displays (e.g., tables, picture graphs, dotplots, bar graphs) Describe key features of distributions for quantitative variables, such as: ◦ center: mean as the equal share, and median as the middle-ordered value of the data ◦ variability: range as the difference between the greatest and least value, and dispersion as how many units from the equal share value ◦ shape: number of clusters, symmetric or not, and gaps Recognize distributions can be used to compare two groups Observe whether there appears to be an association between two variables	Represent the variability of quantitative variables using appropriate displays (e.g., dotplots, boxplots) Learn to use the key features of distributions for quantitative variables, such as: ◦ center: mean as a balance point, and median as the middle-ordered value ◦ variability: interquartile range and mean absolute deviation (MAD) ◦ shape: symmetric or asymmetric and number of modes Use reasoning about distributions to compare two groups based on quantitative variables Explore patterns of association between two quantitative variables or two categorical variables: ◦ measures of correlation: quadrant count ratio (QCR) ◦ comparison of conditional proportions across categorical variables	Use technology to subset and filter data sets and transform variables, including smoothing for time series data Identify appropriate ways to summarize quantitative or categorical data using tables, graphical displays, and numerical summary statistics, which includes using standard deviation as a measure of variability and a modified boxplot for identifying outliers Summarize and describe relationships among multiple variables Understand how sampling distributions (developed through simulation) are used to describe the sample-to-sample variability of sample statistics Develop simulations to determine approximate sampling distributions and compute ***p***-values from those distributions Describe associations between two categorical variables using measures such as difference in proportions and relative risk Describe the relationship between two quantitative variables by interpreting Pearson's correlation coefficient and a least-squares regression line Use simulations to investigate associations between two categorical variables and to compare groups Construct prediction intervals and confidence intervals to determine plausible values of a predicted observation or a population characteristic

Table 1: The Framework

Process Component	Level A	Level B	Level C
IV. Interpret Results	Use statistical evidence from analyses to answer the statistical investigative questions and communicate results through structured answers with teacher guidance Make statements about the group or population from which the data were collected, recognizing that conclusions are limited to these groups and cannot be generalized to other groups Describe the difference between two groups with different conditions	Use statistical evidence from analyses to answer the statistical investigative questions and communicate results with comprehensive answers and some teacher guidance Acknowledge that looking beyond the data is feasible Generalize beyond the sample providing statistical evidence for the generalization and including a statement of uncertainty and plausibility when needed Recognize the uncertainty caused by sample to sample variability State the limitations of sample information (e.g., a sample may or may not be representative of the larger population, measurement variability) Compare results for different conditions in an experiment	Use statistical evidence from analyses to answer the statistical investigative questions and communicate results through more formal reports and presentations Evaluate and interpret the impact of outliers on the results Understand what it means for an outcome or an estimate of a population characteristic to be plausible or not plausible compared to chance variation Interpret the margin of error associated with an estimate of a population characteristic Acknowledge the presence of missing values and understand how missing values may add bias to an analysis Use multivariate thinking to understand how variables impact one another Communicate statistical reasoning and results to others in a variety of formats (verbal, written, visual) Understand how to interpret simulated p-values appropriately