About This Book

> When we see how the branching of trees resembles the branching of arteries and the branching of rivers, how crystal grains look like soap bubbles and the plates of a tortoise's shell, how the fiddleheads of ferns, stellar galaxies, and water emptying from the bathtub spiral in a similar manner, then we cannot help but wonder why nature uses only a few kindred forms in so many different contexts. Why do meandering snakes, meandering rivers, and loops of string adopt the same pattern, and why do cracks in mud and markings on a giraffe arrange themselves like films in a froth of bubbles?
>
> —Peter S. Stevens, *Patterns in Nature*

Have you ever wondered why bees store their honey in hexagonal rather than octagonal honeycombs? Or why a three-legged stool doesn't wobble? Have you considered why you need a mirror that is at least half your height in order to see your entire body? Indeed, from the alignment of the solar system to the structure of an atom, from rocks to crystals to flowers to rings on a snake, from architects to mechanics to artists to musicians, from bike gears to curve balls to Snowboards and Rollerblades, geometry pervades our world.

The study of geometry in the middle grades is crucial in the mathematics education of our children. During these years, students begin to develop the cognitive structures that allow them to reason within a linear, deductive system of thought. Simultaneously, they continue informally to observe the boundless activity of their environment, tapping into native curiosities and intuitions that can, if nurtured, provide the foundation for inductive discoveries and reinventions of many of our fundamental mathematical concepts. Teaching geometry in the middle grades requires delicate pedagogical navigation: How does a teacher draw students toward more-sophisticated, formal systems of thought while recognizing their still wide ranging cognitive abilities and experiences? How does a teacher facilitate the transition between hands-on learning experiences and more-formal, abstract approaches to learning geometric concepts? These questions and many others like them will be addressed through the activities in this book.

As learners progress through the middle-grades curriculum toward a more formal study of geometry, it is important that they begin making the transition from empirically based, inductive reasoning to the deductive use of rules and abstract thinking that are the hallmarks of more-advanced geometry. As *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM] 2000) suggests, students in the middle grades must begin to "create and critique inductive and deductive arguments concerning geometric ideas and relationships" (p. 232). The development of mathematical arguments in the middle grades promotes the transition from informal to more-formal thinking, which leads to an emphasis on mathematical reasoning, including inductive and deductive processes, formulating and defending conjectures, and classifying and defining geometric objects. This book focuses on the central concepts of middle-grades geometry and students' geometric thinking.

The van Hiele framework can provide insights into the development of cognitive and spatial processes in middle-grades students. The van Hieles described five levels of thought that characterize the development of the reasoning abilities needed to engage in geometric thinking. In an essay on the CD-ROM that accompanies this book, Carol Malloy supplies an introduction to these levels and a discussion of their relevance to designing instruction for middle-grades students. Related readings also appear on the CD-ROM.

The four chapters in this book emphasize geometric thinking as an expected outcome of the mathematics experiences of students in grades 6–8. The chapters present important ideas and relevant activities that focus on the "big ideas" of geometry—shape, location, transformations, and visualization. Paralleling the Geometry Standard, the chapters emphasize the expectations for middle-grades students in each of these areas:

- Chapter 1, "Characteristics and Properties of Shapes," emphasizes the development of geometric reasoning.
- Chapter 2, "Coordinate Geometry and Other Representational Systems," explores the use of multiple representations as tools for analysis.
- Chapter 3, "Transformations and Symmetry," emphasizes the use of transformation geometry as another lens for investigating and interpreting geometric objects.
- Chapter 4, "Visualization, Spatial Reasoning, and Geometric Modeling," emphasizes the development of reasoning and the ability to visualize relationships.

Each section of a chapter presents one or more sample activities, many of which have blackline masters, which are signaled by an icon and can be found in the appendix, along with solutions to the problems where appropriate. They can also be printed from the CD-ROM that accompanies the book. The CD-ROM, also signaled by an icon, contains four applets for students to manipulate and resources for professional development. An icon also appears in the margin next to references to *Principles and Standards for School Mathematics.*

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Key to Icons

Principles and Standards

CD-ROM

Blackline Master

Three different icons appear in the book, as shown in the key. One alerts readers to material quoted from *Principles and Standards for School Mathematics,* another points them to supplementary materials on the CD-ROM that accompanies the book, and a third signals the blackline masters and indicates their locations in the appendix.