

Chapter 1

The Common Core State Standards for Mathematics: How Did We Get Here, and What Needs to Happen Next?

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ETYMOLOGICALLY, the word *standard* comes from the Anglo-French word *estandard*, referring to a flag displayed on a battlefield to rally the troops (Oxford English Dictionary n.d.). Over time, the term evolved in two ways. First, instead of referring to a king's authority, it came to mean a consensus among experts. Second, it evolved to mean improved technical specifications that promote efficiency and make measures of that efficiency easier. Standards in education serve two similar purposes: they express a consensus among experts of what to teach and when to teach it, and they make measuring students' proficiency easier through assessments.

For the first time in U.S. history, states across the nation are rallying behind a common set of voluntary state mathematics standards. Spirited debate among varied experts—mathematicians, mathematics educators, teachers, statisticians, and policy leaders—has been undertaken to reach consensus on a negotiated set of goal statements and to build common assessments. Led by the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO), 45 states, the District of Columbia, and the U.S. Virgin Islands have adopted the Common Core State Standards for Mathematics (CCSSM), released June 2, 2010.

These states, the District of Columbia, and the U.S. Virgin Islands have an historic opportunity to ensure that school mathematics programs are based on “fewer, clearer, and higher” standards (Common Core State Standards Initiative [CCSSI] 2010, p. 1) designed to prepare students with the knowledge and skills they need to succeed in college and work.

This chapter traces content standards' evolution in mathematics education, summarizes the CCSSM's current state, and discusses the next steps to ensure the proper implementation of the CCSSM. Typically mathematics educators have defined *content*

This chapter is based on two papers written for the Center for the Study of Mathematics Curriculum, Confrey (2007) and Confrey and Krupa (2010).

standards as a specification of “what a person should know or be able to do” (National Research Council 2002, p. 2), and have stated that standards should “indicate the topics and skills that should be taught at various grades or grade spans and are intended to guide public school instruction, curriculum, teacher preparation, and assessment” (Goertz 2010, p. 52). We use a revised definition reflecting the evolution of standards (Confrey 2007, pp. 6–7):

Content standards consist of a negotiated settlement among authorized experts concerning the specification of what a person should know or be able to do, with consideration of how that is to be measured and/or documented, and as a means of modulating or effecting change within the system of education and restricting excessive variation.

Both definitions recognize the importance of standards functioning collaboratively with other elements in the system—curriculum, instructional teaching capacity, professional development, and assessment.

Part 1: The History of Standards in Mathematics Education

The U. S. Constitution prohibits federal agencies from regulating content standards and assessments, leaving states in charge of their own educational system (Fuhrman 2004). Attempts to standardize education date back to the Committee of Ten in 1892, which sought to standardize secondary school curriculum in order to prepare college-bound and workforce-ready students. Ten years after the committee’s inception, Dexter (1906) questioned its influence, wondering how closely the changes in high school curriculum coincided with the committee’s recommendations. He concluded that the report had little influence on the pedagogy and content taught in secondary school classrooms, raising what has become a perennial question of how to influence education in the classroom.

When the Soviet Union launched the *Sputnik* satellite in 1957, the United States began another concerted effort to produce content standards, focusing on “high quality mathematics for college-capable students, particularly those heading for technical or scientific careers” (National Advisory Committee on Mathematics Education 1975). These standards produced curricula dominated by formal structures, such as set theory and deductive proof. Then in 1975, the Conference Board of the Mathematical Sciences reexamined these ideas and recommended that content standards should (a) maintain the emphasis on the logical structure of mathematics, (b) integrate concrete experiences, (c) include applications, and (d) foster the use of symbols and formal notation. They further advised that mathematics instruction include (a) calculators by eighth grade, (b) the metric system, and (c) statistics (National Advisory Committee on Mathematics Education 1975, pp. 136–39).

The following paragraph, taken from Confrey (2007, pp. 13–14), summarizes the state of mathematics standards at the end of the twentieth century.

In 1989, after three years of work led by Thomas Romberg, together with mathematics teachers, researchers, and administrators, the National Council of Teachers of Mathematics produced the *NCTM Curriculum and Evaluation Standards*, with a set of *Professional Standards for Teaching Mathematics* (1991) and *Assessment Standards for School Mathematics* (1995). Only the *Curriculum*

and Evaluation Standards exerted considerable influence on other factors in reform, curriculum, state changes, [and] assessment. The standards were intended to ensure quality, identify explicit goals, and promote change. They were purposed to create mathematically literate workers, encourage lifelong learning, provide opportunities for all, and support an informed electorate. They were structured by grade bands (K–4, 5–8, and 9–12), and each addressed standards of problem solving, communication, reasoning, connections, and estimation. They addressed the content strands of number and numeration, geometry, measurement, statistics and probability, algebra and trigonometry, and discrete mathematics. Issues of pedagogy were integrated into issues of content, emphasizing the importance of active participation in learning by students. The standards drew heavily on research on student thinking, student misconceptions, and how students learned particular ideas as they encountered challenging tasks. They warned against relying too heavily on memorization and procedural understanding, based on numerous studies documenting disintegration of students’ apparent knowledge when asked for reasons and explanations, and stressed conceptual understanding. (Erlwanger 1973; Ginsburg 1991; Kamii 1985)

In response to the development of standards in mathematics and science, the federal government chose to use standards as a means to establish a more rigorous educational system, known as “the standards movement” or “systemic reform” (Cohen 1995; Lewis 1995). As noted by Goertz (2010, p. 54):

The Improving America’s Schools Act of 1994 required states to develop challenging content standards in at least reading and mathematics, create high-quality assessments to measure performance against these standards, and have local districts identify low-performing schools for assistance.... With the enactment of the NCLB Act of 2001, the federal government expanded its role significantly, requiring states to test more frequently and set more ambitious and uniform improvement goals for their schools, and prescribing sanctions for schools that fail to meet these goals.

Increased accountability pressured states to align their standards with ones produced by national organizations. Nonetheless, the mathematics frameworks in the 50 states and the District of Columbia still varied significantly (Reys 2006). The first ten years of the twenty-first century saw numerous standards released, from prominent national organizations, to help states develop more coherent frameworks. NCTM published *Principles and Standards for School Mathematics* (2000), *Curriculum Focal Points* (2006), and *Focus in High School Mathematics: Reasoning and Sense Making* (2009); Achieve, *Mathematics Benchmarks: Grades K–12* (2004); the American Statistical Association, *Guidelines for Assessment and Instruction in Statistics Education (GAISE)* (Franklin et al. 2007); the College Board (2006), “College Board Standards for College Success: Mathematics and Statistics”; and the National Mathematics Advisory Panel (2008), *Foundations for Success*.

Among these standards-setting efforts a consensus was emerging on the need for specificity in grades and courses, and that these specifications need to be “important and challenging” for all students (Confrey 2007). Usiskin (2010, p. 34) makes the following point:

[These standards] brought algebra and some data analysis into the elementary school, algebra into grade 8 and earlier for many students, applications into the algebra and geometry curricula, graphing calculators into the study of functions, and a major increase in the number of high school students taking calculus. It promoted active learning, classroom discourse, alternative algorithms, and multiple ways of approaching problems.

The progress made in creating and refining national standards for mathematics is undeniable. In less than twenty years, we have progressed as a nation from no standards, to multiple sets of standards, to a voluntary set (CCSSM), each created with clear attention to salient features affecting students' learning. In a distributed system of education, we have crafted a means to advise diverse constituencies on what students should know and do. We have seen many of the states—the national unit of educational change—adopt or adapt this means for local consumption. We have also clearly embraced the complexity involved in acknowledging diverse groups of experts and their crucial roles in establishing standards.

Part 2: The Current State of the CCSSM in Mathematics Education

The development of the CCSSM began in July 2009. The CCSSI based the document's content on benchmarking of U.S. standards to high-performing countries on international exams, research on students' mathematical knowledge and skill from the National Mathematics Advisory Panel, and research from students on student learning (CCSSI 2010, p. 1). The lead writers valued information on how students learn mathematics and, as Confrey (2007, p. 33) stated, the development of "sequenced obstacles and challenges for students ... absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise." The writers also had to weigh competing sources of research and decide what to emphasize and what to delete. They chose to separate content and process standards and did so by proposing eight mathematical practices. These eight practices, derived from a combination of NCTM's process standards and mathematical proficiencies from the National Research Council's (2001) report *Adding It Up*, serve as a foundation for grades K–12 mathematics instruction:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

These eight practices offer instructional habits of mind for teachers to use as they teach students how to understand the content standards. Confrey and Krupa (2010, p. 9) emphasized the importance of linking content and practice standards as a means to strengthen understanding, arguing that "the practices sustain mathematics as the content

evolves. As such, they make what students learn enduring and they ensure that students will continue to be prepared to learn new mathematics.”

Figure 1.1 depicts the content standards’ design and organization. The *standards* contain the first component of Confrey’s (2007) definition, “what a person should know or be able to do.” The document groups these standards into *clusters* of related standards, which it further groups into larger *domains* of related clusters.

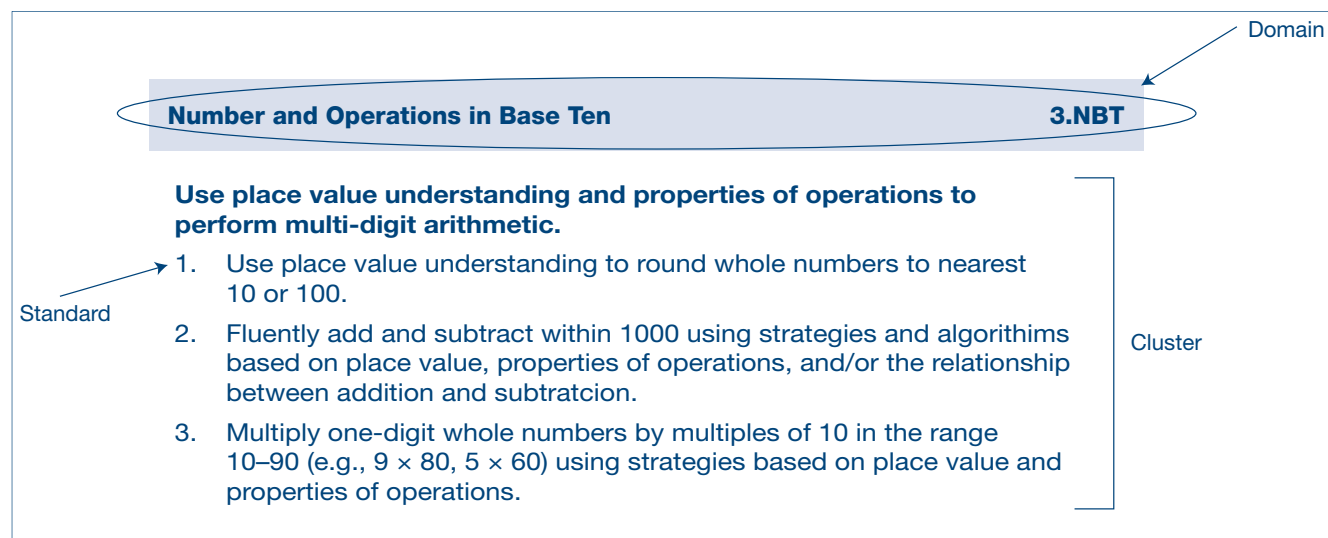


Fig. 1.1. An example of a content standard, taken from the CCSSM’s introduction (CCSSI 2010, p. 5)

For high school, the CCSSM clusters the standards into conceptual categories: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability. These clusters contain the mathematical content all students should learn for college and career readiness, but the CCSSM does not mandate the order in which the content should be taught.

The CCSSI designed these content standards to support learning trajectories, which describe the transition in students’ thinking from novice to sophisticated views of “big ideas” on the basis of empirical research, and identify common obstacles and landmarks in that process. Learning trajectories recognize the pivotal roles that instruction, careful sequencing and selecting of curricular tasks, and proper use of tools and language to support learning all play. Not all “big ideas” have equally solid research bases. Further, some decisions by the lead writers represent “logical thought experiments” on how trajectories might proceed: readers should view these as conjectures subject to further research. As the CCSSM states (CCSSI 2010, p. 5), “one promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today.” Learning trajectories are important because they allow teachers to know what to expect about entering students’ preparation, what to teach in recognition of what they will expect of their students, and how to relate concepts across strands at each grade. Teachers report that these help them recognize a variety of students’ ideas, support discourse, and engage in rich uses of classroom assessment. Confrey, Maloney, and Nguyen (2011)

have created a “learning trajectories display” of the CCSSM, which shows how concepts evolve over time across grades (K–8) and difficulty levels (high school). Because of the two-dimensional structure at high school, the learning trajectory display supports either a siloed or an integrated curriculum, remaining agnostic about the curriculum or order in which it delivers content.

Table 1.1 details several major shifts in the CCSSM in grades K–5 for most states. These grades K–5 standards emphasize numeration and operation, along with many concepts and skills introduced at earlier grade levels. They also expect students to learn three systems of measurement simultaneously—nonstandard, English, and metric. An increased emphasis exists on fractions as numbers and on using the number line to give students structure and visualization of the number system. These standards have deemphasized or removed early algebra—as introduced through patterns; statistics and probability; and percent, ratio, and proportion—from grades K–5. The writers’ arguments were (a) having fewer topics means some must be eliminated, and (b) emphasizing number mastery would lead to quicker and more secure learning of these eliminated topics in middle grades. Many educators question these assumptions and either will argue for their inclusion in future revisions on the basis of empirical studies or choose to address the topics in the 15 percent discretionary content left up to states.

Table 1.1
The CCSSM Grades K–5 Domains

Domains	Grades
Counting and Cardinality	K only
Operations and Algebraic Thinking	K–5
Number and Operations in Base Ten	K–5
Number and Operations—Fractions	3–5
Measurement and Data	K–5
Geometry	K–5

The most extensive changes in the CCSSM occur at the middle grades (see table 1.2), beginning with a strong foundation for early algebra in ratio and rate, and with an introduction to statistics. Grade 6 alone houses the beginning of ratio, proportion, percents, and statistics. Grade 7 introduces rational number. One-third of the regular grade 8 curriculum consists of algebra topics for all students. Phil Daro, one of the writers of the CCSSM, said, “The CCSS were written to assume 100% mastery, in any given year, of the preceding year’s standards” (Confrey and Krupa 2010, p. 2). Thus, middle grades teachers should anticipate that, by having fewer elementary school standards, the richness of the grades K–5 standards will produce more students who understand the concepts and will result in less need for remediation and repetition.

Table 1.2
The CCSSM Grades 6–8 Domains

Domains	Grades
Ratio and Proportional Relationship	6–7
The Number System	6–8
Expressions and Equations	6–8
Functions	8
Geometry	6–8
Statistics and Probability	6–8

Table 1.3 presents the standards for grades 9–12 by conceptual categories that schools must address over the four years a student is in high school, without referent to a particular curricular approach. The major changes at the high school level are the masteries students must show beyond traditional Algebra 2 content. These topics include periodic functions, polynomials, radicals, advanced probability and statistics, and mathematical modeling. In table 1.3, note the absence of any topics in the domains column for the “modeling” conceptual category. The writers of the CCSSM did not incorporate a specific modeling standard, but instead suggested that modeling be woven throughout other standards.

Table 1.3
The CCSSM Grades 9–12 Conceptual Categories and Domains

Conceptual Categories	Domains
Number and Quantity	The Real Number System, Quantities, The Complex Number System, Vector and Matrix Quantities
Algebra	Seeing Structure in Expressions, Arithmetic with Polynomials and Rational Expressions, Creating Equations, Reasoning with Equations and Inequalities
Functions Overview	Interpreting Functions; Building Functions; Linear, Quadratic, and Exponential Models; Trigonometric Functions
Modeling	
Geometry	Congruence; Similarity, Right Triangles, and Trigonometry; Circles; Expressing Geometric Properties with Equations; Geometric Measurement and Dimension; Modeling with Geometry
Statistics and Probability	Interpreting Categorical and Quantitative Data, Making Inferences and Justifying Conclusions, Conditional Probability and the Rules of Probability, Using Probability to Make Decisions

The CCSSM is leading to some significant activity in developing new assessments, an approach that builds on the view of standards as a means to consider “how that [what students should know] is to be measured and/or documented” (Confrey 2007, pp. 6–7). The U. S. Department of Education is funding two state-based assessment consortia, the Partnership for Assessment of Readiness for College and Career (PARCC) and the Smarter Balanced Assessment Consortium (SBAC), to align assessments to the CCSSM by the 2014–15 school year. Having these two groups create, test, validate, and disseminate innovative assessment systems in less than four years is an ambitious goal; collaborations across states, however, will be more efficient and should foster greater coherence than the current accountability system.

Finally, the third component of Confrey’s (2007) definition of *standard* describes standards’ role in “restricting excessive variation.” Adopting the Common Core State Standards (CCSS) means that states agree to implement, word-for-word, 100 percent of the English language arts and mathematics standards. Each state, however, has the option to incorporate up to 15 percent self-selected standards in addition to the CCSS. According to the criteria used to write the CCSS (CCSSI 2010), adherence to these basic standards ensures coherence across states to assist with students’ mobility and provides rigorous content and application for global competition. In addition, the CCSSM’s writers have benchmarked their standards internationally and built them on the strengths of current state standards to give students the knowledge required for success in the twenty-first century. Although the standards’ implementation will be key, agreement by states to follow the CCSS word-for-word is a first step in decreasing variation among teachers, schools, districts, and states. Mathematics teachers should also review the English language arts standards listed under “Scientific and Technical Reading and Writing.” These standards, which lend the mathematical practices standards considerable support, express the importance of all students being able to apply mathematics, scientific and logical reasoning, and argumentation.

Part 3: Current and Future Recommendations

“While the development of the CCSS and widespread adoption is an accomplishment, in fact its passage marks only the beginning of the work to be done through professional development, creation of instructional materials and related tools, and phased implementation” (Confrey and Krupa 2010, p. 2). States will have numerous items to consider as they begin to incorporate the CCSS into instruction. Two frameworks can help in understanding the complexities involved in the implementation of the standards. First, the National Research Council’s (2002) *Investigating the Influence of Standards: A Framework for Research in Mathematics, Science, and Technology Education* provides a comprehensive display of forces and channels that need examination in order to achieve the desired impact on students’ learning (see fig. 1.2).

Second, Confrey and Maloney’s (in press) framework presents standards and high-stakes tests as the bookends of the instructional process. The framework connects the two to show that the data from the high-stakes exams guide both instructional practice and standards’ development. In figure 1.3, the triangle indicates that effective classroom practice, the heart of quality instruction, operates through the interactions among implemented curriculum, instructional practices, and forms of classroom assessment. The

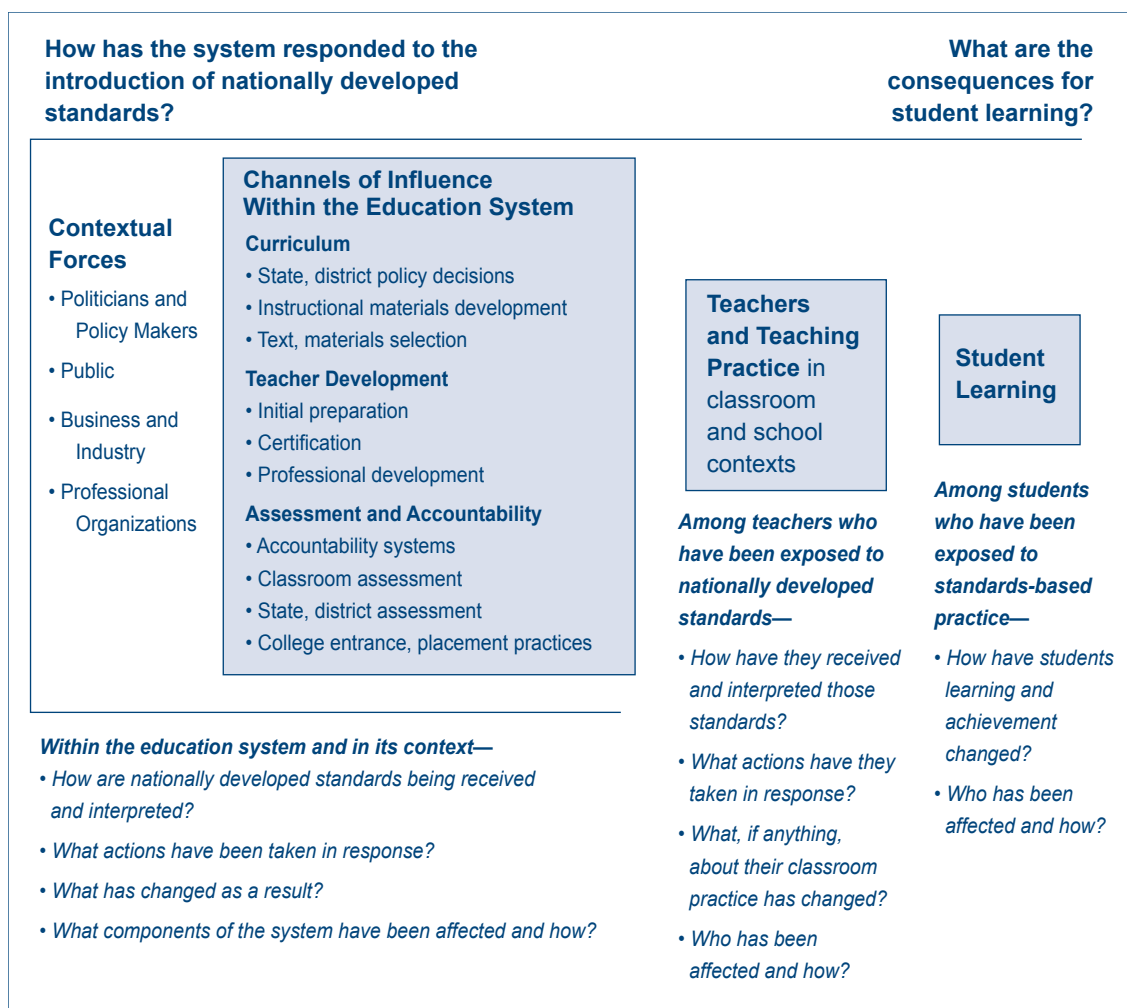


Fig. 1.2. Factors to consider when implementing standards (NRC 2002)

double arrow to Professional Development signals that fostering rich classroom practices is enhanced when professional development is provided by teachers and when “best practices” of experienced teachers, along with research, guide the professional development.

The second framework complements the first with its focus on the effects of the CCSS on classroom practice in order to effect student outcomes. Although each framework centers on the instructional core, each framework’s outcome is on students’ learning, documented through assessment. The rest of this section will focus on our recommendations for what should be done now to implement the CCSSM properly, followed by future considerations to keep the current momentum moving in a positive direction.

The most crucial initial steps, to be taken now to ensure the proper, widespread implementation of the CCSSM, are considering phasing models and launching professional development. Compared to most of the current state standards, the CCSSM contains significant modifications of topics taught at each grade level; thus, local and

state mathematics education communities should take appropriate phasing models into account. These should begin with awareness, by all stakeholders, and proceed toward specific guidelines for coordinated implementation across grades (Confrey and Krupa 2010), concentrating on the trajectories. We consider important stakeholders to include district-level administrators, principals, coaches, teachers, parents, university faculty in mathematics and mathematics education, external professional development providers, and students. The phasing model discussions should include strategies for the transition grades, 5–6 and 8–9, where changes in schools and schooling practices necessitate careful attention to students' success.

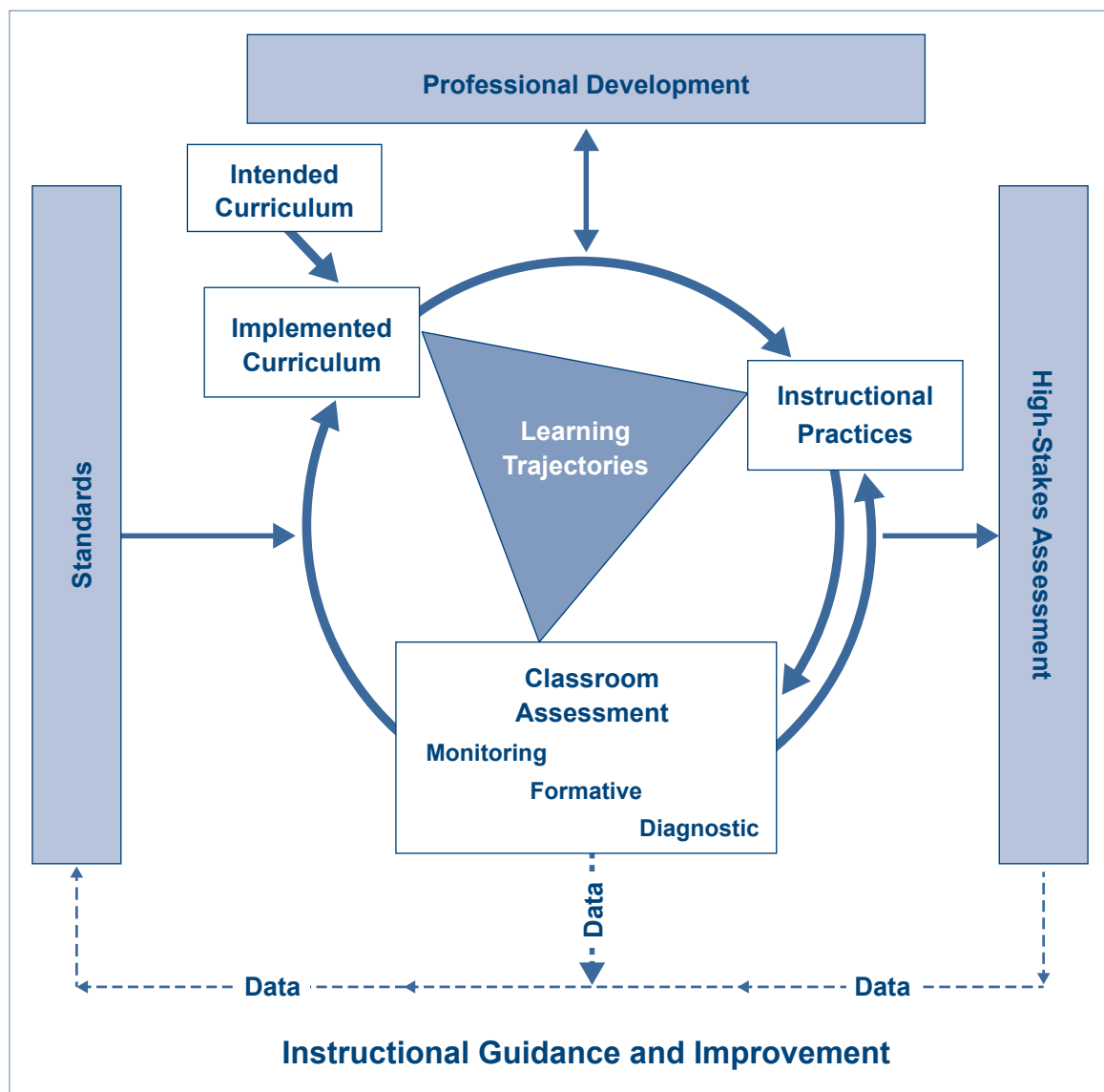


Fig. 1.3. Embedding the instructional core in an accountability framework (Confrey and Maloney in press)

Adequate professional development is imperative to ensure teachers are knowledgeable not only about the major shifts in the CCSSM, but more importantly, so that teachers can begin to lead with the mathematical practices. Since teachers typically use the same instructional practices used by their teachers (Ball 1988; Tyack and Cuban 1995), quality professional development should be designed and disseminated to help teachers negotiate the eight mathematical practices in relation to the content standards. This will help to improve and strengthen teachers' mathematical content and pedagogical knowledge (Loucks-Horsley et al. 2003).

Long-term goals, to be immediately convened, include the development of new summative assessments (under the direction of the two consortia), the development of effective formative and diagnostic assessment systems, the selection and revisions of curricular materials aligned with the content and practices of the CCSSM, and the creation of a process to ensure that the CCSSM remains a living document (Confrey and Krupa 2010; Krupa 2011). Work at the national level has already begun on the summative assessments; as this work continues, sharing insights and expectations with teachers will be important, so that they can incorporate assessments seamlessly into the 2014–15 school year.

To strengthen the instructional guidance teachers provide to each student, formative and diagnostic assessment systems should be embedded into teachers' instruction. These should be designed so that student understanding of main topics is apparent, providing teachers with the information they need to take the next steps with their instruction. These should also be designed to help teachers think critically about students' thinking and problem-solving strategies. Together these systems will help determine which steps need to be taken, each year, for a student to move through a successful learning progression. For more information on specific steps to be taken to create a formative and diagnostic assessment system, see Confrey (2011).

Curricula designers and publishers, front-line implementers from states and school districts, policy experts, and mathematics education researchers all share a fundamental assumption—that *curricula matter!* The scale of the change that the CCSSM proposes requires designated curricular materials that align with the content standards and mathematical practices (Confrey and Krupa 2010). Since the mathematical practices are presented independent from the content standards, there is a risk that the practices will be underemphasized. Curriculum developers need to align current materials to the CCSSM to produce high-quality mathematics curricula. Textbook adoption committees need to pay careful attention to distinguishing between materials that genuinely align to the CCSSM and those that claim alignment as a marketing strategy. Further, both communities need to ensure that their product or textbook selection embeds the mathematical practices throughout.

Finally, and most important, to guarantee the CCSSM's long-term success, and consequently students' understanding of mathematics, the CCSSM should remain a living document. Incorporating all the components of implementation depicted in figures 1.2 and 1.3, the entire mathematics education community should dedicate themselves to the standards' continual improvement. This improvement should include short-term fixes, medium-term adjustments, and long-term review and modification (Confrey and Krupa 2010). Revising the standards over time, on the basis of evidence collected from the field, is imperative. The success of the CCSSM will rest on whether they increase

equity in outcomes, because clear delineation of curricular targets and better forms of testing should permit more effective instruction delivery. In addition, implementing the CCSSM should improve the United States' relative performance on international assessments.

Part 4: Conclusions

This is an exciting time for the future of mathematics education, but one that will require experts to collaborate to accomplish our shared goal of strengthening students' learning. The history of mathematics standards development in the United States suggests that even in a view recognizing states' rights, a coordinated effort to improve mathematics education should serve our highly mobile population better. Reviewing the CCSSM content suggests that needs will exist for teachers' professional development and for using communities of practice to converse and collaborate on plans for implementation. Furthermore, the position outlined in this chapter argues for the importance of teachers' attention to practices to ensure students not only learn the content specified at grade level but have adequate preparation to succeed at more advanced levels because of their facility with the practices. It also argues that the CCSSM must be a living document, owned by our professional community, to ensure that future revisions reflect what we learn from the standards' implementation process.

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