Whiz Kid

Rose Is Rose by Pat Brady and Don Wimmer

1. Add 47 + 21 + 16 mentally, without writing the numbers. Explain how you did it.

2. Add 47 + 21 + 3 + 16 + 9 + 4 an easy way, without writing the numbers. Explain how you did it.

3. Do this problem mentally, without writing the numbers. Explain how you did it.
   $47 + 21 + 16 + (321 - 79 + 17) \times 0 + 10 \times (11 - 1)$

4. Do this problem mentally, without writing the numbers. Describe how you did it.
   $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19$

5. These problems are called "'Round the World" problems because you can do them the long way and go "around the world" or you can find a shortcut and do them mentally. Try these problems mentally.
   a. $[(542 + 679) \times (45\ 1/2 - 40 - 5\ 1/2)] + (2 \times 17 \times 5) - (20 \div 2)$

   b. $(3\ 1/3 \times 13 \times 3) + (-1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9 + 10)$

Challenge

6. Make up your own 'Round the World problem. Show how to do it an easy way.

Solutions to the Cartoon

1. It is hoped that some students will use a method other than the traditional algorithm and that a class discussion will bring this to light. They might, for example, add the tens first, then add on the ones in chunks.

2. Some students will put together “friendly numbers,” adding 47 + 3, 21 + 9, and 16 + 4, then adding 50 + 30 + 20 to get 100.

3. An easy way to add it mentally, since students know that 47 + 21 + 16 = 84, is to notice that the middle of the expression gives 0 and the last part gives 10 × 10, or 100. So the sum is 184.

4. Students might notice that 1 + 19 = 20, 2 + 18 = 20, and so forth, down to the 9 + 11 = 20. That is a total of nine 20s, or 180, with the 10 left in the middle. So the final sum is 180 + 10 = 190.

5. a. Since (45 1/2 – 40 – 5 1/2) = 0, the result of the product with (542 + 679) in the first part of the expression is 0. In the next expression, (2 × 17 × 5) is 17 × 10 or 170, and (20 ÷ 2) = 10. So the right half of the expression, and thus the entire expression, is 160.

   b. (3 1/3 × 13 × 3) gives 130, since 3 1/3 × 3 = 10 and 10 × 13 = 130. The expression (–1 + 2 – 3 + 4 – 5 + 6 – 7 + 8 – 9 + 10) can be added in pairs, working right to left, giving +5. So the answer is 135.

6. Answers will vary. Students might enjoy viewing others’ problems to see if they can find their shortcuts.

Field-Test Comments

(Note: This activity was field-tested with advanced students from the fifth to the eighth grades, but the activity is appropriate for all students, based on their individual needs.)

I used “Whiz Kid” with 54 gifted fifth graders during the third nine weeks of school in our pull-out resource classroom. The students laughed out loud when they first read the cartoon, which added to the interest level and helped them visualize the concept—mental math—at the start.

The students had already studied basic operations, order of operations, and so forth. The trickiest problem for them was 3. Some of my brightest students missed that one because they didn’t realize the “times zero rule” was involved. Almost 100 percent of the children who missed it answered “100” for that one. They thoroughly enjoyed the activity and discussed their problem-solving strategies afterward for quite a while.

The students particularly enjoyed the ’Round the World problems. Currently, I am planning to create an assignment using their own such problems to give them more practice using the idea of finding shortcuts to computation.

—Tina Goggans Gay
Taylor Elementary School
Lawrenceville, Georgia

I used this cartoon in January with fifth graders who are studying seventh- and eighth-grade mathematics. We had not previously discussed strategies to use in solving problems mentally. After several students had questions, I stopped the class and held a discussion that proved extremely helpful to the students. After the discussion, the students understood the need to read an expression first to see if there are any shortcuts to take. The students really enjoyed the idea of friendly numbers. This assignment was completed in class and in pairs. In the future I will discuss strategies first and use the assignment as an exit card or homework assignment.

—Ellen Horlick
Wyngate Elementary School
Bethesda, Maryland

I did the "Whiz Kid" cartoon with 60 eighth-grade Honors Algebra students. They took it as an assignment, read it, and followed the directions on their own. They enjoyed it and did better when they followed the problems already made up than when they wrote their own problems. I asked them to comment on the activity; here are some of their comments:

- I was confused until I realized you could cancel stuff out or get 10; that made it so much easier.
- This was fun, but it was kind of hard.
- It was fun and interesting. It got me to realize I could do much more in my head than I thought!
- It was an enjoyable, sometimes intimidating experience.
- I didn't really understand the cartoon, but I do like the mental math problems.
- This can make students realize that there may be a shortcut in solving math problems.
- I enjoyed it because it makes you use your brain in creative ways.

I then used the concept of the 'Round the World problems with my on-level algebra classes, and they liked the fact that there was a shortcut way to do the problem in your head. It took quite a few tries and working with them so they did not get discouraged, but the cartoon was a success in my classes.

—Ann Henley
River Trail Middle School
Duluth, Georgia

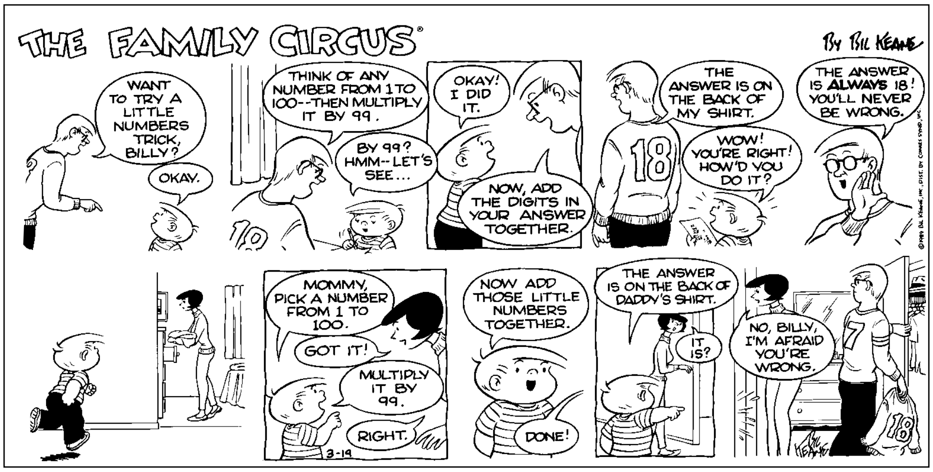
Other Ideas

- Using problems like these on a regular basis will teach students to "look ahead" in computational situations for ways to make mathematics easier.
- Advanced students might enjoy making up such problems using algebraic language.

Adapted from Cartoon Corner, Mathematics Teaching in the Middle School, September 2007, page 92, edited by Andy Reeves and Mary Lou Beasley.

What's My Number?

Family Circus by Bil Keane



1. Billy's dad says he knows a trick. Verify the "trick." Pick several numbers between 1 and 100, multiply your number by 99, then add the digits of your answer. Do you think this is a trick? Can you explain why it works?

2. Billy's mom multiplied her number by 99 without using paper and pencil or a calculator. Find and explain a method for multiplying any number by 99 "in your head" as she did.

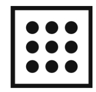
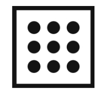
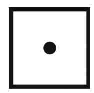
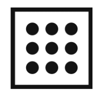
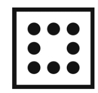
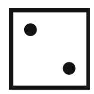
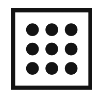
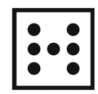
3. To understand why tricks with 9s work, let's look at the easier problem of multiplying by 9. Make a list of the first ten multiples of 9 ($1 \times 9 = 9$, $2 \times 9 = 18$, and so on). What do you observe about the digits of the products in your list?

4. Make a simple visual model of the multiples of 9 by using a table (such as the one started below) for the units, tens, and hundreds columns. Use markers or drawings to represent the first ten multiples.

Multiples of 9	10s	1s
$1 \times 9 = 9$		
$2 \times 9 = 18$		
$3 \times 9 = 27$		

5. Use your model to explain why the digits of the multiples of 9 sum to 9.

6. Generate a new table for multiplication by 99. (You will need to add a thousands column, as in the table below.) Show the first ten multiples of 99 ($1 \times 99 = 99$, $2 \times 99 = 198$, and so on). What do you observe about the digits of the products in your list?

Multiples of 99	1000s	100s	10s	1s
1 × 99 = 99				
2 × 99 = 198				
3 × 99 = 297				

7. Add some additional rows to show the products of numbers you used to test the trick in the cartoon.

8. Use the patterns you observed in the table to explain why the trick works.

Challenge

9. Extend the trick to multiples of 999 or 9999 or more.

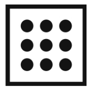
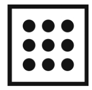
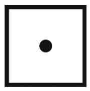
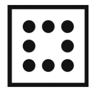
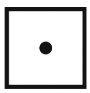
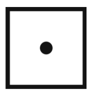
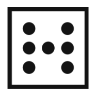
Solutions

1. The result is the same for any number.

2. Answers may vary. Students should recognize that since 99 = 100 – 1, an easy way to multiply any number by 99 is to multiply it by 100, then subtract the original number ($99 \cdot n = 100 \cdot n - n$).

3. The digits of the product always sum to 9.

4.–5. Each successive multiple of 9 adds 9 to the previous multiple. In the model, the number of markers in the 10s column increases by 1, whereas the markers in the units column decrease by 1. For the first 10 multiples, 9 markers are systematically rearranged.

100s	10s	1s	Multiples of 9
			11 × 9 = 99
			12 × 9 = 108
			13 × 9 = 117

6.–8. Eighteen markers are required to model 99 (9 tens and 9 units). Each successive multiple of 99 adds 1 marker in the 100s column and removes 1 from the units column. All multiples of 99 from 99 × 1 through 99 × 100 can be represented with rearrangements of the 18 markers. The sum of the digits will always be 18.

9. Similar patterns arise. The first 100 multiples of 999, from 999 × 1 through 999 × 100, can be represented with rearrangements of 27 markers. The digital sums of these multiples are all 27; the digital roots are all 9.

Field-Test Comments

I used this activity with my sixth-grade prealgebra students. I was teaching divisibility rules, so I thought that using this cartoon would be a fun way for students to learn the rule for 9 but also learn how to multiply by 99 or 999. I introduced them to the activity by asking them to give me a two-digit number, which I would then multiply by 99 in my head. They were impressed that I had a quick answer for them and wanted to learn the method.

They found the questions easy to complete and recognized the pattern. They realized that the trick would work with multiples of 999 or 9999 because the digital roots are 9. Some students were able to figure out how to apply these patterns to rules for divisibility. Some students recognized that it would be in decimal form but had difficulty verbally expressing what it would look like. There was an "a ha!" moment when I modeled 57/99.

We officially learn the 9s concept later in the year, so it will be interesting to see if they remember this lesson.

—Lynn Prichard
Williams IB Middle Magnet School
Tampa, Florida

I presented this set of problems to my students as a reformatted activity packet, with the tables and questions in place. Since many of my students have special needs and require additional scaffolding, large spaces for writing are necessary. For some of my special needs students, a review of place value was a prerequisite for success as well as for living in our base-ten society.

Five Math 8 classes (approximately 150 students) completed the activity. They were easily engaged after I discussed the cartoon scenario with them. "Wow, that is cool!" was the overall sentiment expressed as students worked with partners and calculators to explore the patterns. They were able to discern the digit patterns with multiples of 9 and 99 and to explain why the patterns occurred. They found easy ways to multiply by 9 and 99, which worked correctly.

Students for the most part predicated correctly what would happen if the process was extended to multiples of 999 or 9999, but were surprised as they identified the different patterns that emerged when one-digit numbers were divided by 9 and when two-digit numbers were divided by 99. Very few could explain even vaguely why the divisibility test for multiples of 9 works.

—Deborah Regal Coller
Pathfinder Middle School
Pinckney, Michigan

Other Ideas

- Investigate what happens if you take any one-digit number and divide it by 9 or any two-digit number and divide it by 99 (or any three-digit number divided by 999, and so on).
- Determine if the pattern observed in question 3 continues if you include 11 × 9 and 12 × 9.

Adapted from Cartoon Corner, *Mathematics Teaching in the Middle School,* March 2010, page 382, edited by Stephen P. Smith and Peggy House.