Introduction

A second-grade class was collecting lunch money; lunches each cost $2. Because one student had brought in a $20 bill to pay for future lunches, students began to offer ideas about how many lunches it would buy. Ezra thought it would pay for 10 lunches. Micah said that $10 would pay for 5 lunches. Jared asked how many lunches $5 would pay for. After a few moments of silence, Leah, who had been deep in thought, said, “Two and a half. You could get 2½ lunches for $5.” Then she giggled at the silliness of half a lunch.

Students in this class came upon a problem that involves two quantities—a number of dollars and a number of lunches—that vary in relation to each other. For any number of dollars, there is a certain number of lunches that it will buy: $2 pays for 1 lunch, $4 pays for 2 lunches, $6 pays for 3 lunches, and so on. This problem involving a correspondence from one set (number of dollars) to another (number of lunches) is an example of a function.

In this seminar, *Patterns, Functions, and Change*, you will explore a variety of examples of different kinds of functions: If you drop the same number of pennies into a jar each day, the number of pennies in the jar depends on the number of days you have been collecting pennies. If you are building squares out of square tiles, the number of tiles in your square depends on the number of tiles in one row. If you burn a candle, the height of the candle depends on the length of time it has been burning.

As you work with different kinds of functions, you will explore how such situations can be represented. You will consider conventional forms of representations—tables, graphs, and formulas written algebraically—as well as representations invented by students to keep track of a phenomenon that involves change. For example, how do second graders represent the money-lunch relationship? How does a change in the amount of money you have change the number of lunches you can buy?

You will explore a variety of features of a function and examine how these features appear in the different representations. Is the function increasing, decreasing, or staying the same? Is it increasing or decreasing at a steady rate, or is the rate of change varying?

The cases in the casebook present students in elementary and middle school engaging with these and other ideas about functions. Like other DMI modules, the cases were written by elementary and middle school teachers recounting episodes from their classrooms. Most teachers had inclusive classrooms; a few worked in pullout programs exclusively with students with special needs. The range represents schools in urban, suburban, and rural communities. The teacher-authors, who were themselves working to understand the “big ideas” of the elementary- and middle-grade mathematics curriculum, wrote these cases as part of their process of inquiry. They came together on a regular basis to read and discuss each other’s developing work.

The cases are grouped to present students in classrooms who are working on similar mathematical issues related to functions. Through the cases, you will examine students’ initial work with repeating patterns and explore how this work can be extended to support the study of functions; you will see students' early encounters with conventional and informal representations; and you will identify students' insights and confusions as they explore functional situations of greater complexity.

In the cases in chapter 1, students work with repeating and growing patterns, a precursor to work with functions. Chapters 2 through 5 explore various aspects of linear functions: How are they represented? How can they be compared? What are the connections between linear relationships and direct proportions? Chapter 6 presents students' work with functions that can be defined by formulas but when graphed, are not straight lines. In chapter 7, the functions are not defined by formulas but instead by gathering data, such as the temperature outside the classroom at 9:00 a.m. on different days of the year.

Chapter 8, the last in the casebook, is the essay "The Mathematics of Patterns, Functions, and Change for the Kindergarten–Grade 8 Classroom." It reviews the ideas explored in the seminar, identifying recent research findings that touch on the issues explored in the cases (chapters 1 through 7).