Companion Sheet

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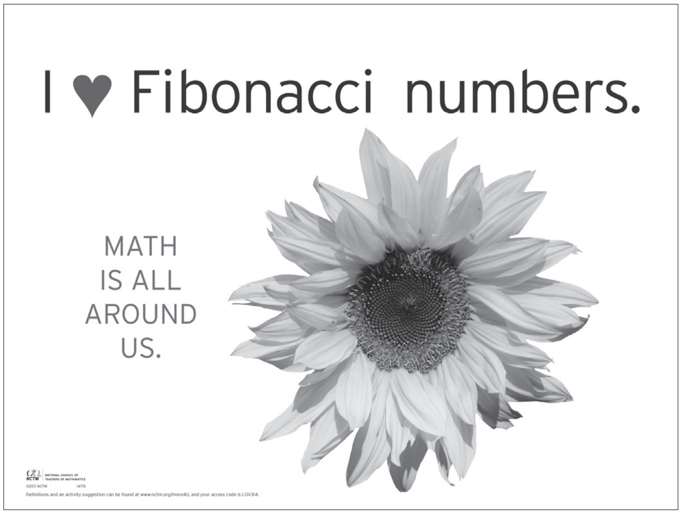

DEFINITION

The Fibonacci sequence, 1, 1, 2, 3, 5, 8, 13, 21, 34, ..., where the first two terms in the sequence are each 1 and every successive term is the sum of the two previous terms, can be defined recursively as follows:

$$F_n = \begin{cases} 1 & \text{when } n = 1 \\ 1 & \text{when } n = 2 \\ F_{n-2} + F_{n-1} & \text{when } n \geq 3 \end{cases}$$

The ratio of successive terms approaches the golden ratio, $ø = \dfrac{1+\sqrt{5}}{2} \approx 1.61803399$.

MATH IS ALL AROUND US

Fibonacci numbers occur often in nature, in particular in phyllotaxy, the study of leaf arrangement on a stem. The spirals on sunflower seed heads, on pineapples, and in pine cones are invariably Fibonacci numbers or multiples of Fibonacci numbers.

ACTIVITY

Much has been written on Fibonacci numbers and the golden ratio (see, for example, *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number* by Mario Livio; *Divine Proportion: Phi in Art, Nature, and Science* by Priya Hemenway; or *The Golden Section: Nature's Greatest Secret* by Scott Olsen). The following activity is less well known than other facts about the Fibonacci sequence or the golden ratio.

Whytoff's Game is a strategy game similar to Nim. There are two piles of matchsticks, with the number of matchsticks in pile A not equal to the number of matchsticks in pile B. A move consists of taking any number of matchsticks from one of the two piles or an equal number of matchsticks from both piles. The object of the game is to take the last matchstick.

Playing a few rounds of the game will indicate that several winning combinations exist, denoted by the ordered pair (a, b), where the values in the pair represent the number of matchsticks in the two piles: (1, 2), (3, 5), (4, 7), (6, 10), (8, 13), (9, 15), and so on. The nth winning combination can be found recursively if we know the first pair, that the difference between successive pairs increases by one, and that the first element in each pair is always the first natural number that has not previously been used in a pair. Thus, the fourth pair starts with 6 because 5 has already appeared (as the second term in the second pair). The nth pair can also be expressed explicitly as ($[nø]$, $[nø] + n$), where ø represents the golden ratio and $[x]$ has the usual meaning of the greatest integer function.