

About This Book

In the discussion of the Algebra Standard in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM] 2000), NCTM takes the stand that by explicitly working to develop algebra concepts and algebraic thinking from the prekindergarten years, many, in fact most, students can complete a reasonable equivalent of algebra 1 by the end of grade 8. In the elementary grades, algebraic reasoning is developed informally. This initial development provides the background for a more systematic study of algebra in the middle grades. This book focuses on the development of algebraic reasoning and algebra concepts in grades 6–8. Two themes span the content of algebra at this level: (1) using mathematical models to represent and understand quantitative relationships and (2) representing and analyzing mathematical situations and structures. The concept of mathematical function, which encompasses both themes, receives considerable attention at these grade levels.

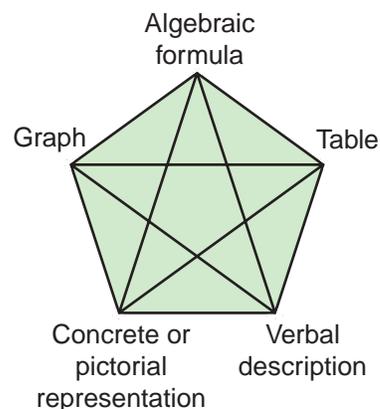
Explorations that develop from problems that can be solved by using tables, graphs, verbal descriptions, concrete or pictorial representations, or algebraic symbols offer opportunities for students to build their understandings of mathematical functions. The relationships among the five representations can be shown by the framework in the margin (Bowman 1993, p. 40).

Each representation highlights some aspect of the concept of function, but no single representation can help students develop the deep understanding of functions that is needed. In fact, it is the processes connecting one representation to another that help students make sense of the concept of function. For example, moving from a table of data to a graph of those data may be done by constructing the graph by hand. The manual process involves such actions as drawing and scaling the x - and y -axes and plotting pairs of points. Alternatively, using a graphing calculator to move from the data to a graph involves choosing an appropriate graphing window and making sense of the axes as seen on the screen of the graphing calculator. Students may not be explicitly aware of the process of plotting points, even though they can identify specific points on the graph by using the trace key, for example. Students' understanding involves an awareness of the similarities and differences in the processes that occur when they construct a graph of tabular data by hand and with technology. Such an awareness helps them develop a richer understanding of the relationship between a table and a graph of a set of data—an important component in building an understanding of the concept of function.

The four chapters in this book consider some of the topics related to integrating the themes of using mathematical models and representing and analyzing mathematical situations and structures. The activities and problems require students to use multiple representations related to work with functions and highlight some of the interactions that may occur among these representations.

In chapter 1, “Understanding Patterns, Relations, and Functions,” the topics include—

- clarifying the different uses of variables, and
- exploring, representing, analyzing, and generalizing a variety of



The development of new vocabulary tied to algebraic concepts is important in the middle grades. Terms such as rate of change, variable, and continuous data label new ideas. A formal introduction of terminology is not initially as important as the modeling of its use by the teacher. As students gain experience with new ideas, they should be encouraged and expected to integrate the appropriate terms into their oral and written mathematical communication.

Key to Icons



Principles and Standards



Blackline Master



CD-ROM

Three different icons appear in the book, as shown in the key.

One alerts readers to material quoted from *Principles and Standards for School Mathematics*, another points them to supplementary materials on the CD-ROM that accompanies the book, and a third signals the blackline masters and indicates their locations in the appendix.

patterns using tables, graphs, words, and, when possible, symbolic rules.

Chapter 2, “Analyzing Change in Various Contexts,” includes—

- investigating how changes in one variable relate to changes in a second variable, and
- describing or portraying situations using graphs that may involve both constant and varying rates of change, with attention given to linear relationships.

In chapter 3, “Exploring Linear Relationships,” the topics include—

- identifying functions as linear or nonlinear and contrasting the properties of these two types of functions by examining tables, graphs, or equations;
- investigating relationships between symbolic expressions and the graphs of lines, paying particular attention to the meaning of intercept and slope; and
- using graphs to analyze the nature of changes in quantities in linear relationships.

Chapter 4, “Using Algebraic Symbols,” includes—

- using symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;
- recognizing and generating equivalent forms for simple algebraic expressions; and
- solving linear equations.

Each section in a chapter begins with a short discussion of important mathematical ideas. Then one or more strategies are presented that may provide insights into what students already know about those ideas. These strategies include examining the content of the curriculum from earlier in the year or previous years, as well as using preassessment tasks that may serve both to yield evidence of students’ understanding and prior knowledge and to introduce a topic.

The remainder of each section presents one or more sample activities, many of which have blackline masters, which are signaled by an icon and can be found in the appendix. The CD-ROM, also signaled by an icon, contains two applets for students to manipulate and resources for professional development. There are many ways to help students develop the important mathematical ideas in the activities, but no matter how the curriculum is configured, it is important to pay attention to the mathematical content and to select activities that afford students an opportunity to develop a conceptual understanding of important mathematics and, at the same time, to accommodate students’ ways of making sense of information. Problems need to be sequenced carefully and taught according to a supportive instructional model that contributes to learning.

Although sequencing may be suggested by the order of the chapters, the interaction among the topics in these chapters is dynamic and is related to developing an increasingly deep and more sophisticated understanding of algebra. Thus, for example, although the study of patterns is used to stimulate the use of symbolic language and to introduce linearity, the topic may be revisited frequently, particularly as new families of functions are introduced and explored.