



Foreword

THAT I WOULD BE writing a Foreword for a book on teaching math seems at once both ironic and absolutely appropriate.

The irony stems from my long and generally unhappy life as a math student. In elementary school, math was neither easy nor hard for me. It was simply something I did. It evoked neither pleasure nor pain. I suppose it was a bit like household chores—something necessary to get through the day, but certainly nothing to be coveted. Through those years, math was as rote as household chores. I memorized the required number sets, counting, and multiplication, absorbed algorithms I watched my teacher scribe on the blackboard in the front of the room, and repeated what I saw. There was no joy in it, but it was doable.

Once I entered the world of algebra, however, math became a sinister thing. Letters invaded the numbers. Equal signs took on a super power. While I could occasionally duplicate what my teacher chalked in front of us, I could no longer commit the strings to memory. There was no reason to them, and worse, I didn't even really grasp the notion that they were all about reasoning. I simply knew that I could no longer succeed with math. The conclusion I drew, of course, was that I was no good at math.

I received a one-year reprieve from my self-imposed verdict when geometry intervened between Algebra I and Algebra II—and before trigonometry had its way with me. In that one year, math was glory. In that one year, math was about reasoning. There was order to it, like the order in the universe. There were words that marched along with the figures and problems. I loved it in the way that I loved good writing. It revealed beauty and it dignified my possibilities as a learner rather than eroding them.

But the interlude didn't last long enough to erase the damage done earlier or to reconfigure my sense of myself as outpriced by math. It took many years for me to realize that the dismay I felt in the presence of math was neither a fault in me nor an indictment of math as a content area. Rather, I came to see myself as one among a legion of students who had abandoned a content area—or even school—because the way a subject was taught drained all the life from it, even as it drained life from the learners.

The appropriateness of my writing this Foreword is, of course, not completely separate from the irony. I have spent my career as a teacher (an English teacher, to be clear) and then as a professor with an evolving belief, then a conviction, and

finally a passion for teaching that opens up possibilities and a life for every student in a classroom. In time, I applied a name to the system of ideas that pointed the way to achieving that sort of success in academically diverse classrooms. That name, of course, is *differentiation*.

I have read and continue to read many books related to differentiation. I learn from many of them, and I turn away from a good share of them as well, when they appear to trivialize differentiation or minimize the complexity of teaching a broad range of students in a meaningful way.

In Marian's work, I find an educational soulmate.

The two strategies she presents in this book are streamlined—they even seem spare. And yet in her writing about them, I see all of the components I have come to value in classrooms that serve academically diverse learners in a powerful way. The five elements I write about as core to effective differentiation are: an invitational learning environment, a meaning-rich curriculum, assessment that informs both teaching and learning, consistent attention to learner differences through responsive instruction, and creating classroom routines that balance predictability and flexibility.

The focus of this book is on meaning-rich curriculum and responsive instruction through open questions and parallel tasks. In both areas, I not only see reflected the best of what our field knows about curriculum and teaching in response to students' varied entry points, but I see what math should always be—should always have been—to students: a language of reason, a catalyst for developing every learner's capacity to think, a way forward in the world. This book lays out those elements through explanation and example.

Woven more subtly, but no less powerfully, throughout the book are the other elements I've come to champion as well. When a teacher asks questions that every student can contribute to answering, and presents work that provides options for student exploration, the learning environment becomes more invitational for every learner. The atmosphere of winners and losers is diminished. Community develops.

The book speaks, too, to the reality that the teacher who consistently observes and listens to students as they work derives insights that contribute to more informed and effective teaching. And in a very interesting way, the use of open questions and parallel tasks (rather than adherence to fixed and coverage-focused routines) requires teachers to become more focused on supporting success for a range of students.

Freeman Hrabowski, a noted mathematician and president of the University of Maryland Baltimore County, says that math gives him goose bumps, and always has. If I almost never understood that reaction as a student of math, I do now when I read Marian Small's work. Not only do I see the power of math revealed, but I also see her work opening that power to so many students who might never otherwise experience it—and to teachers whose own math experiences, like mine, were one-dimensional at the very best.

I am a disciple of Marian Small's approach to differentiating math!

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Preface

FOR THE PAST 15 years, I have had the opportunity to develop a number of resources that support teachers who seek to differentiate instruction in mathematics. Much of the work I do focuses on differentiation in terms of student readiness and highlights two strategies: open questions and parallel tasks.

These strategies have been employed effectively by thousands of teachers throughout Canada and the United States, as well as overseas. What we have learned is that to differentiate instruction in math, it is important for teachers to think about “bigger ideas,” as they do when teaching other subjects, rather than focus intensely on the usual very tight content pieces in mathematics that capture their attention.

ORGANIZATION OF THE BOOK

The introductory chapter of this resource describes the rationale for differentiating math instruction and explains the two principal strategies that are employed throughout the book: open questions and parallel tasks. Nine content chapters built around the typical organization of mathematical content standards used by many states—as well as a chapter focusing on the mathematics of financial literacy—then illustrate applications of these strategies, followed by a final concluding chapter, an appendix listing the mathematical practices addressed throughout the content work, an appendix containing a template for teachers wishing to develop their own materials, a glossary, a bibliography, and an index.

Chapters 1–10 focus on the following content topics.

- Counting & Cardinality / Number & Operations in Base Ten
- Number & Operations—Fractions (including decimals)
- The Number System
- Ratios & Proportional Relationships
- Operations & Algebraic Thinking
- Expressions & Equations / Functions
- Financial Literacy
- Measurement & Data
- Geometry
- Statistics & Probability

The content strands are not developed sequentially, so Chapters 1–10 can be approached in any order. Within each of the content chapters, a set of big ideas is described that underlies the increasingly sophisticated teaching of that content throughout the grades. In this edition, each question or task is tagged to one or more of those big ideas, as well as to applicable standards for mathematical practice. Alternative versions of practice standards or process standards are used in various jurisdictions. Those applied here are listed in Appendix A.

The suggested differentiating questions and tasks are organized according to the primary, elementary, and middle-level grade bands set out in many states' standards. The object of differentiation is to teach the same broad concepts to students at different developmental levels. The multilayered organization of material within Chapters 1–10 is intended to help make this possible.

Appendix B features a template worksheet that will assist teachers in developing their own materials in support of differentiated instruction through use of open questions and parallel tasks. An example of application of the worksheet appears in the Introduction.

The Glossary defines technical terms used throughout. Each word that appears in the Glossary is shown in boldface type at its first occurrence in the text, and each Glossary entry is annotated with the chapter and page number of the term's first occurrence.

The Bibliography highlights three types of resources: those that are referenced in other parts of the text, those that speak to the issues of teaching to big ideas and differentiating instruction, as well as a number of excellent sources for activities that can be used as-is or used as a starting point for creating open questions and parallel tasks.

The Index focuses on educational concepts—standards, student development, teaching methods and principles, and such—as opposed to mathematical concepts. To facilitate user access to the mathematical topics covered, an Index of Big Ideas is provided, listing all big ideas covered in the content chapters.

MATHEMATICAL PRACTICE STANDARDS

The eight standards for mathematical practice that were addressed in the third edition are also addressed in this edition. These practice standards reflect “processes” that should be elicited from students and goals for orienting and developing students as mathematical thinkers as content is learned.

Some of these practices may or may not be brought out, depending on how a teacher handles the presented questions or tasks, but others are implicit no matter what direction the discussion of the question or task takes.

For example, almost all of the questions and tasks posed in this book require students to make sense of them and persevere in solving them. This is true with open questions, which are often deliberately vague and require students to make sense of the question before they can choose the direction in which to go. This is also true with parallel tasks, where students are likely to try to make sense of both options to decide with which one to proceed.

Students are frequently asked to explain their rationale, requiring them to construct viable arguments. Ideally, teachers would encourage students to critique each other's reasoning, although that would not be implicit in the question itself.

A great many of the questions provided require students to reason, either abstractly or, in the case of number or measurement questions, quantitatively.

Some questions bring out the other practices: modeling, using appropriate tools strategically, attending to precision, looking for and making use of structure, and/or looking for and expressing regularity in repeated reasoning. There are fewer questions attending to precision in this particular resource than in others precisely because differentiation must often allow for varying levels of precision.

Listed with each question and task are the practice standards that are clearly evoked in the question or task. This does not mean that teachers might not also evoke others, depending on their approach to student responses or the student responses themselves.

CHANGES IN THE FOURTH EDITION

Readers familiar with the third edition of this book will note that there are about 60 new questions. The biggest change, though, is the inclusion of a new chapter on financial literacy. This is a topic formally addressed in only some states' standards, but is of value for all students.

Financial literacy is an interesting topic since there are both social issues and mathematical issues involved. Different jurisdictions may emphasize one over the other; this resource blends both, addressing essential understandings about both, although emphasizing the mathematics, since this *is* a mathematics resource. For example, one essential understanding focuses on different ways to show the same amount of money, while another focuses on the fact that financial decisions are based on considering many factors.

IT IS MY HOPE that teachers will embrace the two core strategies—open questions and parallel tasks—that are explained and demonstrated in this book, and find them, as I have, to be helpful to the many children who come into classrooms with highly differentiated mathematical preparation, skill, and confidence. Seeing a child who has been struggling in mathematics start to feel successful is an important goal in my teaching. I have seen the use of the strategies described in this volume make that happen over and over again.