NAVIGATIONS SERIES

NAVIGATING *through* ALGEBRA

GRADES 6–8

Chapter 2

Analyzing Change in Various Contexts

Discuss what is changing and how it is changing in the following situations. Describe what the change depends on.

1. You are riding your bike for 30 minutes and cover 3 miles; you keep riding your bike for another 30 minutes and cover a total of 6 miles.
2. The volume of the sound you hear from the TV as you move away from it.
3. The temperature of a cup of hot coffee that is left for two hours on the kitchen table.
4. The speed of a car approaching a red light.

Important Mathematical Ideas

Thinking about change is not simple. It requires thinking about several things at once: What is changing? Over what time period is the change occurring? What is the rate at which the change is occurring? Is the change following a pattern? Middle-grades students need to learn to think about these questions mathematically. They need to encounter situations that involve change in order to develop a language for describing and talking about change and to develop representations for the types of change that they may encounter.

In a mathematical context, a variable is a quantity that changes. Quantifiable characteristics that change (vary) might include distance traveled, the height of an object from the ground, elapsed time, and speed. Some important relationships may remain constant (e.g., 60 minutes in an hour). Middle-grades students need opportunities to detect both constant and changing relationships in problem situations by asking both What is changing? and What is not changing? Several important ideas need to be addressed when students explore and analyze change situations (Lamon 1999):

- Change in one variable may or may not be related to change in a second variable (the *dependence* or *independence* of one variable in relation to another).
- Situations may demonstrate constant or varying rates of change. The *direction* in which change is occurring may be *increasing*, *decreasing*, or, in some instances, both increasing and decreasing.

- How quickly one quantity is changing in relation to another (the *rate of change*) also is important to note.
- There are several ways to represent change that involves relationships among variables, including the use of words, tables, graphs, and symbols. It is important to be able to relate and compare these different forms of *representation*.
- A *sketch or graph* shows the relationship of one quantity to another. The shape of the graph provides insights into the nature of the change (e.g., it may be regular, thus reflecting a pattern, or irregular).

What Might Students Already Know about These Ideas?

What do your students know about making and reading tables and graphs? Does the idea that one variable can depend on another have meaning for them?

A variety of activities can stimulate students' interest in studying change and, at the same time, offer insights into students' prior knowledge. Bouncing Tennis Balls, for instance, takes advantage of middle-grades students' physical energy while both assessing their prior mathematical knowledge and introducing them to representing and analyzing change.

Bouncing Tennis Balls

Goal

To assess students'—

- ability to collect data and record data in a table;
- abilty to make a graph to display data using correct labels, scale, and so on;
- recognition of what varies in an experiment;
- ability to name the independent and dependent variables in a problem.

Materials and Equipment

- A copy of the blackline master "Bouncing Tennis Balls" for each student
- Tennis balls, one for each team of four students
- Access to a clock or watch with a second hand
- Centimeter graph paper, a spreadsheet program, or a graphing calculator

Activity

In teams of four, students bounce a ball to solve this problem:

> How many times can each team member bounce and catch a tennis ball in two minutes?

A bounce is defined as dropping the ball from the student's waist. One student keeps the time while the second student bounces and catches the ball, the third student counts the bounces, and the fourth student records the data in a table showing both the number of bounces during each ten-second interval and the cumulative number of bounces. Each trial consists of a two-minute experiment, with the number of bounces recorded after every ten seconds (or twenty seconds for fewer data points). The timekeeper calls out the time at ten-second intervals. When the time is called, the counter calls out the number of bounces that occurred during that ten-second interval. The recorder records this count and keeps track of the cumulative number of bounces.

The same process is followed by each student, with the students rotating roles, so that each student can collect a set of data. All the students must bounce the ball on the same surface (e.g., tile, carpet, concrete) because differences in the surface could affect the number of bounces.

Once the data have been collected, each student prepares a graph showing the cumulative bounces over two minutes. This graph can be constructed by hand, by using a graphing calculator, or by using a spreadsheet, depending on the students' experiences and on what information the teacher wants to gather about what the students know and are able to do.

This activity has been adapted from Jones and Day (1998, pp. 18–19).

p. 76

During the preassessment, one teacher observed her students. She noted which students made graphs correctly, paying attention to how they used the idea of scale to set up the time and distance axes. She also listened to students' conversations about their graphs, attending to comments that indicated the students realized that the number of bounces depends on the length of time the ball is bounced and that patterns develop when the ball is bounced in a consistent way.

Discussion

The data from one student's experiment are recorded in table 2.1. The graph in figure 2.1 was made using the graphing calculator, and the graph in figure 2.2 was made using a spreadsheet for the sample data set.

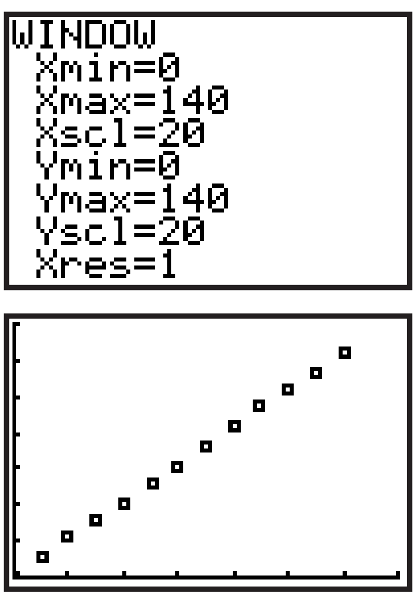
Table 2.1
A Sample Data Set for Bouncing Tennis Balls

Time (Seconds)	Number of Bounces during Interval	Cumulative Number of Bounces
0	0	0
10	11	11
20	11	22
30	9	31
40	10	41
50	11	52
60	10	62
70	11	73
80	11	84
90	10	94
100	10	104
110	10	114
120	10	124

Fig. **2.1.**

A graph made using a graphing calculator

The window shows the scale.

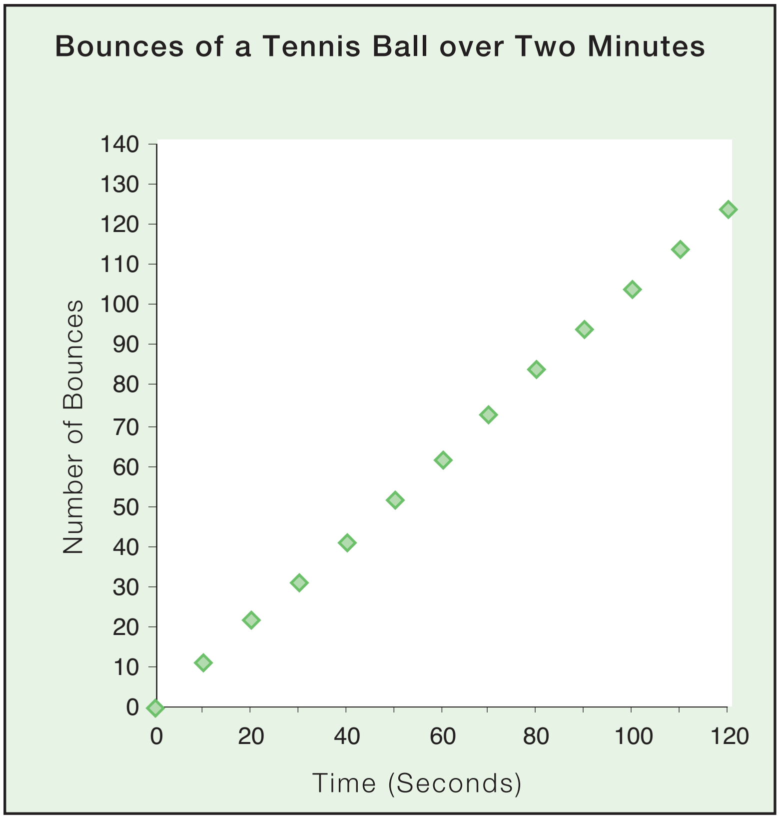


Students present their results to classmates by showing their graphs. The discussion can involve what the students found easy and what they found difficult in completing this task. Students' discussions can be revealing: Can the students identify what varies in the experiment? Do they comment on the dependent and independent variables either implicitly, in their conversations about the graphs, or explicitly, using correct terminology? Do they discuss whether the points should be connected with a line? The numbers of bounces are discrete data, so they should not be connected. Decisions about the scale for each of the axes are important; do the students understand what the graphs would look like if the scales changed? When directed to sketch lines on their graphs in order to notice trends, do they demonstrate some sense that the steepness of a line is related to the number of bounces per second? Your observations related to these and other questions will yield information about what your students appear to know and are able to do that will guide you in making instructional decisions.

An extension of this activity would be for each student to conduct an experiment using, for example, concrete floors and then carpeted floors to investigate the effect of differences in the surfaces.

Selected Instructional Activities

Highlighted here are some fundamental components of a curriculum that addresses content and develops students' understanding by focusing on analyzing change.



	A	B
1	Time (s)	No. of Bounces
2	0	0
3	10	11
4	20	22
5	30	31
6	40	41
7	50	52
8	60	62
9	70	73
10	80	84
11	90	94
12	100	104
13	110	114
14	120	124

Fig. **2.2.**

A graph constructed using a spreadsheet

Building a Sense of Time and Its Relation to Distance and Speed

Initially students need to become aware of their own understanding of time, change over time, and the use of new kinds of measure (i.e., rates). Posing such questions as those listed below focuses their attention on these ideas (adapted from Kleiman et al. 1998).

- How do you measure time? Distance? Speed?
- Give an example of something that might be able to travel at two feet per second.
- What is the difference between traveling at two feet per second and two feet per minute or two feet per hour?

Students can also explore different activities that test their sense of time. They can do the following activities in pairs; in each instance they may want to observe if they overestimate or underestimate the time and try the task again.

- Clap your hands so you clap exactly one clap per second for ten seconds.
- Turn a page in a book at exactly one page every two seconds for twenty seconds.
- Sit still for thirty seconds, letting the timer know by raising your hand when you think thirty seconds has passed.
- Walk at the speed of one foot per second for fifteen seconds.
- Walk the length of your classroom in exactly ten seconds. At what speed were you traveling?

In this context, distance is how far the object or person moves (travels). Speed is how fast the object or person is moving (traveling). Both are described in terms of direction. Distance is measured in such units as feet, miles, or kilometers. Speed is measured in relation to time using units such as meters per second or miles per hour.

One teacher reported that many students initially misjudged time. Before she let them explore the activities in teams, she had them sit for an undisclosed time (e.g., 30 seconds) and make guesses about the amount of time that had elapsed.

Connecting Graphs to Stories

In "Walking Strides," students examine how the time required to walk a given distance varies as the length of their stride varies.