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Numeracy in Nineteenth-Century America

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WHEN twelve-year-old Warren Burton returned to school in the fall of 1812 in his small town of Wilton, New Hampshire, he commenced the formal study of arithmetic. Young Burton was no late bloomer in education: he had been attending his district school since the age of three and a half, first in the summer sessions and then progressing to the winter school. Early on he learned his ABCs and the rudiments of reading, but took up ink and quill only at age nine to start learning the dexterous skill of writing. At twelve the manly art of arithmetic was first set before him: “The entering on arithmetic was quite an era in my school-boy life,” he later reflected, “placing me decidedly among the great-boys, and within hailing distance of manhood” (Burton 1850, pp. 147–48). The youngster already knew how to count: “No man is willing that his son should be without skill in figures” (p. 147), so Burton’s Yankee father had already drilled him on enumeration to 100. But formal arithmetic was a far loftier subject, commanding respect, requiring diligence and memory, and signifying manhood in the mastery of it. The proud boy lettered ARITHMETIC in one-inch high characters on the opening page of his copybook.

Burton’s experience of arithmetic education came right on the eve of major transformations in mathematics instruction in the United

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States. His teacher taught him the old-fashioned way, as the subject had been transmitted for a century and more in the American colonies. Whether taught in New England's rural district day schools or in the urban fee-for-service evening schools dotted in Boston, Newport, New York, Philadelphia, and Charleston, arithmetic was regarded as a vocational subject, a skill whose chief application was in the world of commerce. The appropriate pupil for such study was the twelve-to-fourteen-year-old boy, judged to be mature enough to absorb the arcane techniques of computation as well as sufficiently competent in writing to create a permanent copybook. The method of study involved transcribing formulaic rules out of printed textbooks word for word into the pages of individual copybooks, together with the exact examples offered by the text to illustrate each rule.

Young Burton encountered great difficulty with the concept of carrying tens in addition, "a mystery which our arithmetical oracle, our schoolmaster, did not see fit to explain. It is possible that it was a mystery to him" (Burton 1850, p. 149). One of the most popular texts of the day, Pike's *Arithmetick* (1809), offered this sole explanation:¹ "After adding up every figure in that column, consider how many tens are contained in their sum, and placing the excess under the units, carry so many as you have tens to the next column, of tens" (p. 10). Borrowing in subtraction was an even deeper mystery that failed to resolve for Burton for another year. Again, Pike's baffling algorithm was less than helpful (p. 12): "If the lower figure be greater than the upper, borrow ten and subtract the lower figure therefrom: To this difference add the upper figure, which being set down, you must add one to the ten's place of the lower line, for that which you borrowed." Little wonder that a twelve-year-old might spend weeks or months pondering this perplexity.

By the end of that first term's exposure to arithmetic, Warren Burton had copied the chapters up through reduction descending

1. Pike's first edition was issued in Newburyport, Massachusetts, in 1796; this essay cites the seventh edition published in Boston in 1809. The material up to the rule of three consumed only a third of the 300-page text. The remainder mostly concerned specific business applications—fellowship, tare and tret, stocks and brokerage, insurance, compound interest—along with more-advanced concepts of roots, permutations, and combinations, ending with two pages on the use of logarithms.

(ways to reduce denominate numbers like five hours to the equivalent in minutes or seconds). At age thirteen, he began with addition again, reviewing the three other basic rules (subtraction, multiplication, and division), mastering borrowing, and finally arriving at the rules of interest. In his third year, he recapitulated the whole and at the end conquered the rule of three, “deemed quite an achievement for a lad only fourteen years old” (Burton 1850, p. 151). The single rule of three came in two forms, direct and inverse. Its basic principle, according to Pike’s *Arithmetick*, was that it “teacheth, by having three numbers given, to find a fourth, that shall have the same proportion to the third, as the second hath to the first” (Pike 1809, p. 100). This was “arithmetic enough for the common purposes of life,” concluded Burton (1850, p. 151), no doubt relieved to be done with his study.

The low state of arithmetic instruction in early America reflected the long-standing Anglo-American assumption that numeracy was a business skill needed chiefly by boys heading for careers in commerce and trade. Simple counting in arabic numerals probably was routinely passed on from most fathers to most children, just as Warren Burton indicated, allowing a substratum of primitive reckoning skills to reside in the American population, eminently useful for handling money, paying taxes, toting up firkins of butter, selling excess eggs, and in general thinking about prices in the typically static colonial economy. Unless one were going to buy and sell goods for profit, keep complex account books, enter into business partnerships, loan money, buy stock, survey land, or navigate on ship, arithmetic beyond addition and subtraction was simply not necessary. Advanced arithmetic rules were specialized, difficult, and very narrowly associated with commerce. Interestingly, this meant that not only the low end but also the high end of the social hierarchy could afford to ignore or even spurn arithmetic. Among the educated elite, whose sons were destined to be ministers, lawyers, and professors, arithmetic was not only not necessary but sometimes disdained as a mere tradesman’s subject.

Between the Revolutionary era and the mid-nineteenth century, a slow evolution in thinking about arithmetic education commenced, picked up speed, and finally produced a spectacular alteration in the

way educators recommended teaching the subject.² The most impressive (as well as controversial) change in method came after 1820, when an entirely new approach to the field, championing inductive reasoning, rendered the heavily memory-based books like Pike's *Arithmetick* obsolete, at least for a time. From the 1820s to the 1860s, experts argued the merits of reasoning versus rote learning in an ongoing debate that to some extent foreshadowed late twentieth-century arguments about the best way to teach arithmetic. Equally significant, the repositioning of arithmetic as a school subject had profound implications for the spread of numerical skills into the female half of the population. By the end of the nineteenth century, beginning arithmetic had become a subject taught to six-year-olds, both boys and girls; and algebra and geometry were routine high school subjects, taken by both sexes.³

This chapter proceeds through four sections, roughly chronological in order. The first quickly surveys the narrow and limited state of arithmetic training in the years up to about 1790, as known by evidence drawn from published textbooks, students' copybooks, apprenticeship contracts, and advertisements for evening schools. The second part explores the impetus given to arithmetic by the American Revolution and early state-building work, most prominently the shift to a decimal-based currency and the increased emphasis placed on developing public education to sustain democratic institutions. The third part takes up the remarkable innovations of the 1820s and 1830s that recast arithmetic as a tool to impart critical-thinking skills to students. For the first time, elementary arithmetic education reached far beyond numbers and rote operations to encompass quantitative and analytic reasoning. Of course, the grand claims of the new methods predictably drew severe criticism, and by the late nineteenth century

2. The history of arithmetic instruction in early America was first sketched out by education scholars in the late nineteenth century (e.g., Cajori 1890, 1896). Works since then include Littlefield (1905), Johnson (1917), Smith (1923), Smith and Ginsburg (1934), Middlekauff (1963), Jones and Coxford (1970), Calhoun (1973), and my own book, *A Calculating People* (Cohen [1982] 1999).

3. Thirmuthis Brookman (1910), a teacher at Berkeley High School in California, declared that one year of algebra and one year of geometry were routinely taken in the ninth and tenth grades, after which four-fifths of the students quit the subject; only one-fifth persisted into a third year of high school mathematics.

a reaction had set in with a return to older methods. And finally, the chapter's fourth section considers gender issues in arithmetic. Warren Burton's assumption that arithmetic mastery betokened manhood came in for serious challenge as American girls attained unprecedented levels of literacy and numeracy in the expanding public schools. And yet the stereotype persisted: even at the century's end, when girls' high school graduation rates exceeded that of boys and the nation's teaching force had been largely feminized, mathematics was still regarded as a masculine domain.

The steady rhythm underneath this crucial transformation of arithmetic over the course of the nineteenth century came from the insistent beat of a capitalist economy that required and rewarded greater degrees of reckoning skills from ever larger numbers of people. The early nineteenth century ushered in a fast-developing market economy, one far from static, with bank stocks and speculative investments for the moneyed few and consumer goods, wages, inflation, and panics for everyone else. Yet, ironically, although the spreading market revolution clearly fostered and favored numeracy, it was precisely in these years that formal arithmetic instruction in the schools began to shed its exclusive association with commerce.

Eighteenth-Century Arithmetic

Arithmetic knowledge in the eighteenth century resided in printed textbooks, usually imported from England or reprinted from English texts, and in students' manuscript copybooks, faithful transcriptions of the texts written out by hand and meant to last as a life-long reference work. The most commonly used texts were little more than a collection of rules, written in sentences for ease of memorization. First the reader was introduced to numeration, the written arabic symbols and the place system that gave the numerals their power. Then came the four basic rules—addition, subtraction, multiplication, division—as applied to whole numbers. After that, the order of rules varied quite a bit. Some books rehashed the four rules with fractions, and then yet again with decimals, whereas others first took up commercial calculations using weights and measures, such as reduction descending and ascending. The four basic rules were taught over again as compound addition through compound division, operations per-

formed with denominate numbers (quantities expressed in the intricate denominations of weights and measures). Still other texts jumped to the single and double rule of three, both direct and inverse, returning later to revisit the basic four rules redone in fractions and denominate numbers.

Several texts prefaced their material with frank disclaimers about the scope and order of their presentations. *The Scholar's Guide to Arithmetic* prided itself on omitting "tedious and inconvenient" explanations for the rules, preferring only to print the minimum necessary "to fix in the memory" each operation. Worse still, this text abandoned any idea that arithmetic had a sequential progression to it: "The Order in which the different Rules should be taught is a matter entirely arbitrary, and therefore no directions could be given for it; however, they are so disposed as to have but little dependence on each other, and consequently every teacher is left to his own choice in that respect" (Bonnycastle 1786, preface). Many eighteenth-century texts concurred that there need not be a set order to the rules of arithmetic. An American text of 1782 boasted of an arrangement of the rules such that "one shall have as little dependence on a succeeding one as possible" (Dearborn 1782, preface).

Every book began with numeration, however, so there was some minimal order to learning. But even here there were striking variations. Some texts held that there were nine figures, whereas others included zero to make the total ten. One highly popular book, Cocker's *Arithmetick*, published first in 1677 in England and reissued in 112 editions over the next century in both England and America, attempted this puzzling definition of zero: "A cypher is the beginning of number, or rather the medium between increasing and decreasing numbers, commonly called *absolute* or *whole numbers*, and *negative* or *fractional numbers*" (Cocker 1677, p. 2). An American text printed by Benjamin Franklin in Philadelphia described the complexity of the arabic place system in this pithy formulation: "We are to note, that every one, or any, of the abovementioned nine Figures, or Digits, have two values; one certain, and another uncertain." In other words, a number standing alone had the certain value of the number itself, but when it was placed next to others, its value was altered by multiples of 10 (Fisher 1748, pp. 56–57). Another text (Dilworth 1743)

elaborated on the place system with this series of questions and answers, designed to anticipate the baffled student seeking guidance (p. 2):

- Q. How many figures are sufficient to express most ordinary concerns?
A. Nine.
- Q. Why does it consist of nine Places, rather than eight or ten?
A. Because they make up three even periods.
- Q. What do you mean by a Period?
A. A Period is a Quantity express'd by three Figures, whereof the first to the right Hand signifies so many Units or single Things; the second so many Tens; and the third so many Hundreds.
- Q. Why are three figures called a Period?
A. Because if the Number be increased above three Places, there is still the same periodical Return of the Value of those Places, and every third Figure to the left Hand, will always be Hundreds, if it be never so far extended.

If a student had any kind of able teacher, perhaps these obfuscatory explanations could be quickly passed through and the place system comprehended. The real devil in eighteenth-century arithmetic lay in the computational complexity of weights, measures, and monetary systems. Arithmetic was the handmaid of commerce; its primary purpose was to help sellers, buyers, and investors calculate quantities and prices. But the denominations of commerce introduced enormous headaches. Books like Dilworth's *Schoolmaster's Assistant* contained page after page of equivalencies in the archaic English system of weights and measures, all replicated in the American reprint editions. Often the denominations of volume or size were specific to the item being measured: so, for example, a firkin of butter weighed 56 pounds whereas a firken of soap weighed 64; a hogshead of beer contained 45 gallons, whereas a hogshead of wine was 63. Prunes were measured by the puncheon, lead by the fother, steel by the faggot. Troy and apothecary ounces totaled 12 to the pound, whereas avoirdupois ounces were 16, a distinction that reflected a concept of expected attrition for some kinds of substances. There were nondecimal weight progressions: a clove equaled 7 pounds, a stone was 14 pounds, and a tod was 28 pounds or 2 stones (Dilworth 1743, p. 13).

A major project for these eighteenth-century texts was to convey techniques for manipulating pounds, shillings, pence, and farthings,

and then to teach students to figure the cost of goods when the latter was also expressed in complicated (and often nondecimal) denominations. A daunting yet actually commonplace problem might present in this form:

What will 9 cwt. 3 qrs. 14 lb. of sugar come to, at 4l. 17s. 4d. per cwt.?

In modern terms, this weight of sugar was 989 pounds, and the price per hundred pounds was 4 pounds, 17 shillings, 4 pence: we can scan it as just under 1,000 pounds of sugar priced at just under 5 pounds per hundred, yielding an answer somewhere under but close to 50 pounds. Precision of course was required, and Pike's *Arithmetick* gave this as the rule to follow: "When the price of one hundred weight, &c. is of several denominations, and the quantity likewise: Multiply the price by the integers, and take parts for the rest from the price of an integer; which, added together, will be the answer." This prescription is not immediately obvious at first reading, at least not to modern ears. Pike took nine lines of figuring to produce the exact answer of 48l. 1s. 2d. (Pike 1809, p. 144). If one studies the nine lines intently, divining the calculation engaged at each step, eventually the rule begins to make sense. But it takes considerable mental energy to fathom it, and the book offers no help beyond stating the rule and working the example. The taxing effort required to decode it is an impressive demonstration of why eighteenth-century educators reserved commercial arithmetic for older male youth.

The celebrated rule of three—three numbers given, to find the fourth—was the most common culmination of a basic arithmetic education. Apprenticeship contracts that specified a master's duty to instruct his charge often formulaically stipulated reading, writing, and ciphering to the rule of three. The vast majority of the scores of surviving eighteenth-century copybooks also ended at the rule of three. This rule and its variations (single and double, direct and inverse) covered the basic commercial problem of proportional relationships. If a man pays 1s. 7d. to pasture a cow for one week, how much will it cost him to pasture 37 cows for two weeks? If nine men can build a house in five months, working 14 hours a day, in what time will nine men do it if they work only 10 hours a day? A trader in Maryland purchased a