into practice

Chapter 1
Counting and Part-Part-Whole Relationships

Big Idea 1
Addition and subtraction are used to represent and solve many different kinds of problems.

Essential Understanding 1*a*
Addition and subtraction of whole numbers are based on sequential counting with whole numbers.

Essential Understanding 1*c*
Many different problem situations can be represented by part-part-whole relationships and addition or subtraction.

Essential Understanding 1*d*
Part-part-whole relationships can be expressed by using number sentences like $a + b = c$ or $c - b = a$, where a and b are the parts and c is the whole.

Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Prekindergarten–Grade 2 (Caldwell, Karp, and Bay-Williams 2011) presents big ideas and essential understandings that teachers need to know well to teach addition and subtraction to students in prekindergarten–grade 2. A very natural way to begin putting this knowledge into practice in the classroom is by focusing on children's early development of number sense for addition and subtraction and the importance of providing young students with a multitude of experiences to compose and decompose number. Building understanding of part-part-whole relationships is central because this understanding is the cornerstone of addition and subtraction concepts, properties, and algorithms.

Common Core State Standards for Mathematics

Related to the Big Idea and Essential Understandings for Chapter 1

Kindergarten (K.OA.1–5)

1. Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

(National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010, p. 11)

We cannot overstate the importance of these early experiences to students, whose understanding develops through these opportunities to represent their thinking and understanding. These experiences help them meet the expectations of the Common Core State Standards for Mathematics (CCSSM; NGA Center and CCSSO 2010), which detail outcomes for students' understanding of addition and subtraction, beginning in kindergarten and continuing into the elementary grades.

Counting as a Foundation for Addition and Subtraction

Acquiring fundamental counting skills is the hallmark of early number development for preschool and kindergarten students. These skills are critical to their acquisition of skills in addition and subtraction. Students need to have experiences to build four fundamental aspects of early numerical knowledge as they work toward the idea of an operation (Clements and Sarama 2014):

1. Number sequence
2. One-to-one correspondence

3. Cardinality
4. Subitizing

Students must have many opportunities to count objects of varying sizes and quantities so that they can develop competencies in these four areas. Students should engage in multiple activities that have the same basic form.

Task

How many _______ do you have?

Through such experiences, students develop a foundation for understanding and giving meaning to number and number relationships. The contexts for these situations influence how they count and what strategies they use to find the whole, or total quantity. These activities include tasks that focus on knowledge of the physical world (physical knowledge activities [Kamii and Rummelsburg 2008]).

One way to link counting with the idea of finding the whole amount is to conduct classroom inventories. You can send children, armed with clipboards, sticky notes, and pencils, off to different corners of the room to count and record numbers of blocks, books, crayons, and so on. Young children find counting a powerful experience and are eager to share their results. These experiences develop their understanding of cardinality and one-to-one correspondence between objects and numbers.

You can build interesting discussions on these counting adventures by asking different pairs of students to count the same objects and report their results. Having young students reconcile differences in the counts—that one pair counted 23 crayons and another pair counted 25 crayons—offers a powerful opportunity for them to develop the concept of accuracy while putting into practice standard 3 in the Standards for Mathematical Practice in the Common Core State Standards for Mathematics: "Construct viable arguments and critique the reasoning of others" (NGA Center and CCSSO 2010, p. 6).

You might also ask each pair to demonstrate their counting techniques for the class. After they have counted, you could ask them to share how they know when they have already counted an object. When young students demonstrate a simple technique such as moving an object to the side as it is counted, they help other students understand that this physical action is a crucial step in developing accuracy while promoting an understanding of one-to-one correspondence. Through multiple experiences, children sharpen their counting skills to include rote counting, one-to-one correspondence, conservation, accuracy, and understanding of magnitude. All of these understandings support their development of a sense of the meaning of the operations of addition and subtraction.

Another way of developing these foundational counting opportunities for children begins with reading counting books aloud to them. Counting books have been written on almost every imaginable topic and can be connected with many curricular areas and units. Figure 1.1 shows the work of kindergarten students who were given a contextualized counting activity after listening to the children's book *Counting in the Garden* (Parker 2005). The students were assigned to groups, and each was given a bag of garden items to inventory. Each group's bag had different numbers of particular items and different total amounts. Group members could decide to sort and count the items in their own way, but they had to figure out how to represent and report to the rest of the class what was in their bag.

Fig. 1.1. Two students' inventories of garden items

Students can enjoy very natural conversations about numbers while counting and comparing the contents of their bags. The following conversation occurred among students in two groups as the students discussed the objects in their inventoried bags. Maria and Marcus were in one group, and Selena and Tavon were in the other.

Maria: [*To Selena*] Our bag has only three butterflies! How many do you and Tavon have?

Tavon: We have five butterflies!

Marcus: Awwww...

Tavon: Yes, we have more!

Selena: It's OK! If you and Maria had two more butterflies, then we would have the same!

Maria: Yay! And besides, I think we have more yellow flowers than you.

Tavon: Oh! How many do you have? Let's count 'em to find out.

This conversation illustrates the way in which young children build additive number relationships naturally by counting items and combining and comparing the numbers of objects that they have—in this case, the contents of their bags. You can facilitate small- and large-group discussions by encouraging your students to share their counting techniques, asking them to total their combinations of inventoried items, and inviting them to compare different numbers of items. Questions like the following encourage this thinking:

- "How did you organize your items?"
- "Why did you organize your items that way?"
- "What item do you have the greatest number of in your bag?"
- "What item do you have the smallest number of in your bag?"
- "Who has the same number of [*particular items*] as [*a particular student*]?"
- "Who has more [*particular items*] than [*a particular student*]? How many more?"
- "Who has fewer [*particular items*] than [*a particular student*]? How many fewer?"
- "How many items do you and [*a particular student*] have together?"
- "Does anyone have two more [*particular items*] than [*a particular student*]?"

Student misconceptions about number often arise from immature counting skills and difficulty in understanding number sequence, one-to-one correspondence, and number magnitude. Students vary greatly in their ability to count and record numbers of objects, but they can progress rapidly when given many rich experiences. Students also need to hear other students count aloud, have opportunities to compare their count of a collection of items with others' counts of the same collection, and then explore ways to reconcile differing totals. Reflect 1.1 invites you to consider ways of ensuring that *all* students have access to these types of opportunities.

Reflect 1.1

Activities that call for conducting inventories in the classroom can be differentiated for students with varying abilities.

How might you differentiate inventories for your students?

How might inventories address several of the common student misconceptions related to immature counting skills and difficulties in understanding number sequence, one-to-one correspondence, and number magnitude?

Many children's books promote counting skills and understanding of relationships among numbers. *The Icky Bug Counting Book* (Pallotta 1991) is a preschool and kindergarten favorite that can be connected easily and directly with a unit of study on insects. After reading the book aloud to your students, you can give them collections of plastic insects or pictures of insects to sort, count, combine, and compare, recording their results. Alternatively, if the technology is available, you might have your students create a collection of insects to count by giving them digital cameras and sending them off on a bug hunt to count, record, and then report their discoveries through number sentences. No matter how students obtain their collections, you can then engage them in a discussion of the numbers of different insects that they have counted and ask questions that will help them use their counting and recording to report their results.

You might also extend this approach to a more advanced counting book, *Counting Jennie* (Pittman 1994), which portrays a young girl who counts her way through the day with a constant stream of addition and subtraction problems, including problems with two-digit numbers.

The Bag I Am Taking to Grandma's (Neitzle 1986) is another children's book that offers rich possibilities for counting experiences. After reading the book to your students, you can have them create inventories of items that they might need for an overnight stay. Students can determine the objects and draw pictures of them inside an outline of an overnight bag (see fig. 1.2). They can then share their work and "read" one another's inventories and report the total quantity in each one. Depending on their level of understanding, students can differentiate the total value by quantity. They can create and record number sentences to represent the combined items in their bags or the difference between the numbers of items in their bags and a partner's.

Fig. 1.2. A student's record of what is in the suitcase

Understanding Relationships among Numbers

It is important for students to compare numbers and build their understanding of relationships among them. Reflect 1.2 asks you to predict how a young student might compare two particular one-digit numbers.

Reflect 1.2

How are 4 and 6 alike?

How would you expect a kindergarten or first-grade student to respond to this question?

What might the response tell you about the child's thinking about relationships between two numbers?

Interviews with first graders who were asked, "How are 4 and 6 alike?" offered insight into their thinking regarding relationships between numbers. Consider the following excerpts from interviews with three students—Sarita, Rema, and Jose—and then respond to the questions in Reflect 1.3.

Student 1: Sarita

Sarita: 4 and 6 are not alike, because 4 has straight lines in it, and 6 has a circle in it!

Student 2: Rema

Rema:	I see 4 on my way to 6.
Teacher:	What do you mean?
Rema:	[*Stands up*] 1, 2, 3; hi, 4. Then there's 5 and 6.
Teacher:	What was going on here?
Rema:	I was walking on a number line, and I saw 4 on my way to 6.

Student 3: Jose

Jose:	Well, 4 is part of 6.
Teacher:	It is? How?
Jose:	Well, if you have 6 things, then you have 4 things, of course. Actually, you have 4 things in all numbers bigger than 4.

Reflect 1.3

What do you notice about Sarita's, Rema's, and Jose's responses to the question, "How are 4 and 6 alike?"

Which responses reveal a naïve understanding of the relationship between the numbers? Why?

Which responses are more sophisticated? Why?

Students' misconceptions about relationships among numbers indicate the sophistication of their mathematical thinking. Students who focus on the formation of the numerals may not fully understand their values. To build understanding of relationships among numbers, students need to be able to represent quantities to compare.

Exploring "Both Addends Unknown"

Kindergartners can begin to think about problems of particular types. Several additive types are easier for them to interpret than others. One is rooted in situations with both addends unknown. Tasks that support students' understanding of this type of problem have a common form, with the blank filled in by a number up to 10.

Task

How many ways can you show ______ ?

Students' work with this task connects with and supports their understanding of relationships among numbers. At the same time, this work takes their understanding to the level of early algebraic thinking as the children find all the whole number combinations that yield a given total and justify that they have found all possibilities.

To develop flexible thinking about part-part-whole relationships, students need to understand that numbers can be represented in many ways. You should provide many opportunities for students to construct multiple representations of number values, using a variety of materials and a range of values. Initially, students should compose and decompose these values by using a given total so that they focus on the parts that create the same whole. If the total values are constantly changing within a lesson, students often completely fail to understand how the parts are composed to create the same whole and may develop misconceptions about how many ways they can represent a given value.

Also, if you encourage students to be systematic in their representations, you can help them develop an understanding of the patterns related to the combinations. After students have composed and decomposed many variations for a given value, they can discuss, compare, and share their results. Students are often amazed at the variety of number combinations that they and their classmates can create for a given quantity.

Another children's book, *Jack and the Beanstalk* (Kellogg 1997), presents an engaging context for a productive activity with two-sided (two-colored) counters or beans that have been spray-painted on one side. Building on this fairy-tale context, students can explore two-number combinations that yield a particular sum up to 10, given in the blank in the following task statement.

Task

If you toss _______ of Jack's beans, how many different ways can they land?

(Based on *Jack and the Beanstalk* [Kellogg 1997])

After students have tossed a particular number of Jack's beans multiple times, they can investigate and represent the two-color results. Initially, students can color in outlines of beans on paper and then progress to recording the values of the beans. Students should advance to larger quantities as they develop their understanding. Figure 1.3 shows student responses in vertically arranged progressions, from pictures to spoken words to written numerical symbols.

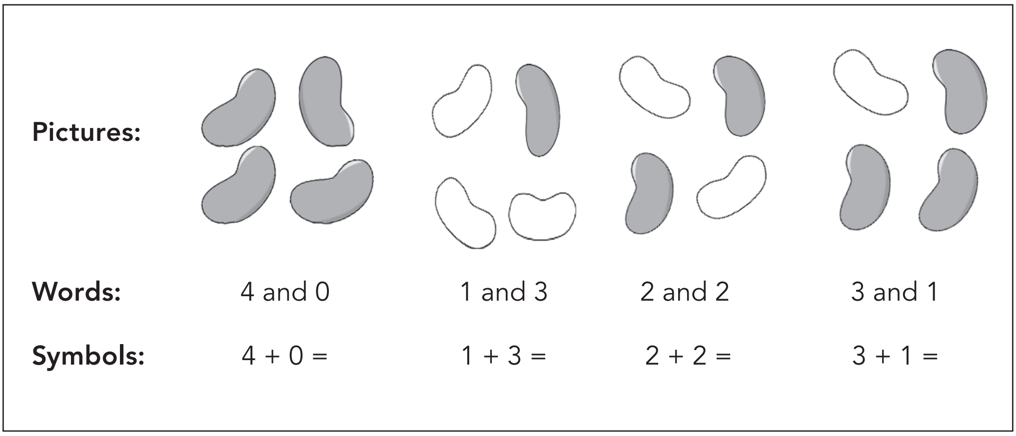


Fig. 1.3. Progressions of student responses, from visual to oral to numerical records

The children's book *There Is a Carrot in My Ear and Other Noodle Tales* (Schwartz 1982) describes the escapades of the silly Noodle family. This book offers another context for an exploration of unknown addends, the Macaroni Squeeze game. To engage your students in this game, you will need uncooked, small noodles and several small, clear sandwich bags with self-sealing closures. Affix a piece of colored tape or draw a line with a permanent marker down the middle of each bag, as shown in figure 1.4a. An alternative, shown in figure 1.4b, is to draw a circle on the bag.

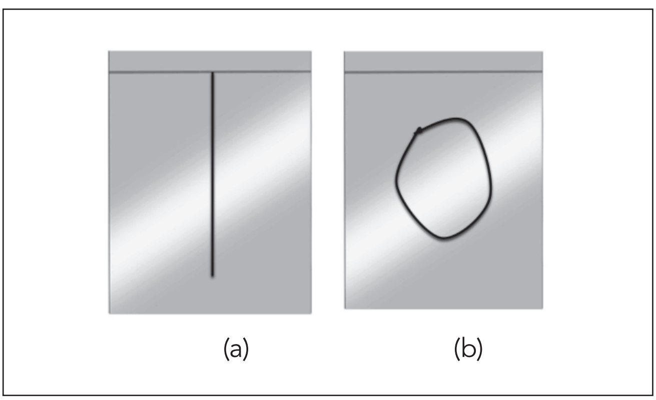


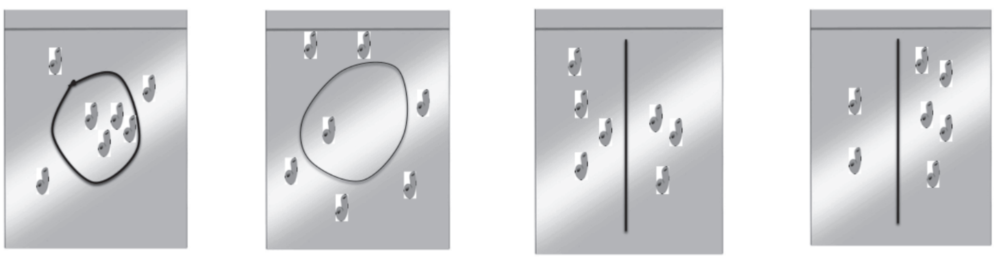
Fig. 1.4. Two options for preparing clear sandwich bags
for use in the Macaroni Squeeze game

Put 7 noodles (or another number of noodles, up to 10) in each clear plastic bag, and then seal it. Place the bags flat on a table or desk for your students to work with them on the task.

Task: Macaroni Squeeze

How many ways can you arrange the noodles in the bag?

Students move noodles to either side of the line (or in or out of the circle) and record their results. They can approach this task in a variety of ways, exploring different compositions of the given number of noodles. Observe how students find all the combinations. Do they use a random approach or a systematic strategy? As students find and record the different combinations, they can begin to understand that the whole remains constant while the parts change. Figure 1.5 shows different ways in which students might represent their work with 7 noodles.



4 inside and 3 outside	1 inside and 6 outside	4 on the left and 3 on the right	2 on the left and 5 on the right
4 + 3 = 7	1 + 6 = 7	4 + 3 = 7	2 + 5 = 7

Fig. 1.5. Several ways in which students might represent arrangements of noodles

Another children's book that offers a motivating opportunity for exploring situations where both addends are unknown is *Guinea Pigs Add Up* (Cuyler 2010). This book tells the story of a classroom in which an active guinea pig population grows and decreases as babies are born and pets are distributed to interested adoptive families. This context presents an ideal situation for a variety of additive tasks; the following is one example.

Task: Where Are the Guinea Pigs?

There are 10 guinea pigs in 2 cages. Write equations to show how many guinea pigs are in each cage. How do you know that you have all the possible options?

(Based on *Guinea Pigs Add Up* [Cuyler 2010])

Start with two illustrations of cages (see fig. 1.6) serving as mats for your students to use in sorting counters (see fig. 1.7) to assign guinea pigs to cages. (Appendix 3 at More4U provides templates for cage mats and guinea pig counters for students' use in testing different arrangements of the guinea pigs in the cages.) Distribute the cage work mats and the guinea pig counters, and say to your students, "These 2 cages have 10 guinea pigs in all. Where are the guinea pigs?"

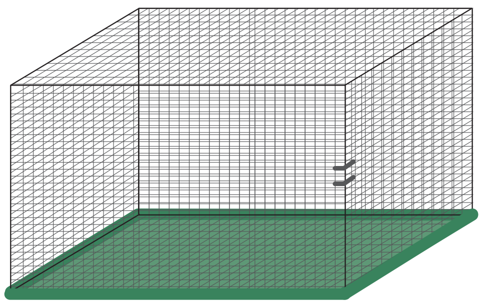


Fig. 1.6. Two cages; template for a student work mat available at More4U (Appendix 3)

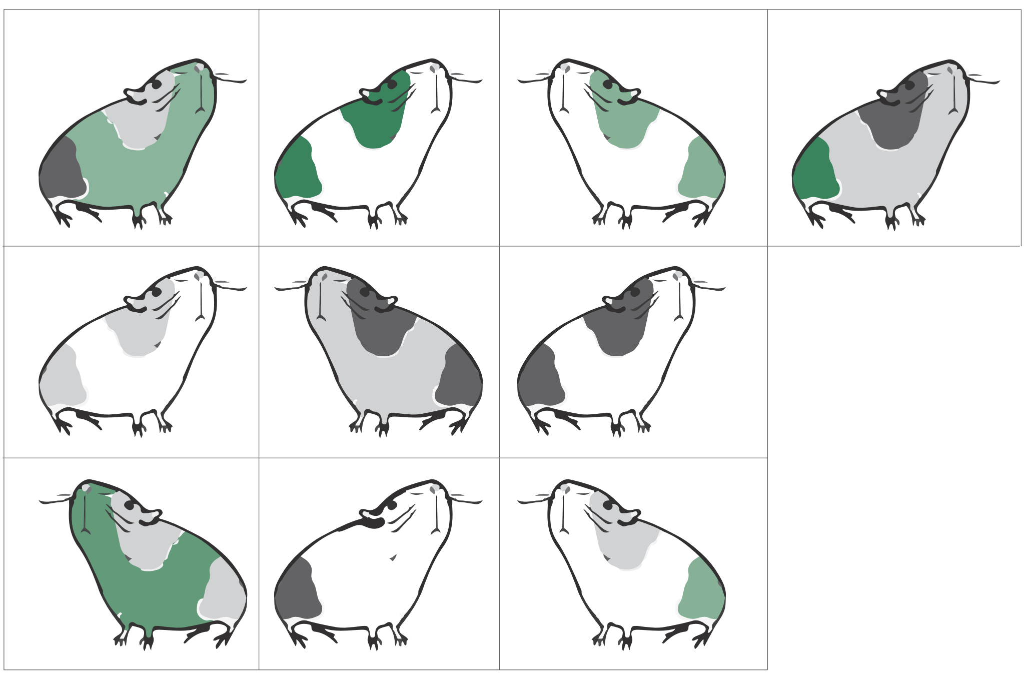


Fig. 1.7. Ten guinea pigs; template available in Appendix 3 at More4U

Figure 1.8 shows one student's work on the task; a link to a student's pencast (Livescribe file) is available at More4U. The task starts as a simple decomposition activity, supporting students' ability to "Decompose numbers less than or equal to 10 into pairs in more than one way" (NGA Center and CCSSO 2010, K.OA.3, p. 11). However, as students determine all possible options for showing 10, the task deepens, moving to offer an early experience in algebraic thinking. It also lends itself to use as a first step in exploring the structures of problems.

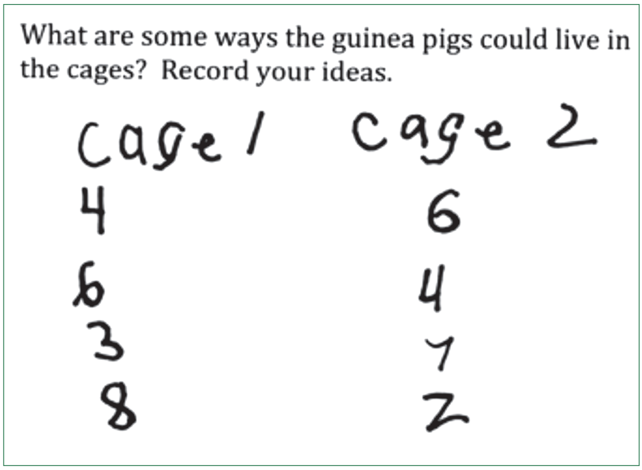


Fig. 1.8. A sample of student work on the task Where Are the Guinea Pigs?

Subitizing for Efficiency

Subitizing is naming a quantity without counting. It is a compelling counting strategy that supports an understanding of number. By developing your students' skills in subitizing, you can foster their understanding of cardinality (Benoit, Lehalle, and Jouen 2004). When students count objects one by one, over and over, they often focus on counting and making the one-to-one correspondence between each number and each object, but sometimes they do not make the transition to the principle of cardinality. By using subitizing as a tool, you can help your students learn that naming the quantity as they see it and then counting the parts to explain their thinking also names the whole set.

Subitizing also supports students' understanding of the part-whole relationship (Clements 1999) because students see the whole, but depending on the representation of objects or dots in use, they can decompose the parts in different ways. As students explore multiple representations of a number, subitizing helps them use spatial organization to visualize and instantly recognize quantities in many different arrangements, building flexibility in the way in which they think about number. Students can work with subitized values by using dot cards with varying numbers of dots and visual representations. In the classroom dialogue that follows, the teacher shows students the dot card that appears in figure 1.9 and asks them to name the total number of dots and then explain how they found their answer.

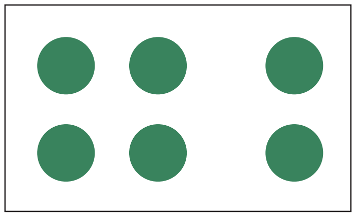


Fig. 1.9. A dot card used to develop students' skills in subitizing

Teacher: Can you name the number of dots that you see?

Allie: I see 6.

Teacher: Can you explain what you see?

Allie: I see 4, and then I counted, 5, 6.

Teacher: Did anyone else see it differently?

Roberto: I see 3 on the top row, and 3 on the bottom row makes 6.

To help your students think about the part-whole relationship, you can ask them to share the total number that they see by using the language of parts and wholes. Reflect 1.4 provides an opportunity to explore this idea in the context of the ten frame shown in figure 1.10.

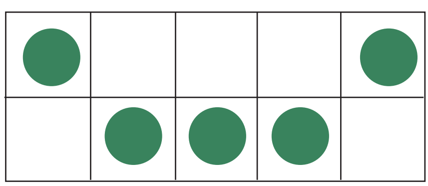


Fig. 1.10. A ten frame with counters

Reflect 1.4

Look at the ten frame in figure 1.10, and think about how you would describe what you see, using the language of parts and wholes.

Make a prediction about how students might interpret the ten frame.

Compare your prediction about students' interpretations of the ten frame with the responses of two students, Rita and Jenna, in the following classroom exchange:

Teacher: Name and explain what you see.

Jenna: I see 5! I see 2 on the top and 3 on the bottom!

Teacher: What is the whole number of dots on this card?

Jenna: 5!

Teacher: How do you know?

Jenna: Because there are 5 dots!

Teacher: What are the parts that you see, Jenna?

Jenna: One of my parts is 2 and the other is 3.

Teacher: Does anyone else see other parts?

Rita: I see one of my parts as 1 and the other part as 4.

Teacher: Can you explain that, Rita?

Rita: I see 1 on the first box and then 4 more.

Teacher: So what is your whole?

Rita: 5.

Teacher: Wow, we talked about two ways to make 5 just by using this one ten-frame card.

Frequent use of subitizing is essential to developing this technique. Short, recurrent sessions along with powerful discussions using explicit vocabulary help students develop understanding of the part-part-whole relationships that they are seeing and increase their ability to retain the information that they are learning (Resnick 1983).

Often when students struggle in describing their thinking about subitizing, they just need more experience. Gradually, they become more able to describe what they see and how they see the values. Initially, some students may need to try smaller quantities and then ramp up. The children's book *10 Black Dots* (Crews 1995) can set the stage for a fruitful task. The student fills in the first blank with a number up to 10 and the second blank with the name of an object that has that many black dots.

Task: Picturing with Black Dots

______ black dots can make a ______________________ . See my picture!

(Based on *10 Black Dots* [Crews 1995])

Students can create a picture by using a designated quantity of dots but breaking it into two parts or combining or comparing the dots in two different pictures. The whole class can create pages of black-dot pictures by using combinations that make the same quantity or different quantities. You can put a classroom book of the pages in a learning center and let students subitize one another's creations. Figure 1.11 shows two students' work on the Picturing with Black Dots task.

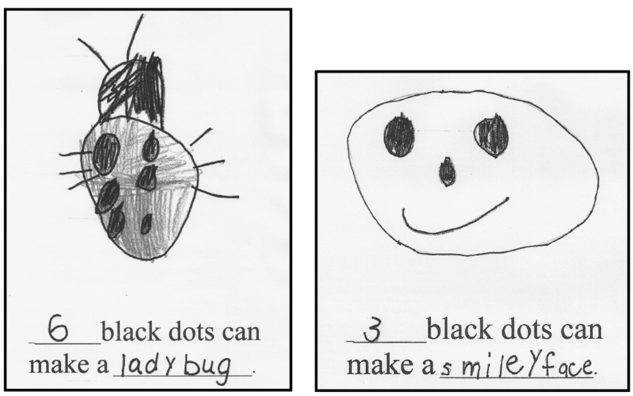


Fig. 1.11. Two sample pages for Picturing with Black Dots

Linking Contexts with Concrete Representations

Concrete representations are just one type among the variety of types of representations for additive problems. Yet, their value cannot be overemphasized. Whether students use beans, plastic counters, dot cards, ten frames, Rekenreks, Cuisenaire rods, part-whole graphic organizers, tape diagrams, or, eventually, number lines, they need to develop flexibility in using these representations. Some contexts lend themselves to concrete representations with particular tools. Consider the Rekenrek as an example.

The Rekenrek was developed by researchers in the Netherlands and consists of a rectangular frame supporting two wires strung with 10 beads apiece (see fig. 1.12). Each group of 10 beads is broken in the middle into two groups of 5, distinguished by color. This arrangement encourages students to see and think in groups of 5 or 10. In the classroom vignette that follows, a teacher is working with students with a Rekenrek.

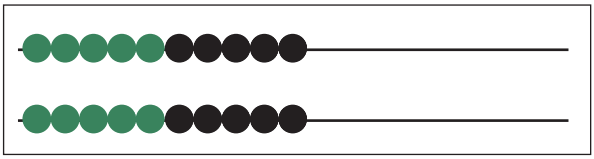
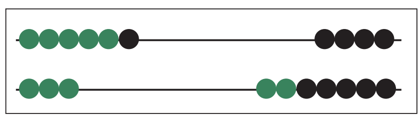


Fig. 1.12. A Rekenrek, or two-part bead frame

Teacher: I am thinking of a way to show 9 on my Rekenrek. Can you show me a way?

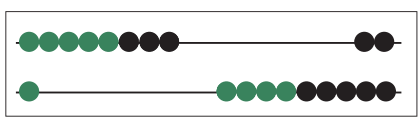
Students: Yes!

Amelia: I have 6 on the top and 3 on the bottom.



Teacher: [*Writing* 6 + 3 = 9 *on the board:*] Let's show Amelia's answer by writing it here.

Jocie: I have 8 on the top and 1 on the bottom!



Teacher: So, Jocie, if the whole is 9, can you tell me what your two parts are?

Jocie: One of my parts is 8, and the other is 1.

Teacher: How can we write that as a number sentence?

Jocie: 8 + 1?

Teacher: Let's write that down [*writes* 8 + 1 = 9 *on the board*]. Now, we have two ways to make 9. How many ways do you think we can find?

[*The teacher waits, letting the students think before speaking again.*]

Teacher: Is there a way we could use Jocie's idea of 8 + 1 to help us find other combinations?

Many students begin with an unorganized list. If your students start in this manner, record their results in an organized list as they report them. Lead them in a discussion about any patterns that they notice, and then prompt a discussion about using logical thinking. When students move to making an organized list, the mere construction of such a list requires some sophistication in their thinking.

By noticing a pattern and recording the combinations as they discover the pattern, students can begin to develop their understanding of part-part-whole combinations. Students should watch a classmate model the development of the pattern by consciously moving one item at a time. When students make an unorganized listing, the combinations may remain mysterious and numerous (with some repeated at times) because they do not see the relationships of the parts to the whole, the power of the commutative property, or the structure of the part-part-whole model.

A number of contexts offer rich possibilities for use of the Rekenrek to link a context with a concrete representation. Some appealing contexts are available in children's literature. For example, you might read *Anno's Counting House* (Anno 1982) to your students and use a Rekenrek to represent a two-story house (or two stories in an apartment building if you have a book or plotline that links to that scenario). Link the two stories of the house to the two levels of the Rekenrek, as illustrated in figure 1.13. The number 8 in the peak of the house is the total number of people in the two-story house, with the Rekenrek representing 2 people downstairs and 6 people upstairs.

Other children's books also offer contexts that lend themselves to the use of a Rekenrek to reinforce the connection between context and concrete representation. This tool works well with storybook contexts involving bunk beds; books such as *How Many Feet in the Bed?* (Hamm 1994) and *The Napping House* (Wood 2009) can set the stage for this work. Also consider reading *The Memory Cupboard: A Thanksgiving Story* (Herman 2003) and using the Rekenrek to show the placement of different numbers of items on two shelves in a cabinet. Each of these stories presents a context that you can model nicely on the Rekenrek, double ten frames, or other two-part representational tools. (Another possibility is to visit the school library and create situations involving the placement of books on two shelves.)

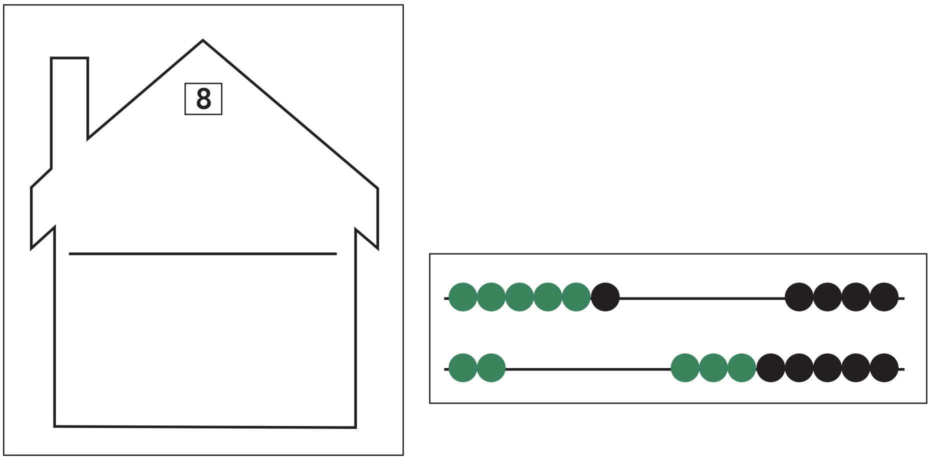


Fig. 1.13. Linking the context of *Anno's Counting House* with the concrete representation provided by the Rekenrek

Also be sure to link representations on the Rekenrek with part-part-whole situations as students use the color change in the beads to link a semi-concrete representation and an abstract representation in an equation. For example, ask, "If there are 10 people in the house, how many people are upstairs and how many people are downstairs?" Then ask the students to use the Rekenrek to show how many people might be upstairs and downstairs, recording their responses in number sentences.

The next section focuses on investigating missing addends. Reflect 1.5 leads into that discussion by probing the use of representations to build understanding of part-part-whole relationships.

Reflect 1.5

How might the use of multiple representations support student understanding of part-part-whole concepts?

What questions could you ask students to promote this understanding?

Investigating Missing Addends

After students have had multiple and varied opportunities to find parts for quantities and compare and explain the parts and wholes, they will be ready to make the transition to situations with a missing addend. You can build the concept of the missing addend by using the very same tools that you used to build their understanding of multiple representations of a number.

Consider the Hen and Egg game, for example. To play this game with your students, you will need beans or other counters to represent eggs and a cup or paper cutout of a chicken to represent a hen. Decide on a target number up to 10—say, 8—and place 8 beans, as eggs, on the desk (see fig. 1.14).

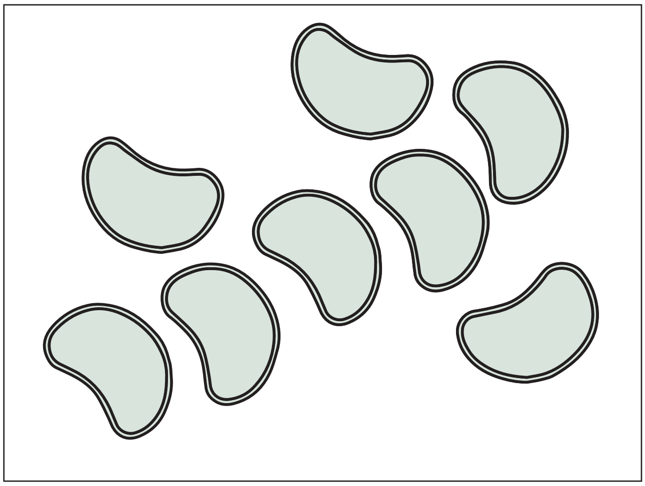


Fig. 1.14. Eight beans representing 8 eggs in the Hen and Egg game

After directing the students' attention to the 8 eggs, move the hen cutout to "sit on" some number of eggs, hiding them completely from view (see fig. 1.15). First say, "There are 8 eggs in the whole nest." Then ask, "How many eggs is the hen sitting on?" Relate the language of part-part-whole by saying to your students, "You know what one of the parts is, and you know what the whole is, but now you need to find the value of the missing part."

Ask the students to name the part and the whole and find the missing part. Students can play this game in pairs, with one student using the cup or hen cutout to hide some of the eggs and then asking the other student to name the missing part.

Dot cards like the sample shown in figure 1.16 also give students strong visual support for finding a missing addend when they know the whole and one of the parts. Filling in both blanks with the same number sets the task shown on the next page.

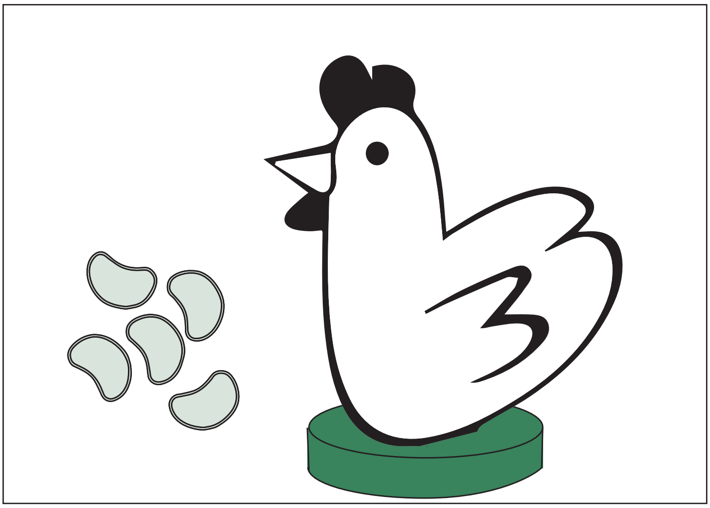


Fig 1.15. A "hen" "sitting on" some of the 8 "eggs" in the nest

Task: Missing Addend on a Dot Card

Can you make ______ by adding two parts so that this sentence is true?

The part on the card + the missing part = ________.

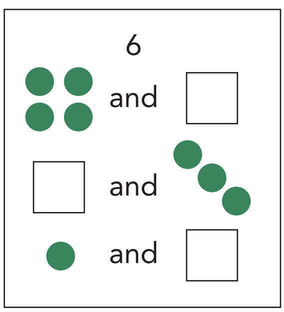


Fig. 1.16. A dot card showing a sample task

Students who are unable to find a missing part or give incorrect missing parts, or parts that do not match the total value, may have misconceptions about how to find a missing addend. Using smaller quantities or representing the missing parts with physical models can scaffold their thinking.

Another useful way to support students' exploration of missing parts is by playing a hiding game that comes very naturally to young students. Small children frequently clutch some small object, known only to them, in a closed fist. Find a number of small items, such as buttons or pennies, that you can easily hide in your hand. Show the whole collection to your students in your open hand (or state the total number) before closing your hand. (With very young children, have them count and place the counters in your hand.) Once students know the whole, carefully transfer some of the counters to your other hand, keeping them hidden from view. Then reveal the remaining part of the collection in your first hand, as shown in figure 1.17. Ask, "How many am I hiding in my other hand?" Start with very small quantities and gradually increase them. Students can state the missing part or represent it in front of them with different counters or cubes, matching what is in your hands. Once again, precise and explicit language on your part will help your students connect the representation with the part-part-whole language.

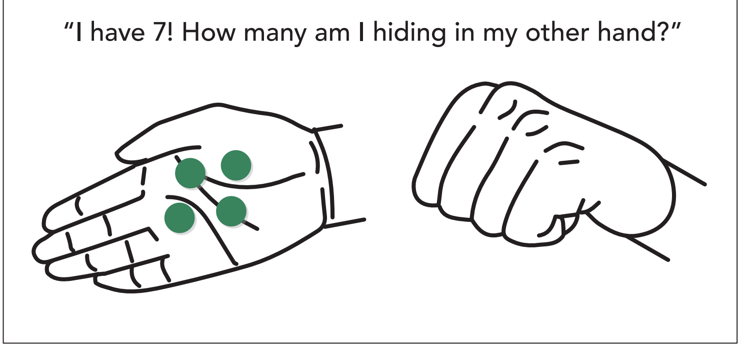


Fig. 1.17. An example of the hiding game

A number of children's books, including *A Pocket for Corduroy* (Freeman 1978), *Ten Apples on Top!* (Dr. Seuss 1961), *Caps for Sale* (Slobodkina 1940), and *Sheep in a Jeep* (Shaw 1986), can reinforce young learners' development of part-part-whole concepts. After reading the book aloud to your students, you can present part-part-whole tasks that engage them in finding missing parts. For example, the following task related to *A Pocket for Corduroy* asks students to explore a situation in which both addends are unknown; figure 1.18 shows a task card and a sample of student work.

Task: Corduroy's Pocket, Inside and Out

Corduroy has 8 pennies. Some are inside his pocket, and some are out. What could be inside his pocket?

(Based on *A Pocket for Corduroy* [Freeman 1978])

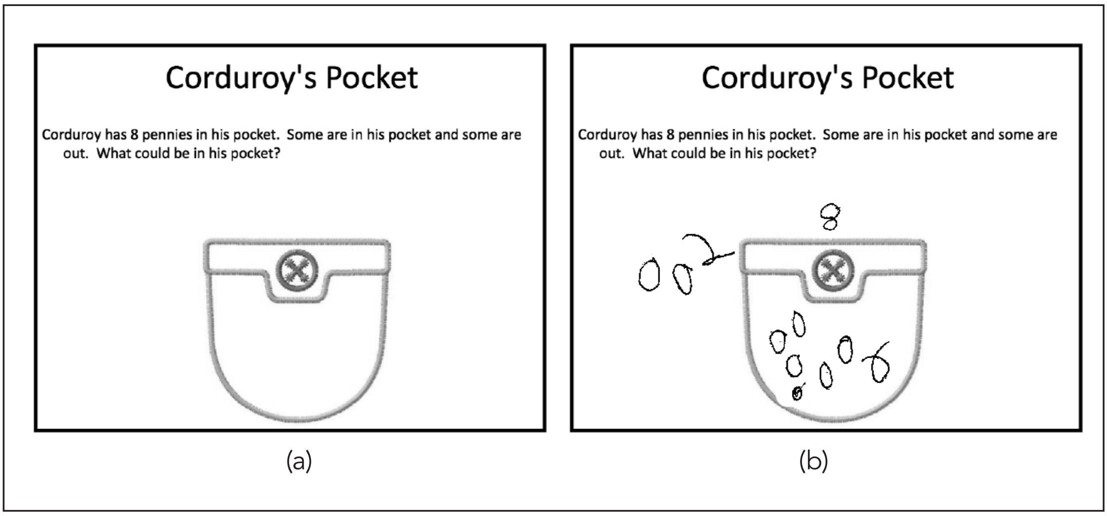


Fig. 1.18. Corduroy's Pocket, Inside and Out: (a) task card and (b) student work showing 2 + 6 = 8

Summary: Learners, Curriculum, Instruction, and Assessment

Teaching counting and part-part-whole concepts requires teachers to have an understanding of learners, curriculum, instruction, and assessment. Understanding learners helps in structuring meaningful tasks for them as they learn to count, compare, and determine part-part-whole relationships for given wholes. Sequencing and designing tasks is a significant component of implementing the curriculum. The associated instructional strategies should support students' construction of knowledge as they build their understanding of counting, parts and wholes, contexts linked to representations, and missing addends. These strategies include asking questions that scaffold students' progress to more complex ideas.

Assessing students' thinking throughout the process involves creating opportunities and investing time to probe their understanding as well as to uncover any misconceptions. You can examine how your students compose and decompose values, and how they find and record the parts for a given whole, to help you determine which quantities you might introduce next. You can evaluate whether your students use systematic ways to find all the parts for a whole or whether they rely on random or incomplete strategies. You can also assess your students' strategies for determining missing addends and then design tasks that support their thinking and understanding.

By consciously designing classroom tasks that integrate the four components—learners, curriculum, instructional strategies, and assessment—you can provide a full range of classroom experiences that will challenge and support all learners.

Conclusion

This chapter has discussed the importance of foundational counting experiences, rich tasks using literature, and questioning to promote a deep understanding of counting and part-part-whole relationships. Students need many opportunities to compose and decompose quantities to develop their understanding of the various ways to construct a whole and determine the relationship of parts to wholes. We have emphasized the importance of giving students many opportunities to count, record, and report inventories of many different types of objects, preparing them to progress to decomposing the whole into multiple parts. We have shared real conversations to illustrate student thinking about numbers to show how teachers and students engage in communicating about part-part-whole relationships. We have also included many examples of student work to demonstrate how students think about constructing parts and wholes within rich tasks. Students need a great variety of these experiences so that they can explore, discuss, and share their ideas. Although many of these experiences are open-ended, you should think carefully about the questions that you ask and the scaffolding that you provide so that your students can make important discoveries about how to compose and decompose given values.