Preface

A yearbook gives an organization an opportunity to take stock, at one moment in time, of the status, concerns, understandings, and expectations regarding a specific topic. Such is true of this Seventy-first Yearbook of the National Council of Teachers of Mathematics (NCTM). Only three of the seventy yearbooks published by NCTM prior to this one have had "geometry" in their titles. The following look at previous yearbooks highlights several changes in how our profession has attended to geometry over nearly eight decades.

NCTM's Fifth Yearbook, *The Teaching of Geometry* (Reeve 1930), said in its preface that the yearbook was intended to supplement and assist a committee appointed by NCTM and the Mathematical Association of America to study the feasibility of a combined one-year course in plane and solid geometry. Many articles in that yearbook advocate such a course; others suggest that to prepare students for this course in the tenth year of high school, groundwork in "demonstrative" geometry (i.e., informal proofs) should be laid in grades 7, 8, and 9.

The basic framework for the Fifth Yearbook was the assumption that the tradition of teaching some variation of Euclid's geometry should be continued. The major departure from Euclid's approach was to allow some propositions that Euclid proved (such as the equality of vertical angles) to be taken as postulates. In one article, George Birkhoff and Ralph Beatty of Harvard University argued for a system of postulates based on measurement of distance and angle (1930, p. 92). These eventually became the ruler and protractor postulates incorporated into many textbooks from the 1950s onward (Sinclair 2008, p. 60).

By the time the Thirty-sixth Yearbook, *Geometry in the Mathematics Curriculum* (Henderson 1973), appeared, a major upheaval in mathematics education had occurred with the introduction of "new math" curricula. No longer was it assumed that geometry should be taught as a one-year course in synthetic, Euclidean geometry. Instead, the core of that yearbook was a series of articles proposing a variety of ways to organize the high school geometry curriculum. These included a modification of the synthetic approach; courses based primarily on coordinates, transformations, or vectors; developing affine properties (e.g., incidence and parallelism) prior to introducing distance and angle measure; an eclectic approach; and spreading geometry over six years of an integrated program.

Almost every article focused on identifying an appropriate set of axioms for the approach it discussed. However, one author reported studies indicating that few students completing a formal course in geometry could reliably identify axioms, definitions, and theorems (Brumfiel 1973, p. 102):

> What can I conclude? Students of 1954 who studied an old-fashioned, hodge-podge geometry had no conception of geometric structure. Students of today who have studied a tight axiomatic treatment also have no conception of geometric structure.

He cautioned, "We need to listen to students and learn what they really think. If we do listen, what we hear will provide useful guidance as we experiment in the years ahead with various approaches to the teaching of geometry" (ibid).

Ironically, aside from this observation, the Thirty-sixth Yearbook focused largely on curriculum with virtually no discussion of what students "really think" or how they come to understand geometry. In a departure from its predecessor, however, it included two articles that dealt with informal geometry (one for grades K–6, the other for grades 7–12) and contained sections on the place of geometry in modern mathematics and on the preparation of teachers.

The most recent previous yearbook on geometry was *Learning and Teaching Geometry, K–12* (Lindquist 1987). The opening article in that yearbook, "The van Hiele Model of the Development of Geometric Thought" (Crowley 1987), represented a fundamental shift away from an almost exclusive focus on content and curriculum to a consideration of issues related to students' learning.

In contrast with its predecessors, the 1987 Yearbook placed little emphasis on axiomatics, although a few articles offered suggestions for helping students learn how to construct proofs. Rather, geometry as a vehicle for problem solving was highlighted, in keeping with the *Agenda for Action* (NCTM 1980) and anticipating the centrality of problem solving in the later *Standards* documents (NCTM 1989, 2000). One entire section of the yearbook was devoted to "activities," including those appropriate at the elementary and middle school levels. Another section emphasized geometry's relationship to other branches of mathematics including algebra, calculus, probability, and combinatorics, anticipating the Connections Standard. As in 1973, the yearbook closed with two articles on the preparation of teachers.

Only two of the twenty articles in the 1987 Yearbook dealt with computers. One of them advocated giving increased attention to such topics as matrices, parametric equations, and homogeneous coordinates that are applied in computer graphics (Smart 1987). The other described how the program Logo with turtle graphics could be used to enrich the secondary school mathematics curriculum (Kenney 1987). At the time the yearbook was written, interactive geometry software lay in the future.

A lot has changed in the past twenty-two years. First and foremost, all mathematics education has been influenced by the Standards movement (NCTM 1989, 2000). In *Principles and Standards for School Mathematics* (NCTM 2000), geometry is given continual emphasis throughout all grade levels; in fact, the graph on page 30 (NCTM 2000), reproduced on the cover of this yearbook, suggests that geometry is the one content standard that should receive relatively constant attention from prekindergarten through grade 12.

Research on students' learning of geometry has continued to inform curriculum developers. Some textbooks, for example Serra's *Discovering Geometry* (1989), were guided by the van Hiele model. Projects supported by the National Science Foundation produced curricula at all grade levels aligned with the Standards and emphasizing developmentally appropriate activities, real-world applications, and the integration of algebra and geometry (Sinclair 2008, pp. 80–81).

Perhaps the most significant change was the development and dissemination of interactive geometry software, specifically such products as Cabri (Laborde and Bellemain 2005) and The Geometer's Sketchpad (Jackiw 1991). That change is reflected in this yearbook, in which nine of twenty-three articles refer to interactive geometry software as a tool for teaching and learning geometry.

Parallel with these developments in our understanding of students' learning and the availability of new tools for teaching, the field of geometry itself has experienced a revival. After languishing during of the early twentieth century as a field peripheral to mainstream mathematics (Sinclair p. 46), in the latter half of the century geometry again emerged as an area of research. A conference on "Geometry's Future" held in 1990 assessed the implications of this development on grades K–12 and university curricula (Malkevitch 1992).

Nevertheless, the state of the geometry curriculum remains unsettled much as it was twenty-two years ago. At that time Usiskin (1987, p. 20) observed, "There is lack of agreement regarding not just the details but even the nature of geometry that should be taught from elementary school through college" (Usiskin 1987, p. 20). This volume contains numerous articles with insights about teaching and learning but few that take a more global curricular perspective. A need still exists for a detailed discussion in the mathematics education community on what school geometry ought to be. We hope that the insights provided by the articles in this yearbook will contribute to that discussion.

This yearbook is divided into three parts. The first, "Expanding Visions of Geometry" attempts to bring us up to date in the work that today's geometers do. It focuses on topics in geometry that are current but not traditionally part of the curriculum. The section opener by Editorial Panel member Malkevitch gives an overview of problems contemporary geometers are working on. Schattschneider engages us in detective work to establish the existence of exactly seven different types of frieze patterns. Handa, James, and Mattman share the beauty and intrigue of the well-known Möbius strip, its extension to Möbius tori, and its applications in art and architecture. Iseri shows how to make the concept of curvature accessible to school students. Camp and Hauenstein reveal how fractal geometry can be used to model the structure of plants. Finally, O'Rourke poses an engaging question about folding polygons that is simple to state but remains a challenge.

Articles in the second section, "Learning Geometry," give important attention to the ways students perceive shape, location, angle, and other geometry concepts and processes. Editorial Panel member Battista opens the section with a summary of current research on learning school geometry, including an explanation of the van Hiele model, which appears in several subsequent articles. Yu, Barrett, and Presmeg describe research on how interactive geometry software affects students' ability to reason about categories. Browning and Garza-Kling report on studies of how school students and preservice teachers develop the concept of angle. Sack and van Niekerk share activities to develop children's spatial visualization abilities. Driscoll, Egan, DiMatteo, and Nikula detail a professional development program based on identifying and fostering students' "geometric habits of mind."

The third section, "Teaching Geometry for Understanding," brings us to the actual teaching of geometry. The section opener by Editorial Panel member Paniati relates one geometry teacher's evolution as a practitioner of discovery learning. DeVilliers, Govender, and Patterson share perspectives on how teachers can develop and give to students a nuanced view of definitions. Casa and Gavin then give examples of how to develop elementary school students' understanding of definitions for quadrilaterals. Hollebrands and Smith provide an overview of research on the use of interactive geometry software. Contreras and Martinez-Cruz collaborate in two distinct articles to show how interactive geometry software can be used to help students become better problem solvers and to invent their own theorems. In a similar vein, Quesada reports on discoveries students have made using interactive geometry software and implications for professional development. Blair and Canada demonstrate how one carefully chosen, open-ended problem can lead to a very rich exploration. Flores shows how interactive computer-generated figures can be used to develop and show connections among area formulas.

In the one article that discusses an "integrated" approach to the secondary school geometry curriculum, Wilson discusses the advantages and challenges of structuring a course around a set of carefully chosen problems. Davis shows how a traditional geometry lesson can be redesigned to promote students' more active involvement. The final article by Todd looks to the future, in which software integrating algebra and geometry may further extend teachers' capacity to use technology effectively to stimulate students' thinking.

At the outset I mentioned the three previous yearbooks (Reeve 1930; Henderson 1973; Lindquist 1987) that specifically refer to geometry in their titles. In addition, in the Thirteenth Yearbook, *The Nature of Proof* (Fawcett 1938, 2001), Harold Fawcett described an experiment in which a class of high school students constructed their own system of geometry—undefined terms, definitions, postulates, and theorems—from scratch. Many of the articles in the third section of this Seventy-first Yearbook emphasize the power that students gain when they are encouraged to “invent” their own mathematics—much in the spirit of Fawcett.

The preparation of this yearbook would not have been possible without the work of an outstanding editorial panel. Each person brought his or her unique perspective, and together they functioned as an effective team. The following individuals served as editorial panel members:

Michael Battista, Ohio State University

Earlene Hall, Detroit Public Schools, Detroit, Michigan

Joseph Malkevitch, City University of New York—York College

James Paniati, Northwestern Regional High School, Winsted, Connecticut

Ann Spinelli, Bristol Public Schools, Bristol, Connecticut,

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