Foreword

Over the last three decades, there has been a growing emphasis on the role of discourse in learning mathematics. Beginning with the publication of two landmark standards documents from the National Council of Teachers of Mathematics (NCTM)—*Curriculum and Evaluation Standards for School Mathematics* (1989) and the *Professional Standards for Teaching Mathematics* (1991)—and continuing with nearly every mathematics standards document established since, discourse has been identified as a central component of classroom instruction.

Mathematics classrooms, however, have historically featured students sitting silently in rows listening to the teacher and working individually on scores of problems using just learned procedures. This image is captured in a description that appeared in the *Professional Standards for Teaching Mathematics* (NCTM 1991) and was taken from a series of case studies written by Wayne Welch more than 40 years ago (Welch 1978, p. 6; quoted in NCTM 1991, p. 1). Surprisingly perhaps, this image still describes instruction in many high school classrooms today:

> In all math classes that I visited, the sequence of activities was the same. First, answers were given for the previous day's assignment. The more difficult problems were worked on by the teacher or the students at the chalkboard. A brief explanation, sometimes none at all, was given for the new materials, and the problems assigned for the next day, The remainder of the class was devoted to working on homework while the teacher moved around the room answering questions. The most noticeable thing about math classes was the repetition of this routine.

By contrast, consider a description that appeared in a recent NCTM publication (Boston et al. 2017, p. 2) that is aligned with what reformers have been advocating since the late 1980s:

> [In discourse-rich classrooms] students are active learners, constructing their knowledge of mathematics through exploration, discussion, and reflection. The tasks in which students engage are both challenging and interesting and cannot be answered quickly by applying a known rule or procedure. Students must reason about and make sense of a situation and persevere when a pathway is not immediately evident. Students use a range of tools to support their thinking and collaborate with their peers to test and refine their ideas. A whole-class discussion provides a forum for students to share ideas and clarify

understandings, develop convincing arguments, and learn to see things from other students' perspectives.

The contrast between these two descriptions is striking and raises two questions: "What precipitated the calls for such radical change?" and "Why has it been difficult to see the vision for discourse-rich classrooms become a reality in the United States?"

By the early 1980s, there was a growing concern about the performance of U.S. students on international comparisons, standardized tests, and on SATs. In addition, there had been a significant increase in remedial mathematics courses offered at the college level. Essentially, students were not performing well, and folks began to worry about the U.S. losing its competitive edge. It was clear that the traditional way of teaching mathematics was not working. But what is the alternative?

A growing body of research suggested that complex knowledge and skills are learned through social interaction. Social interaction provides students with the opportunity to use others as resources, to share our ideas with others, and to participate in the joint construction of knowledge. High-quality discussions, it was argued, would give students the opportunity to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other peoples' perspectives (NCTM 1991). Since the previous approach to teaching and learning mathematics was not adequately preparing all students, it was time for a new approach—one that had its foundations in how people learn.

Despite the growing evidence that engaging students in discussion is central to learning mathematics with understanding, such discussions present considerable challenges for teachers, many of whom learned mathematics in classrooms that looked more like what Welch described in 1978 than what Boston and her colleagues described in 2017. While there are many reasons why teachers may be reluctant to engage students in discussions, a key factor is teachers' lack of familiarity with discussions as a vehicle for learning and, therefore, their limited confidence that students will learn what is intended (and assessed) through this process.

Many resources are currently available that are intended to help teachers meet some of the challenges they face in making discussions a central feature of classroom instruction. For example, *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith and Stein 2011, 2018) provides a five-step model that is intended to make whole-class discussions more manageable by limiting the amount of improvisation needed during the lesson. By focusing on the need to carefully plan a lesson prior to ever setting foot in the classroom, Smith and Stein argue that the teacher is then better positioned to listen carefully to students during the lesson and guide students' thinking in disciplinary productive directions.

In addition, several authors (e.g., Chapin, O'Connor, and Anderson 2009; Herbel-Eisenmann et al. 2017; Kazemi and Hintz 2014) have described ways to hold students accountable for listening to and making sense of the thinking of others during a discussion. These "talk moves" (e.g., adding on to what has been said, restating what has been said in your own words, comparing your reasoning to someone else's reasoning) invite students to contribute to a discussion, thus ensuring that a discussion is not simply a conversation between one student and the teacher and that students are understanding and making sense of the ideas of others.

While these resources and others have been helpful in supporting teachers in their efforts to engage students in classroom discussions, they do not address all of the discussion-related challenges that teachers face. One such challenge, that is the focus of *this* book, is how to foster and improve small-group, student-to-student discourse. This is not to say that the previously mentioned resources ignore small-group work, but rather that previous work does not provide the step-by-step process encountered herein. Here the reader will find a blueprint for how to improve small-group, student-to-student discourse from initial observations and data gathering to assess current discourse, to implementing specific plans for encouraging and supporting student-to-student conversations.

At the heart of the book is a set of instructional strategies that teachers are encouraged to study and try out in their own practice. The features of the book make it both readable and practical. The numerous vignettes provide concrete grounding of the ideas and strategies discussed. The Opportunity for Engagement and Your Turn features provide teachers with the opportunity to consider what they have learned in terms of their own practice. Hence, the book requires teachers to actively engage with the text by investigating the carefully constructed examples and then considering the implications for their own teaching.

Throughout the book, two key messages are clear. First, if you want students to engage in discussion, you need to give them something worth talking about. That is, a first step in fostering student-to-student discussion is to give students a high-level, cognitively challenging mathematical task that has the potential to engage students in thinking, reasoning, and problem solving. Because such tasks can often be entered and solved in different ways, they provide access to all students by drawing on different strengths and resources of knowledge. In addition, such "group-worthy" tasks can serve to narrow the achievement gap by broadening the view of what counts as competence and who is seen as competent (Lotan 2003). While high-level tasks have been part of the lexicon of mathematics education for more than 20 years, they are still not a central feature of most classrooms.

Second, fostering and supporting student-to-student discourse does not happen overnight. It takes sustained commitment and effort. Many teachers express frustration that their students do not talk to each other and that small-group discussions simply "don't work." This book will help teachers carefully consider what they can do to help their students develop the skills to talk with each other in ways that lead to improved mathematics learning.

Getting students to talk to each other is not the goal of the work described herein—it is a means to an end. The goal is to improve student learning outcomes by providing students with opportunities to share ideas and clarify understandings, develop convincing arguments regarding why and how things work, develop a language for expressing mathematical ideas, and learn to see things from other peoples' perspectives.

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