CHAPTER 3

Instruction: Yesterday I Learned to Add; Today I Forgot

Jeffrey Shih,
William R. Speer,
and Beatrice C. Babbitt

A primary consideration in the effective teaching of mathematics is that students bring a wide range of abilities and learning approaches. Teachers must recognize, reveal, and address these differences among learners. We frame this chapter according to the National Council of Teachers of Mathematics Teaching Principle (NCTM 2000):

Effective Mathematics Teaching Requires Understanding What Students Know and Need to Learn and Then Challenging and Supporting Them to Learn It Well

To truly embrace the vision and promise of the *Principles and Standards for School Mathematics* (NCTM 2000), we must revitalize mathematics programs and rethink teaching and learning for the benefit of all students (Cawley and Reines 1996; Miller and Mercer 1997). The following example shows the complexities of teaching and learning for the benefit of *all* students:

> Ms. Sanchez knows that even with the best of planning, things do not always turn out as expected. However, today's results were particularly discouraging. Ms. Sanchez had been excited about the mathematics activity that she had planned for her fourth-grade mathematics class. It was a relatively simple activity requiring students to explore the relationships among proper fractions by using fraction bars, a ruler, and possibly some measuring cups. Students were supposed to work in groups and determine the order of magnitude of a given set of fractions. They were also encouraged to add to the set of fractions and incorporate these new fractions in the sequence. Students were to record any work and describe the accompanying experiments that supported their conclusions. Ms. Sanchez believed that she had accommodated the lesson to meet the special needs of her students: she had incorporated several concrete materials and ensured that each group included a natural leader and a student with good writing skills. They had spent the last few days learning how to represent fractions. Surely the students had all they needed to succeed with today's activity. As Ms. Sanchez moved from group to group, however, she saw several problems

developing. Sara had taken over group one and was busily marking the position of the fractions on the ruler while the other students made designs with the fraction bars. Jeff decided to place fraction bars on top of one another to determine size, but since others could not see what he was doing they busied themselves scooping rice with the measuring cups. Lori and Kyle were intently arguing about the relationship between 2/3 and 4/5 while their groupmates created fences with the fraction bars. What Ms. Sanchez had designed to be an interesting exploratory activity had turned into an objective-driven learning activity for only a few. What went wrong, and why? How could Ms. Sanchez have more effectively designed and carried out the lesson for this, or any, diverse population?

When one considers differences among students, the crucial questions include the following:

- How should teachers motivate and encourage every student to effectively explore and continue studying mathematics?
- How should schools and school districts develop and correct policies, programs, and practices that may contribute or lead to mathematics avoidance?
- How should mathematics educators make individual and collective commitments to eliminate barriers to studying mathematics?
- How should educators explore and implement effective means of convincing students and stakeholders of the importance of mathematics as a field of study?

Consistent with this book's purpose, this chapter will focus on challenging and supporting all students in learning mathematics—particularly children who often struggle to understand mathematical concepts, tend to expend great effort to calculate accurately, and seem to lack persistence in problem-solving situations.

Reform documents rarely give much guidance on modifying circumstances for at-risk students or those with a diagnosed learning disability in mathematics. Researchers in mathematics education often focus on anecdotal accounts of the effects of reform-based pedagogy and curricula on low achievers (Fennema et al. 1993). Some researchers seem to imply that reform-based mathematics pedagogy and materials are effective for all students, without a need for any special adaptations to curriculum, instructional techniques, or classroom organization (Resnick et al. 1991).

Some researchers in special education express doubt that proposed methods and materials associated with reform mathematics are appropriate for students with learning disabilities or those at risk (Carnine, Dixon, and Silbert 1998; Carnine, Jones, and Dixon 1994; Hofmeister 1993). For example, special educators have long recommended using a clear set of procedures to reduce ambiguity when teaching mathematics (Carnine, Jones, and Dixon 1994). Believing that multiple approaches to solving problems can lead to confusion, these researchers view alternative strategies and invented algorithms, a common approach in reform-based mathematics instruction, as problematic for low achievers. These researchers see one simple set of rules as the best approach to teaching these students.

Research on attempts to achieve inclusion for special education students, particularly students with learning disabilities, also suggests that general education teachers have a hard time accommodating such students' needs (Baker and Zigmond 1990; Schumm et al. 1995; Scruggs and Mastropieri 1996). Researchers have typically tried inclusion in settings where general education teachers used traditional pedagogy and curricular materials. Two dramatically different interpretations of inclusion have emerged. One view is that traditional pedagogy is, in some sense, responsible for the difficulties that low achievers experience and that, with curricula and pedagogy that emphasize levels of both content and pedagogical reform, many students who formerly struggled in traditional mathematics instruction will thrive. A contrasting view is that students who have difficulties in a more traditional mathematics setting will have even greater problems with the advanced topics and problem-solving activities in a classroom emphasizing a more reform-based approach. We need additional classroom-based research to clarify and perhaps end such debates. The lack of research, coupled with the concerns of the special education community, highlights the need for studies on how reform mathematics instruction affects low achievers.

Bottge (2001) posed the *key–lock model* of teaching mathematics to help guide efforts to improve learning of students with disabilities (fig. 3.1). The model is based on theories of cognition; emphasizes the Equity Principle (NCTM 2000, p. 12); and considers learner, contextual, and task variables essential to adequately describe teaching and learning mathematics. For significant learning to occur, the model proposes, six teeth of the instruction key (meaningful, explicit, informal, [de]situational, social, and teacher specific) must each fit a pin of the learning lock (engagement, foundations, intuitions, transfer, cultural supports, and student specific).

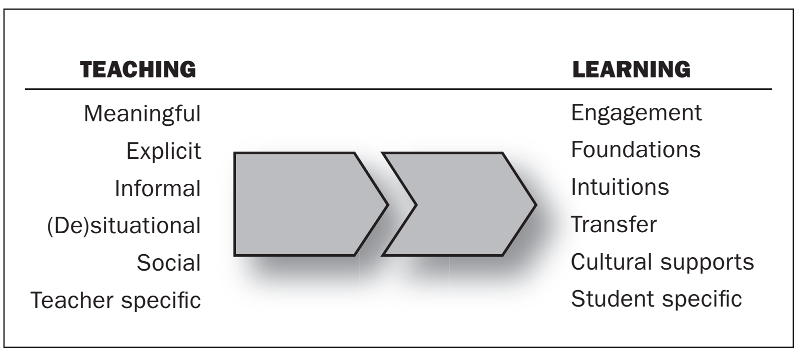

Fig. 3.1. The Bottge key–lock model

The model represents teaching and learning and is not a prescriptive guide. For example, the model leaves to the teacher finding meaningful learning experiences to engage students and ways to teach foundation skills explicitly. The teacher must also use informal methods to encourage students to use the intuitions they bring to the classroom and to (de)situate learning experiences, which students will recognize and transfer to problems in future contexts. And because students often solve authentic problems in groups, teachers should encourage students to work together in social contexts where they can find support for their ideas. Somewhat nonscientifically, the model also acknowledges teacher and learner interpersonal factors that either contribute to or work against higher achievement.

Even so, the Bottge key–lock model gives us a useful metaphor to explore the parameters of teaching and learning in a special-needs construct.

Effective Teaching Requires Knowing and Understanding Mathematics, Students as Learners, and Pedagogical Strategies

Knowing and understanding mathematics is a challenge for far too many elementary and special education teachers. Love of children and interest in developing reading and writing skills may be of greater interest to these teachers. In short, they are far more comfortable teaching subjects other than mathematics. Moreover, some middle school and high school teachers holding initial certification in fields other than mathematics find themselves assigned to mathematics classrooms. Such teachers may find themselves beginning to teach with inadequate content knowledge and skills in mathematics. Teachers may find fewer resources for students with special needs in areas such as algebra and geometry (Babbitt 2006; Witzel, Mercer, and Miller 2003). When they must accommodate students with learning difficulties, teachers may feel more anxiety from lack of awareness or knowledge of student needs and appropriate instructional strategies. Despite these limitations, most teachers want to, and strive to, teach effectively; the challenge is how to meet these diverse needs. This chapter discusses instructional strategies that will help teachers support mathematics learning for students with learning difficulties. These strategies, combined with opportunities for increased teacher content knowledge, are key to meeting the NCTM goals of mathematics for all.

Understanding learners with special needs suggests recognizing that students do not all learn in the same way (Badian 1999; Fox 1998; Keeler and Swanson 2001). Although students across achievement levels often have difficulty understanding concepts and skills, these difficulties often persist longer with special-needs children (Miller and Mercer 1997; Patton et al. 1997; Shalev et al. 1998). Also, these difficulties often resurface when teachers introduce new, more complex ideas and procedures. For example, subtracting from zero can be a problem for many children. Although all students might benefit from place value modeling, students with learning difficulties will initially need extensive place value modeling to understand this concept. They may also require more work on judging the reasonableness of an answer to catch their own errors, as well as systematic practice to overcome the common tendency to write "20 – 6 = 26." Such students will often revert to this error when attempting long division.

One approach to meeting the diverse needs of students in today's classroom is incorporating multiple representations of mathematical ideas. Doing so increases the probability that teachers will reach every student through an efficient and effective personal learning style. Different students benefit from hearing "it," seeing "it," saying "it," touching "it," manipulating "it," writing "it," or drawing "it." Most students benefit from experiencing a mathematical concept in several of these ways. Most students also expand personal understanding of a concept by seeing it represented with different materials or described with different examples. Before using materials and representations, teachers must carefully assess learning styles—both how the students take in the information and how they output it (receptive and expressive modalities). Using multiple representations does not mean bombarding them with these at the same time. Teachers should exercise caution in the number of presentation modalities used, particularly in the early stages of learning. A mathematical concept's seemingly unlimited number of different representations may simply overwhelm many students with learning difficulties. They may perceive each representation as a new concept and fail to see the commonality and relationships among the varied representations. Multiple stimuli and multiple representations could obscure the mathematical concept they were designed to clarify.

Consider the ways to represent decimals with dollars, dimes, and pennies; with place value blocks; and with a metric ruler. Each representation has its advantages: real-life, familiar materials used in new ways. Each representation also has its disadvantages: money relationships, such as relative value, are not obvious from each of the materials; the thousands cube (or, more commonly, the hundreds flat) now becomes the unit block; the metric system may be unfamiliar to the students and thus not serve as a good model for decimals. Each system requires a solid understanding of the unit and the relationship among the objects within the system. Students with learning difficulties often have problems keeping within-system relationships in mind without the added difficulty of comparing across representational systems.

From a cognitive perspective, however, teachers must represent mathematical ideas in multiple ways to expand students' understanding of core ideas and to help them see connections among these ideas. The challenge to the teacher is not only to help students link ideas and concepts superficially but also to establish an ownership of the relationship. Students with learning difficulties rarely learn from seeing or hearing only. In a cooperative group setting, someone else's having discovered a relationship is typically not enough to ensure understanding by students with learning difficulties—nor is having one person in the group summarize the discovery. All students must personalize the learning by demonstrating it with many examples. All students must also be able to describe the newly found relationship in their own words. They must be able to demonstrate understanding in some way that is meaningful to them, perhaps by using drawings or models, orally, or in writing. Moreover, few students, particularly those with learning difficulties, can retain and use a new connection after one encounter. They will need frequent experiences with the new connection on later days for it to become part of their conceptual repertoire.

The *Principles and Standards'* Content Standards (NCTM 2000) and the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM 2006) and, more recently, the *Common Core State Standards* (Council of Chief State School Officers [CCSSO] 2010) all stress the need to understand numbers and the relationships among them, to comprehend the meaning of operations, and to compute fluently and make reasonable estimates. The research literature on instruction often addresses understanding as concept instruction, whereas it often addresses computation under basic skill instruction. Extensive special-needs research supports systematic instruction in basic skills that incorporates appropriate modeling, sustained practice, and planned reviews (Hudson and Miller 2006; Fuchs and Fuchs 2001; Mastropieri, Scruggs, and Shiah 1991). Special-needs students experience enhanced mathematical performance when teachers explicitly model computational procedures (Miller and Mercer 1997), model thought processes used during the procedure (Montague 1997), offer sustained practice on a new skill (Fuchs and Fuchs 2001), and schedule planned reviews that slowly integrate new and old skills while leaving no skill unpracticed for long periods (Carnine 1997).

Although most teachers explicitly model new procedures to the entire class, they may find it necessary to redo this modeling with a small group of students with learning difficulties to make certain that these students comprehend each step as modeled. Materials that help students organize their work are particularly helpful for students who have difficulty organizing problems on a page and keeping numbers aligned. Examples include using lined paper rotated through ninety degrees to align numerals, using black construction paper "windows" to block out the excess stimuli on a page, and simply folding back the page to expose less material at one time. Students with language learning difficulties will also need particular help learning the self-talk that can guide personal movement through the process. Practice on the skill will need to continue over many days, rather than only a few days, with short daily reviews supporting this development. Systematic integration of ever more difficult problems will help students gain the desired competence and fluency.

Conceptual understanding is an important component of mathematical proficiency but has received less attention in the special-needs literature (Baroody and Ginsburg 1991; Ginsburg 1997). However, Miller and Mercer (1997) established the importance of concrete experiences, pictorial modeling, and gradual transitions to abstract thinking. Students with learning difficulties in mathematics often profit from beginning instruction in which they use their own bodies or concrete materials to illustrate mathematical ideas. Students learn a counting sequence as they count themselves while lining up for recess. Students distribute real apples to classmates to illustrate the sharing model of division. Students determine equivalent fractions by using objects that have meaning in their everyday world as fraction manipulatives.

Pictorial modeling helps student thinking become slightly more abstract. Ready-made pictures or drawn pictures can help illustrate a mathematical idea such as proportion. Early on in their use, pictures also should be able to be manipulated since many special-needs students have difficulty imagining a change in the pictured situation. For example, some pictured images that are supposed to convey movement, as in common set-separation pictures for subtraction, may not be easily interpreted as subtraction by the attempt to stagnantly show birds flying away from other birds or koalas climbing down trees or frogs hopping off lily pads.

As instruction moves to the symbolic level, teachers must form a link between the prior concrete and pictorial representations and the now-new abstract representations (Miller and Mercer 1997). Teachers must make this link, or transition, explicit so that the student knows exactly how symbols represent each element of the concrete or pictorial representation. Without this crucial link, students who understand at the concrete and pictorial level will often revert to mindless, and thus potentially meaningless, actions when using symbols. Butler and colleagues (2003) showed the effectiveness of this *concrete–representational–abstract* teaching approach in effectively teaching fraction concepts and operations.

Problem solving

Problem solving has been an area of particular concern for students with learning difficulties (Bottge 2001; Montague, Applegate, and Marquard 1993). Nearly every aspect of problem solving appears to pose some form of difficulty for students with special needs. If the teacher describes a problem to the class, some students may not be able to hold it in mind long enough to begin to solve it. However, if the problem is written, some students do not attempt to read it for comprehension but rather try to solve it by simply operating on the numbers. Students may not read the problem because of problems with decoding or reading comprehension. If the students do read the problem for comprehension, they may have difficulty picturing the problem situation. Even if students understand the problem situation, they may not make the leap to a solution strategy. In such instances, all strategies seem to hold equal value and promise. Strategies that require computation introduce the prospect of computation errors into problem solving. If students do arrive at a solution, they may rarely check to make sure that an answer is reasonable and accurate. Despite these challenges, students with learning difficulties improve problem-solving performance with appropriate instruction (Montague 1997).

Research has suggested a variety of problem-solving approaches for students with learning difficulties. Many approaches suggest following a series of problem-solving steps (Hutchinson 1993). Most of these approaches work well with traditional word problems but are less helpful with the nontraditional problems more common in standards-based classrooms.

Some have proposed anchored instruction as an effective alternative to step-by-step problem-solving strategies. Anchored instruction presents real-life situations as contexts (the anchors). We might consider problem solving within a traditional school store setting to be anchored instruction. Current versions of anchored instruction often use video vignettes, through which any real-life experience with embedded mathematical data might serve as an anchor for problem-solving instruction. For example, some videos focus on discovering measurements embedded in such adventures as fighting a fire or on interpreting medical measurements that emergency teams use. Whereas a typical problem that a teacher or textbook poses depends on the experience and imagination of the students, the videos compensate for different experience levels by giving all students a common picture of the problem situation or setting. The problem situation is reality based and complex enough to incorporate data for several problems. Students can refer to the video at any time to check information. The Cognitive Group at Vanderbilt University reported increased persistence and growth in problem-solving skills when using anchored instruction for students with learning difficulties (Cognition and Technology Group at Vanderbilt 1990).

Problem-solving environments such as those in the *Principles and Standards* are complex, often posing nontraditional problems. These guidelines respect and even encourage multiple approaches to problem solving. Language plays an important role in communicating the problem-solving approaches that students use. Students with learning difficulties may struggle within this environment, and teachers may wonder how to help them find their way.

To assess student needs, teachers must reflect on what happens during these complex problem-solving interchanges. Teachers also need to be open to observation by and feedback from colleagues to help understand the challenges to learning in their classroom (Friend and Cook 1996). A colleague–observer might be able to collect data to determine whether the teacher has structured open-ended questions so that struggling students have an opportunity to lay out the path to a solution. For example, an observer might note whether a teacher moves on after the top five students respond or instead offers all students a chance to respond in ways appropriate to their skills and abilities. Although a third party as an external observer will be beneficial to help make sense of the complexity in the standards-based classroom, entering into this professional growth partnership nevertheless requires collegiality and professionalism from both parties.

Dispositions

Principles and Standards describes the desired dispositions of children learning mathematics (NCTM 2000, p. 54). These dispositions encompass the desire that all students become autonomous learners who define personal goals, monitor personal progress, display confidence in their ability to solve a problem, show eagerness to figure things out on their own, are flexible in exploring mathematical ideas, and are willing to persevere.

Children with learning difficulties in mathematics rarely display these dispositions and in fact often display opposite behaviors. Such children are often adult dependent and display characteristics of learned helplessness (Halmhuber and Paris 1993). They may lack confidence in their ability to perform in mathematics, enthusiasm for mathematical activities, and flexibility or perseverance (Deshler, Ellis, and Lenz 1996). However, as *Principles and Standards* states, “Effective teachers recognize that the decisions they make shape students’ mathematical dispositions and can create rich settings for learning” (NCTM 2000, p. 18).

Two specific teacher behaviors will begin to support positive changes in student dispositions. First, students must feel that they are in an environment where taking risks is safe. Students with learning difficulties—and most students, for that matter—stop taking risks when their approaches are wrong and when others ignore or ridicule them for being wrong. The teacher’s challenge is to help students see that their path is valued and protected, as well as to guide them along that path. . . . Sometimes the path they have chosen may seem wrong to us but actually is one that may take us to a solution if we encourage and nurture the persistence that these students need. Students need to be right more often so that they will try again, and they need help to find the kernel of a useful idea in an incorrect response that can lead to eventual understanding. Second, since standards-based classrooms use many cooperative groups and peer groups, all students must learn these nurturing approaches as well. Even kind peers will tend to ignore a peer’s wrong response and propose a more appropriate solution. Learning theory suggests that punishment, ridicule, or consistent ignoring will extinguish a behavior. Here, such treatment might extinguish the desirable behavior of contributing strategies for problem solving unless teachers and peers alter interactions with students.

Effective Teaching Requires a Challenging and Supportive Classroom Learning Environment

Expanding the range of pedagogical strategies is one important consideration for teaching children with special needs. Teachers must pay greater attention to length and organization of tasks than is the norm for a particular age level. They must also take into account the impact of oral and written language difficulties as students demonstrate what they have learned. If teachers make clear the primary purpose for each mathematical activity, they will be better able to adjust requirements in supportive domains. That is, if the purpose of a given lesson centers on problem solving, then teachers might give aided support of computation for those students who still have a hard time with the required computation.

Challenging students without overwhelming them is an important task for teachers. Teachers need to individualize challenges but to consider them systematically so that all students move forward in the ability to handle ever more complex mathematical tasks. A key to challenging students with learning difficulties is realizing that they can often think about the same challenging problems as classmates but that they may need more support in organizing responses. For example, one activity for all students early on in data collection, organization, and analysis might be to develop several reporting systems that student groups can then use as desired. Students could then compare various data organization systems as far as usefulness in organizing and reporting data. Many students with learning difficulties will find that these organization systems help them approach a problem systematically without getting lost in the details. They will also help teachers and peers give specific feedback about which parts of the task remain to be completed.

Active engagement

Challenging environments require active engagement from all students during whole-group discussion, as well as small-group and paired activities. However, Baxter, Woodward, and Olson (2001) present a sobering picture of low achievers' involvement in whole-class discussions and small-group activities. In the classrooms that these authors observed, low achievers and students with special needs were generally silent during whole-group discussions. When such students did speak, they often gave one-word answers. During the class discussion in which other students might be actively engaged, the low achievers were often perceived not to be listening but instead were involved in off-task behaviors such as staring out the window, playing with a small toy, writing on paper, or arranging materials in their desk. Low achievers were much more involved in small-group work, but they often took nonmathematical roles such as organizing the materials or copying a partner's work. Pairs of low achievers had a difficult time starting the task and needed help from the teacher or aide to reread the directions, start the task, and work through several problems.

Some teachers have found ways to engage students with learning difficulties in meaningful mathematical tasks (Baxter, Woodward, and Olson 2001). Low-achieving students were engaged when the teacher first used the children's own arms to model parallel lines, then used yarn with children as endpoints to create parallel lines, and finally moved students to using a geoboard to model parallel and intersecting lines. The teacher emphasized both the mathematical vocabulary and the conceptual ideas during the lesson. Baxter, Woodward, and Olson speculated that the teacher's use of multiple representations before using the traditional geoboard made this lesson more effective. A special education teacher might recognize that this teacher first had the children experience the concept with their own bodies, then they externalized the concept by using yarn with themselves as endpoints, and finally they transferred this learning to objects totally outside themselves: rubber bands and pegs on the geoboard. Even though these children were third graders, they learned best when engaged in activities that were similar to those used with younger children. They could understand age-appropriate mathematical concepts by using learning approaches that others might consider developmentally delayed.

Adapting instruction

Several effective methods exist to adapt instruction for students with learning difficulties. Those described here include scaffolding; time adjustments; homework adaptations; and using technology in instruction, including assistive technology. We encourage you to seek out other techniques in the professional literature that this and other chapters cite.

Scaffolding

Scaffolding refers to the supports that teachers give during the early stages of learning a concept or skill. Scaffolds furnish temporary support as a student learns a new skill. The idea is often linked to Vygotsky's (1962) zone of proximal development, wherein one chooses instructional levels that are neither too easy nor too difficult for the child. Vygotsky views the social interaction of the teacher and student within this zone of proximal development as crucial for effective learning. In mathematics, scaffolds might include rephrasing a problem to make it more understandable, modeling a problem solution, thinking aloud by the teacher to show the thought process required in the problem solution, and questioning to help guide a student through problem solving. Scaffolds assist all students in developing the metacognitive skills that support problem solving. Scaffolds also often supply the vocabulary to discuss the learning experience, which is particularly important for students with language deficits.

Time needed to learn mathematics

Instructional time is valuable and limited, yet students with disabilities may require more exposure when learning a mathematical concept, more practice mastering a computational procedure, and more experience with each aspect of problem solving than other classmates. Adjusting for variances in needed learning time is essential in every classroom.