Trick or Treat?

Evan is going trick-or-treating with a large group of his friends. He notices several things:

- When they go to a door in groups of 2, there is always 1 trick-or-treater left to go alone.
- When they go to a door in groups of 3, there is always 1 trick-or-treater left to go alone.
- When they go to a door in groups of 4, there is always 1 trick-or-treater left to go alone.
- When they go to a door in groups of 5, everyone is included in a group.

How many trick-or-treaters could be in Evan's group? Create a poster that demonstrates and explains how you solved this problem.

CCSSM Standards for Mathematical Practice

Practice 1: Make sense of problems and persevere in solving them.

Practice 3: Construct viable arguments and critique the reasoning of others.

CCSSM Content Standards

4.NBT.B.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.OA.B.4: Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

4.OA.C.5: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Problem Discussion

This task is an excellent opportunity for students to engage in making sense of problems (SMP 1). Students need to develop an understanding that the number of trick-or-treaters in the whole group will have a remainder of 1 when divided by 2, 3, and 4. However, when the number of trick-or-treaters in the whole group is divided by 5, there are none left over (4.NBT.B.6). This remainder (or lack of remainder in the case of division by 5) needs to be connected contextually to the problem as students make sense of the situation.

This is a measurement division situation (or repeated subtraction), in which the given number of trick-or-treaters in a group is peeled away from the larger group to determine the

number of groups, rather than being distributed into a given number of groups to determine the size of each group (partitive division; fair sharing) (3.OA.A.2). Thus, the remainder is the trick-or-treater who is left when the maximum number of groups has been formed of a specific size.

Once students have made sense of the problem, they can use many different strategies to solve it, presenting an opportunity for them to defend their own strategy and critique others (SMP 3). These multiple strategies also provide an opportunity to make connections among multiplication, division, factors, multiples, divisibility tests, number patterns, remainders, and subtraction (4.NBT.B.6, 4.OA.B.5, and 4.OA.C.6). Although most students may come up with 25 trick-or-treaters as the answer, the problem has multiple answers. Students who are ready for a challenge may be asked to find other solutions; adding 60 (the least common multiple of 2, 3, 4, and 5) to a solution will generate another correct solution. Correct solutions therefore include 25, 85, 145, 205…, although 25 may be the most viable solution in the context of the problem.

Strategies

- Students may use trial and error with multiples of 5 to identify a multiple of 5 for which the number that is 1 less is a multiple of 2, 3, and 4. For example, the student may see whether the number 14 (1 less than 15) is divisible by 2, 3, and 4. When this does not work, he may proceed to the next multiple of 5.
- Students may use logic to determine that the solution must have a 5 in the ones place. This conclusion results from logical deduction:
    - The solution must be a multiple of 5, so it has a 0 or 5 in the ones place.
    - Any number with a 0 in the ones place is a multiple of 2, so it cannot be the solution.
    - Therefore, the solution must have a 5 in the ones place.

  Students may then use trial and error with multiples of 5 with a 5 in the ones place.
- Students may make lists of multiples for 2, 3, 4, and 5. Then they will check the lists for a number that is common to them for 2, 3, and 4 but also has a number that is one greater on the list of the multiples of 5.
- Students may identify the least common multiple (12) of 2, 3, and 4 (although they might not use this terminology). They may then check values that are 1 greater than the LCM and its multiples (12, 24, 36… ; 13, 25, 37…) to see whether the value is divisible by 5.
- Students may make lists of numbers that are 1 greater than multiples of 2, 3, and 4. For example, numbers that are 1 greater than multiples of 4 are 1, 5, 9, 13…. (Note how this strategy aligns with content standard 4.OA.C.6.) Then the students may check these lists for a common value and check to see whether this value is a multiple of 5.
- Students may use divisibility tests (e.g., if the sum of the digits of a number is a multiple of 3, then that number is also divisible by 3) in connection with the strategies above to determine whether numbers are multiples of 2, 3, 4, or 5.

Misconceptions/Student Difficulties

- Students may not recognize the significance of there being 1 trick-or-treater left over when the trick-or-treating friends are grouped by 2's, 3's, and 4's.
- Students may identify a multiple of 5 that satisfies limited criteria. For example, 15 would work for 1 trick-or-treater left over when the trick-or-treating friends are grouped in 2's, but it does not satisfy the criteria for 3's and 4's.
- Students may identify a value that works for 1 trick-or-treater left over when the trick-or-treating friends are grouped by 2's, 3's, and 4's, but is not a multiple of 5 (e.g., 13).
- Students may assume that any multiple of 2 is also a multiple of 4.
- Students may overgeneralize divisibility tests. For example, they may apply the test for divisibility by 3 to divisibility with other numbers, such as 4.

Launch

Begin this task by asking students how many of them are considering going trick-or-treating on Halloween. Ask them if they will be going with a group of friends, with family members, or by themselves. If students have trick-or-treated in groups before, ask them if any groups ever split into smaller groups, and when this might be a good idea.

Display the problem statement of Trick or Treat? to the class. Have a student read the brief introduction aloud. Then, call on different students to read aloud the four criteria of the problem and the question. Ask students to turn to one another and discuss what they know about the problem. Have students share their ideas. If they mention something that is explicitly stated in the problem, underline that portion of the problem. If they mention something that is an inference from the problem, (e.g., the number of trick or treaters is a multiple of 5; 1 more than a multiple of 3; not a multiple of 4), make sure that these are recorded for students to see.

Next, ask students to turn to a different neighbor and discuss *what they want to know*. Have students share their thoughts, highlighting particular questions that are not the final answer. For example, a student might indicate that a question they would like to answer is, "What are multiples of 5?" This question is not the final answer, but it may be a question that they need to answer along the way to finding their final solution, depending on the method they choose.

Finally, have students think to themselves about one last question: "What is your plan?" Do not have them share their thoughts in response to this question until they are in their respective working groups.

Explore

Assign students to small groups to work on this task. First, ask students to share their ideas for a plan, one at a time, without interruptions. Then direct them to discuss which of these plans they would like to pursue. Remind them that they may decide to switch plans as they work, especially if their initial plan is not fruitful.

Allow groups several minutes to work together before asking questions. Circulate throughout the room, and listen to all initial ideas about solving the task. As students continue their work or try different solution strategies, ask questions that check for understanding:

- What do these grouping explanations mean to you?
- What does it mean to you when it says that 1 trick-or-treater is left to go alone?
- What do you know about the solution?
- Why are you considering multiples of 5?
- How you do you know that you're looking for an odd number?
- How did you decide that value would not work?
- How do you know that the solution has to have a 5 in the ones place?
- Can you show me why you think 25 is the answer?

If students have come up with an incorrect solution, ask them to explain their reasoning. If they do not realize their misconception as they explain, ask them to demonstrate how their solution works, using linking cubes or another manipulative. Read each criterion aloud as they demonstrate, as needed.

Provide poster paper and markers so that students can create posters that illustrate and explain their solution strategies. Explain to the class that they will be doing a gallery walk, so the posters should clearly justify their reasoning for students who may have used a different solution strategy.

Summarize

Allow ten minutes for students to circulate throughout the classroom to look at posters. When students come back together as a group, ask them to turn and talk with a neighbor about any number patterns that were important to either their own solution strategy or other groups' solution strategies.

Call on students to share number patterns that were important. As students share a particular number pattern, write this pattern on the board (e.g., 1, 5, 9, 13…) and examine what is important about the pattern. For example, with the number pattern listed above, each of these values will leave a remainder of 1 when divided by 4. Be sure to ask students to connect the number pattern to the problem itself, explaining how the numbers are important to the criteria of the task. Other patterns that might be included in this discussion follow:

- 1, 3, 5, 7, 9…
- 1, 4, 7, 10, 13…
- 5, 10, 15, 20, 25…
- 2, 4, 6, 8, 10, 12…
- 3, 6, 9, 12, 15…
- 4, 8, 12, 16…
- 12, 24, 36, 48…

Next to each number pattern itself, record the meaning of the pattern in relation to the context of the problem if this will be helpful for students. Finally, ask students how the number patterns were used to solve the problem. Emphasize connections between the number patterns as students discuss their strategies. For example, how is the number 12 important in the final three patterns? As the patterns are presented, ask students to share how these patterns connect with their strategies, if applicable, thereby reviewing the strategies that groups used.

To conclude, discuss the following two Standards for Mathematical Practice: "Make sense of problems and persevere in solving them" (SMP 1) and "Construct viable arguments and critique the reasoning of others" (SMP 3). Ask students questions to explore how they demonstrated SMP 1 in their work on this task:

- What was helpful to you as you tried to make sense of this problem?
- What was tricky about understanding this problem?
- How did you or your group demonstrate perseverance in this problem?

In addition, ask students what challenges they faced as they used the skills called for in SMP 3:

- What did you have to keep in mind as you put together your poster?
- What did you find helpful or confusing about others' posters?
- How might you do your poster differently next time?

Differentiation

- For students who struggle to make sense of the remainder of 1 for this situation, allow them to find a value with an unspecified remainder when divided by 2, 3, and 4. That is, the number of trick-or-treating friends will have *some* trick-or-treaters left over when groups of 2, 3, or 4 are made. Solutions for this modification are 5, 25, 35, 55, 65….
- Challenge students to find other solutions. Also ask them if they can find a pattern in the solutions (they are 60 apart), how many solutions there are (an infinite number of solutions—although not contextually), and how this pattern relates to the numbers in the problem (adding the least common multiple of 2, 3, 4, and 5).
- Challenge students to write a similar problem of their own.