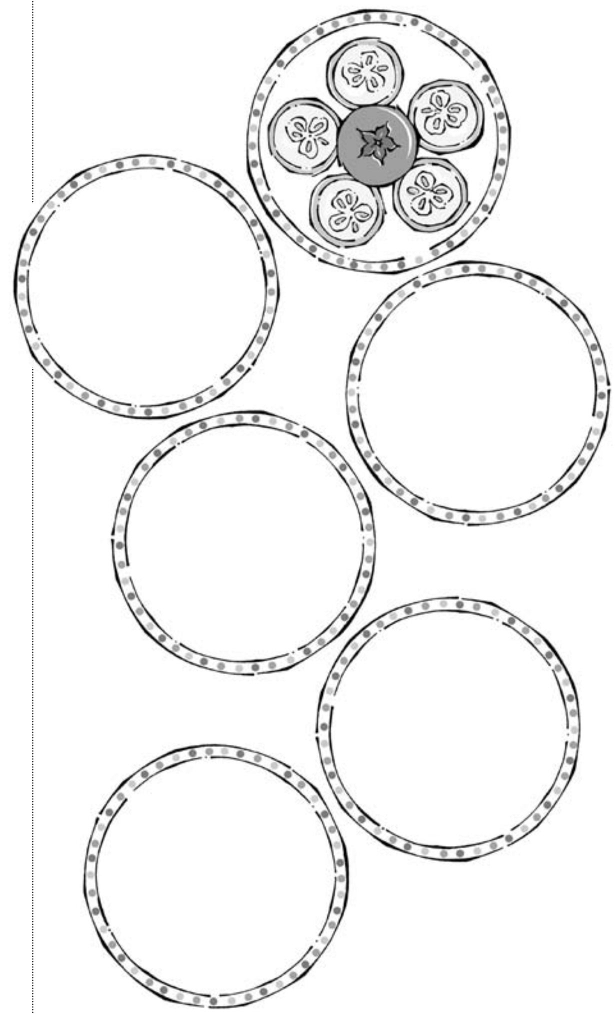
1

Beginning Multiplication
It's Time for the Times Tables

Multiplication is one of the basic arithmetic operations that your child will need to master in grades 3–5. For many people it is the first thing they remember struggling to memorize. You might not think that you use multiplication regularly, but it is hidden in many common activities. You can make these uses more obvious and help your child to become familiar and comfortable with multiplication.

How Can We Use Multiplication at Home?

Multiplication is used in many situations. Preparing lunch, you might say: *If we put two carrot sticks on each plate, how many will I need to cut? What if we put three on each plate? Four on each plate?* If there will be four people eating, your child might notice the pattern of 4, 8, 12, 16 that develops—it's counting by fours. *Let's put away two blocks at a time and see how many blocks we used* leads to counting by twos, which is another way of stating the twos table. Stringing beads in a pattern such as red-blue-yellow-yellow-green leads to estimating how many beads are needed to make a necklace. *If we make the pattern six times, how many total beads will we need? How many blue? How many yellow?* On family driving trips, if you travel at 55 miles per hour, *How far can we go in two hours? In three hours? In half an hour?*



How many cucumber slices in all are needed to decorate six plates?

Important Vocabulary for Multiplication

Product is the name for the result of a multiplication. The numbers multiplied are called **factors** of the product, and are also called **multipliers** and **multiplicands.** A **multiple** of a number is any of the products you get when multiplying it by a whole number. For example, 18, 26, and 32 are all multiples of 2 because they can be formed from 9×2, 13×2, and 16×2, respectively.

Why Is It Important to Memorize the Multiplication Tables?

Learning to count, recognizing the written symbols for numbers and operations, and getting to know basic addition facts all occur at an early enough age that we may not remember the effort they all took. But the multiplication tables, usually a part of third-grade math, required hard work at a late enough age that many people can recall the exertion. You may wonder whether this project is still necessary. Even with calculators readily available and excellent for many situations, memorization of the multiplication tables actually remains an extremely important tool. In order to understand division, fractions, and ratios and be able to spot many patterns, your child must recognize the numbers in the multiplication tables.

Division relates to multiplication in a manner similar to subtraction's relationship to addition. Just as $3 + 2 = 5$ helps us know that $5 - 2 = 3$ and $5 - 3 = 2$, the fact that $3 \times 2 = 6$ lets us see that $6 \div 2 = 3$ and $6 \div 3 = 2$. Also, multiplication and division undo each other just as addition and subtraction do: $4 + 3 - 3$ brings us back to 4 again. $4 \times 3 \div 3$ also yields 4. Operations that can undo each other in this way are called **inverse operations.** When children see these relationships, division becomes an extension of what they already know, and not an entirely new topic to learn.

Multiplication is also needed to understand fractions. Here's an example: If we have $\frac{3}{4}$ of a cake and cut each of the fourths into two equal parts, the result is $\frac{6}{8}$. Cutting the pieces did not change the amount of cake (if we ignore a few lost crumbs), as $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions. It is easy to see this if you know that when each fourth is cut into two parts, the new pieces are each an eighth of the cake. The number of pieces you had, three, has now doubled to six because you have two new pieces for each one you had before. Both the number of pieces that would form the whole cake (the **denominator**) and the number of pieces you had (the **numerator**) were multiplied by 2 to form the equivalent fraction.

However you slice it, $\frac{3}{4}$ of a cake is the same as $\frac{6}{8}$ (except for a few crumbs).

> Many people are uncomfortable working with fractions, but don't be daunted by the fraction example given here. Division and fractions are discussed in detail in later chapters, and you will have the chance to learn more about them when you and your child begin that work.

What Is Your Role in Helping Your Child?

Learning at home can be enjoyable for both parents and children. It is also a necessary companion to the work done at school. Each child learns at his own pace. In the classroom, there is often one teacher for twenty-seven students. You know your child better than anyone, and you are in a position to know exactly which times table your child is ready to learn and, within it, which facts your child needs to practice further. You can ask the same question at different times of the day and on weekends and holidays until it is memorized. When one of my daughters was learning the seven times table, she often forgot the product of seven times eight. I asked her this fact at different times, over several days, until she told me, "You don't have to ask me that one anymore—I know it now." A full multiplication table looks like 144 facts, but by working in an organized way, you will see how we reduce the number to only 39! Knowing this, you can be encouraging and confident about your child's learning.

This complete multiplication table is one of the items you can print out at home from this book's More4U website.

more4U

The Multiplication Table

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

This table has many patterns. See which ones you and your child can find, and look back at the table over time to see if you notice new ones.

What Is Multiplication?

A child's first understanding of multiplication is repeated addition. An example is that 3×5 means $5 + 5 + 5$, or "five, three times." Another way to say this is "three groups of five," which is a wording that will also be meaningful for multiplication of fractions:

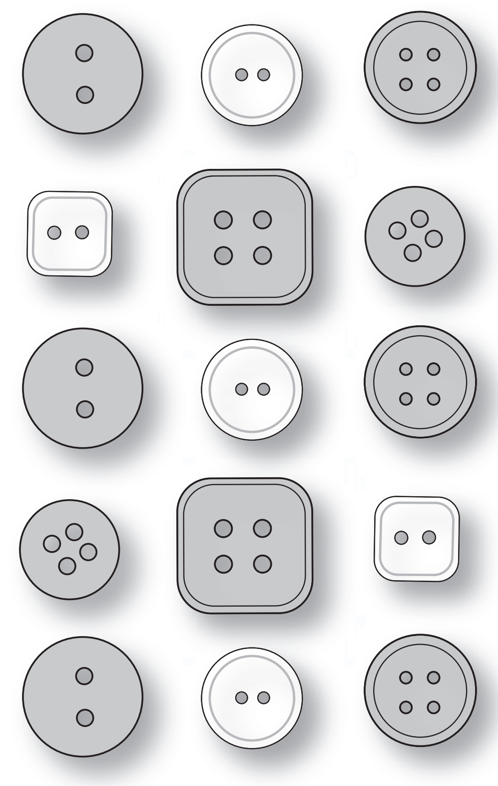
$3 \times \frac{1}{2} = 3$ groups of size $\frac{1}{2}$
$\frac{1}{2} \times \frac{1}{2}$ means "one-half of a group of size one-half"

Multiplication will become more abstract when your child learns to multiply negative numbers and to use exponents, but don't worry about this now. At this point, it is sufficient for him to recognize multiplication as repeated addition and as a group of groups.

Building "rectangles" (technically these are "rectangular arrays") with buttons or other materials is an excellent way to visualize multiplication as repeated addition or groups of groups.

At right, you see five rows of three buttons, which you can also view as three columns of five buttons each. Arrays are extremely useful because they clearly illustrate these four important aspects of multiplication:

- The same amount is added repeatedly: We see that each row has the same number of buttons without needing to count them. This is also true if we look at the columns—each column contains the same number of buttons.
- Multiplying 5×3 and 3×5 yields the same number of buttons; this is a very important property of multiplication that is true for any pair of numbers. It is the **commutative property** of multiplication and is sometimes expressed as $a \times b = b \times a$.
- The close relationship of division to multiplication is clear when we look at the array as 15 buttons divided into 5 rows of equal length. We see that 3 buttons go in each row. Similarly, 15 buttons divided into 3 columns of the same height means we can place 5 buttons in each column.
- In geometry, multiplication is made visible as the area of a rectangle, which can be found by multiplying the length by the width.



A 3 × 5 array of buttons

What's Commutative, What's Not

The order in which we multiply two numbers does not affect the product: 4×5 gives the same answer as 5×4. This is similar to the commutative property of addition: The order in which we add two numbers does not affect the sum. Notice that this is not true for subtraction or division: $6 - 3$ does *not* equal $3 - 6$, and $12 \div 4$ isn't the same as $4 \div 12$.

When a child builds rows, uses repeated addition, or employs a related fact (such as "I know $6 \times 6 = 36$, so 6×7 must be 6 more, so $6 \times 7 = 42$") to find a product of two numbers, he is multiplying. When he learns the multiplication tables, he will be memorizing by rote—immediate recall without having to actually do any arithmetic. Later, he will use these facts to multiply and divide large numbers, work with fractions, solve algebraic equations, and much more.

When Should I Start Teaching the Multiplication Tables?

The first step in helping your child learn the multiplication tables is to make sure that you don't start too early—that is, not until he can easily add the numbers 1 through 12 in his head to any two-digit number. For example, $35 + 7$ and $48 + 12$ are sums he can find without counting up by ones.

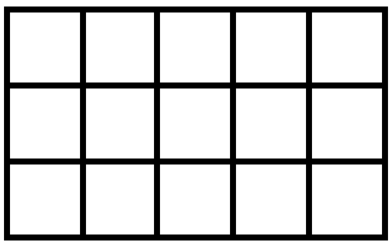
It is appropriate to begin memorizing the multiplication tables when a child has already noticed the following:

- Multiplication is repeated addition. (3×5 means "three groups of five," which is the same as $5 + 5 + 5$.)
- Zero times any number is zero. ($0 \times 5 = 0$, $0 \times 8 = 0$)
- One times any number is that same number. ($1 \times 5 = 5$, $1 \times 7 = 7$)
- The tens table is very easy. (4×10 = "4 next to 0" = 40, and $11 \times 10 = 110$)
- The child knows most or all of the twos table from the playground and other informal sources.
- Multiplication is commutative: $a \times b$ is always the same as $b \times a$ (where "a" and "b" stand for numbers).

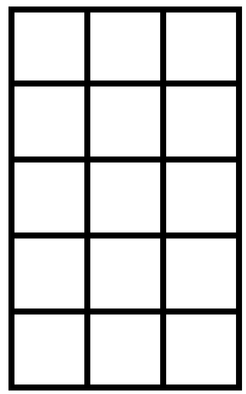
If your child is not yet aware of these facts, work on them in an informal manner. Children often know part, but not all, of the twos table. You can pair pennies, cucumber slices, anything available, and have your child figure out how many there are. Then have him create pairs, count the number of pairs, and find out how many objects there are. Practice counting by twos, and later by threes, fives, and tens.

Building rectangular arrays with buttons or other materials can help your child understand the relationship between 3 × 5 and 5 × 3 as discussed above. Another good material for visualizing multiplication tables is "square grid paper"—what is usually referred to as graph paper.

Using graph paper, your child can color in a 3 × 5 rectangle and also one that is 5 × 3. These can then be cut out and compared so that the sameness of their shape and size (or, as you may remember from geometry class, their "congruence") can be carefully checked.



3 rows with 5 in each row



5 rows with 3 in each row

These rectangles can also be used to find the answer to a multiplication fact, by counting if necessary. If your child needs to count to find the answer, it is not yet time to memorize the tables. He should be able to "add on" to figure out the table initially. For example, for the sevens table:

1 × 7 = 7.
2 × 7 = 7 + 7 = 14.
14 + 7 = 21. That's 3 × 7.
21 + 7 = 28. That's 4 × 7.
And so on.

You can find a sample of graph paper to print out and use at this book's More4U page.

more4U

If adding the next multiple of a number is too hard, make sure your child knows his sums to ten from memory. (You can make it fun and easy by teaching him the base 10 card trick explained in chapter 7 of my earlier book *Adding Math, Subtracting Tension* for children in prekindergarten to grade 2.) Then go on to sums to twenty. Have him look at the pattern of 8 + 5, 18 + 5, 28 + 5, etc.

You'll find a printable multiplication table for 1 × 1 through 12 × 12 at this book's More4U web page.

Getting Started

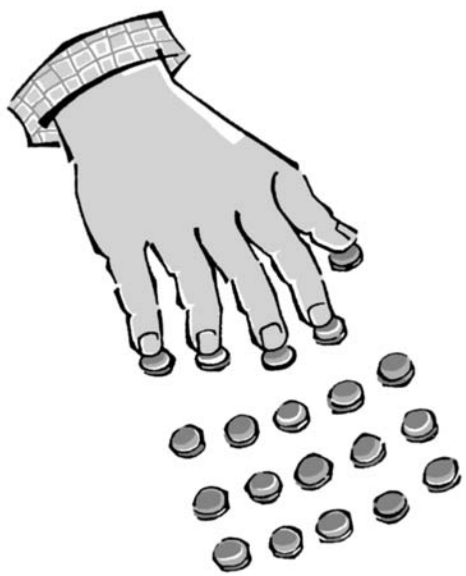
Now that you see how learning multiplication facts is going to be important for your child, and you have checked that he is ready, you can help him to learn 1×1 through 12×12 (and to say the products for them almost as fast as he can for 2×2). Because we still use inches and feet for measurement, it is useful to learn the multiplication facts through twelve instead of stopping at ten.

When you begin to help your child memorize the tables, **do not frighten him by showing him the entire 12 × 12 table!** A traditional multiplication table will be a useful reference for several of the following activities, so you will show it when needed for that purpose. Learning to read the rows and columns to find a particular product is also a skill worth developing. You can use the table and show your child how to use it when you reach the activities for children who know several times tables. To begin, though, activities are given for learning one table at a time. This can look like a daunting task, but we are going to make it simpler by playing games. As always: Remember to be patient with the struggling learner.

Learning the Tables

Most children begin to learn the times tables by repeatedly adding on the fixed amount of the table being learned. For example, with the threes table: 3, 3 + 3 = 6, 6 + 3 = 9, etc. Next, they learn to recite the table in order, getting sufficiently familiar with the numbers to know them without adding: "3, 6, 9 . . ." Finally, when they know the facts out of order, they have mastered that table.

Reciting the table



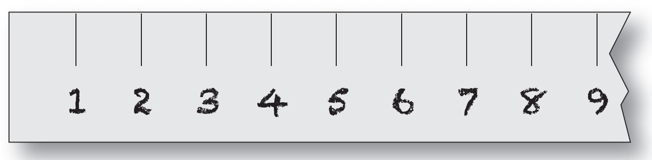
"5, 10, 15, *20* . . ."

The fives table is often the best place to start because your child is probably at least partially familiar with it. If he does not yet know the chant "5, 10, 15, 20 . . . " then repeat it often. You can count a penny or button collection by placing the five fingers of one hand on each object as you move them and count.

Jumping along a number line

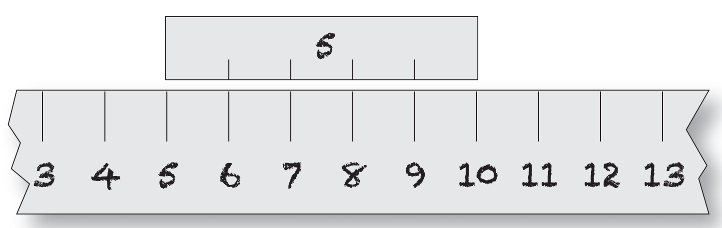
The multiplication table can also be thought of as equal-size jumps along a number line. Visualizing the numbers on a number line is extremely helpful for many parts of mathematics, including such concepts as the multiplication of fractions and negative numbers. By creating this image you both help your child learn the tables and prepare him for more advanced math.

You can make a number line on a long strip of paper, using a ruler to make sure that the "tick marks" for the numbers are spaced equally far apart. Leaving ¼ or ½ inch between tick marks will probably give you enough room to write the numbers. The edge at the far left will be zero. Create a number line that goes to 150 to accommodate the largest fact in the tables, which is $12 \times 12 = 144$. (The paper used in grocery stores to print receipts is good for this purpose. It is called "adding machine" or "calculator roll" paper.)



A homemade number line

For the fives table, cut a piece of cardboard that is the length of five units on your number line and label it "5." Your child can slide the cardboard along the number line to see one five, then two fives, and so on, with one end of the cardboard at each multiple of five.



A piece of cardboard that is five units long makes it easy to find multiples of five.

$6 \times 5 =$

$6 \times 5 = 30$

The front and back of a flash card

In elementary school, when we had to memorize a poem, some vocabulary words, or math facts, my sisters and I would help each other by taking the page of what was to be learned and listening while the other recited. After a while we noticed that the person holding the sheet and thinking about what was supposed to be said ended up memorizing things first, so we switched roles. Letting your child test you is an excellent way to help him learn.

Making flash cards

Help your child to make flash cards by using index cards or cutting up lightweight cardboard into rectangles. Let him write the facts himself—doing so provides practice for learning them. The cards can be created in order, with each having a back and front such as shown at left. Be sure to check that the back and front of each card have the same multiplication fact and that the products are correct.

Let your child find the products by any means he chooses, including referring to a printed multiplication table or using a calculator. All the time he is doing this he is concentrating on the facts he needs to memorize. Later, these cards can also be used to learn the tables out of order.

The best way to begin is to have him test you. You do not need to make errors on purpose: Every time your child hears the correct number helps him to learn.

After they've been played with for a while, the flash cards can be stored in two envelopes, one labeled "multiplication facts I know well" and the other "multiplication facts I'm working on." As the deck of known facts gets larger, your child will have a tangible sense of his progress. He already knows 1×5, 2×5, and 10×5, so he has nine multiplication facts to learn for five. Practice just one or two facts at a time, until he knows them all well. Remember to review the known facts too, so that they are not forgotten.

Playing "Concentration" (also called "Memory")

No matter how you play this matching card game, both you and your child are winning: As he turns over a card with "3×5" and tries to remember "Where was that 15?" he is embedding that fact in his mind. This is the case even if he needed to look up the fact or add up to find it; he now associates 15 with 3×5.

Making the deck for this game is not complicated. You will need twenty-four index cards or halved index cards or any equal-size pieces of cardboard. On each card you will write either a multiplication question or its answer. For example, one card will have "3×5," and another will say "15." Your child can use a crayon or anything that will not show through on the back.

Turn the cards over, mix them up, and then take turns flipping over two cards, looking for matches between the factors and their product. The game can be made easier by playing with a smaller deck, beginning with the facts for 1×5 through 6×5 and another game for 7×5 through 12×5. Just make sure to include the correct answers. Setting up these smaller decks is another way that your child practices the facts.

One of the beauties of this game is that children are often better at it than adults. Perhaps this is because, while playing, the adult is also thinking about the chores that must be accomplished, while the child is completely focused on the game. Children love to win, and you are winning, too, when you are helping to accomplish your goal of having your child know the multiplication facts.

Inventing word questions

Look for situations in which multiplication by five can be done in a practical context, creating "word questions." How many fingers are there in our family? Count each hand as five. How many sticks of gum in three five-packs? If we are filling seven party bags and want to put five stickers in each bag, how many stickers will we need? Apple cores have five sections and phlox have five petals. Together, find things that come in fives and make up questions about them. Then find things that come in the amounts of the next table you work on.

Creating a Multiplication Table Picture Puzzle

The best way to practice a table is by making a picture puzzle. When I show a sample to a class and then tell them that they will be making their own, the children cannot contain their delight—they jump up and down, hug me, and get to work. Several branches of math are involved in the work: measuring, dividing, drawing parallel lines, and multiplication. These may look like a lot of work, but each step is easy, and they can be made in a little over an hour. When your child puts the puzzle together, it is the same exercise as practicing with flash cards, but much more fun.

"I know where that 56 is!"

Before Moving On

When you think that your child knows a table fairly well, find out if there are some facts that he is still uncertain about. Remember that focusing on just one fact at a time might be necessary and more helpful than reviewing the entire table.

For a version of this puzzle that is ready to assemble, go to this book's More4U page.

more4U

A. The picture glued onto cardboard

Marking both edges to draw dividing lines in this puzzle makes the lines more nearly parallel (they won't be perfect) than freehand drawing would. When you do this, you are developing the meaning of parallel as lines that are the same distance apart.

What you will need

- Picture or drawing a little smaller than 6 × 8 inches, made by your child or cut from a magazine
- 6 × 8-inch piece of lightweight cardboard or construction paper
- Scissors
- Glue
- Pencil
- Ruler
- Markers or crayons
- Rubber band
- 6 × 9-inch cardboard folder, made from two 6 × 9-inch pieces taped together on the long end, or one 9 × 12-inch piece scored across the middle and folded

How to make the puzzle

1. Glue the picture on the 6 × 8-inch piece of cardboard.

If the picture is smaller, the cardboard edge can form a little border around it, as shown in drawing A.

2. On the back of the picture (on the cardboard), your child will mark twelve equal-size squares.

Plan with your child where the marks should be made. If your cardboard is 8 inches long, you can make marks along one edge at 2, 4, and 6 inches, and then at the same measurements on the opposite side. Place the ruler by those dots to draw three straight lines that divide the cardboard into four strips.

3. Do the same as the last step with the shorter sides, marking dots at 2 and 4 inches along both edges.

When your child connects these dots, as shown in drawing B, the back of the picture will be sectioned into twelve 2 × 2-inch squares.

4. Have your child trace the 6 × 8-inch cardboard's outline inside the folder with the long side against the fold.

Then have him divide this rectangle inside the folder into twelve equal squares in the same way as he marked the back of the picture. He can use the marks on the back of the picture as guides to make the same lines 2 inches apart, or he can use the ruler as before.

5. In the squares inside the folder, have your child write the times table questions, one per box and *out of order,* from one to twelve times the number being studied.

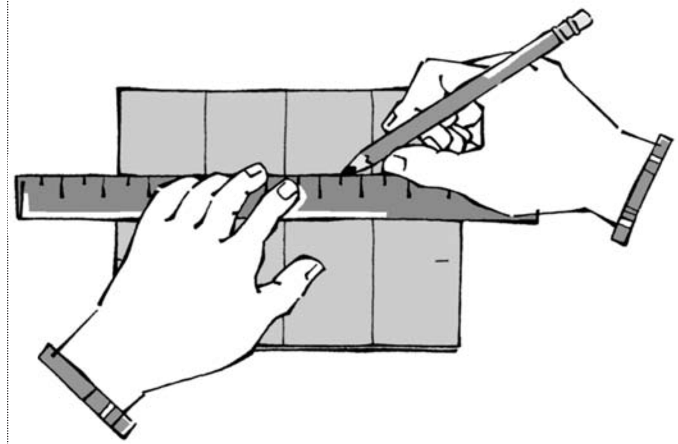
He can write first in pencil and then overwrite in marker or crayon after you have checked that everything is correct.

6. Next, place the picture upside down next to the rectangle in the folder (with the cardboard side facing up) and have your child write the products of each of the multiplications.

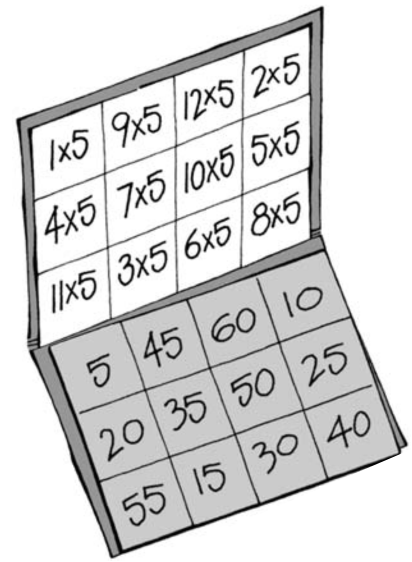
Make sure that the answer to each is in the box that it would cover if it were placed directly over it, as shown in drawing C. This is very important! Many people think that this placement of the answers will not work, but it is correct.

7. After checking to make sure all is correct, cut the picture along the lines, so that you have twelve squares.

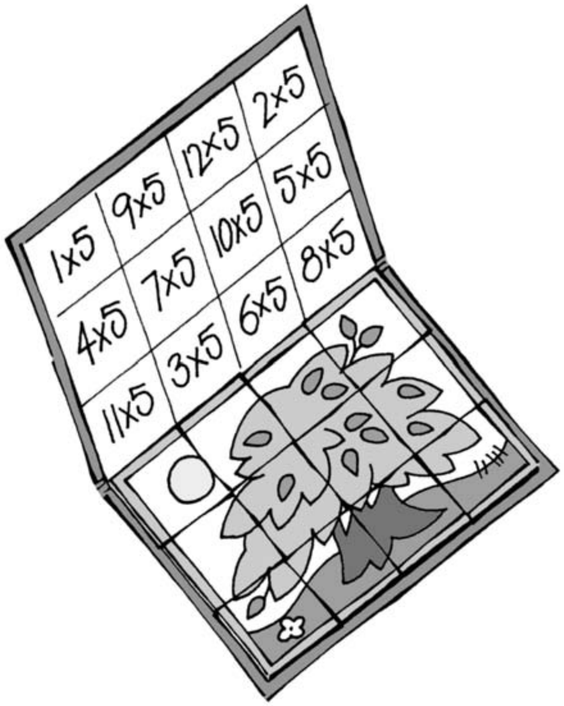
Cutting cardboard can be difficult, so you may need to do this for your child.



B. Marking the squares on the back of the picture



C. The multiplication questions written inside the folder, with the answers written on the back of the upside-down picture



D. The correctly solved puzzle

Solving the puzzle

Now the puzzle is ready for play. Turn the picture pieces so that the number side is up and shuffle them. With the folder open, have your child place the answers on top of the appropriate questions. No need to say anything if he makes an error—the picture will tell him soon enough.

When all twelve have been placed, tightly close the folder over the squares and flip it over. Then open it to reveal the completed picture puzzle, as shown in drawing D. If all the answers are correct, the picture is complete with every square in the right place—a magical result for many children! If any squares are out of place, your child can simply try again.

Solving the puzzle just a few times can be sufficient for memorizing a table. Wait until one table is learned before creating the next puzzle. If you do this with several children, they can each make a puzzle for a different table and then use each other's for practice.

Developing Measuring Skills

Your child will soon be expected to know the meaning of "parallel lines," but learning to draw them is not a usual classroom activity. In an activity like making the picture puzzle here, children often first take a ruler and use its straight edge to draw lines that are approximately parallel, without making any measurements. These lines are not parallel. Children usually won't see the need to measure along both edges that the lines will touch, but you can use a different piece of paper to show them how this is done.

When measuring, children often place the one-inch mark at the edge of the paper, not realizing that the distance from zero to one measures the first inch. Some children think that to measure 2 inches, they must move the ruler and measure from the zero to the two mark each time, not knowing that the marks from two to four and four to six also measure 2 inches. By working together, you can help your child learn these important measuring skills.

Learning the Other Tables

Each of the activities described above for the fives table can (and should) be used for learning the other tables:

- Jumping along a number line
- Reciting the table in order to get familiar with the numbers in the table
- Making flash cards
- Playing "Concentration"
- Having your child test you before you test him
- Creating word questions
- Making a picture puzzle

You might suggest that your child work on the elevens table next, because that is also very easy. Only two facts in it involve memorizing or noticing something new: 11×11 and 12×11. Then try the threes table. Your child will know 1×3, 2×3, 5×3, 10×3, and 11×3 by now, and so for the threes table there are only seven more facts to be memorized.

Continuing in this way with the four, six, seven, eight, nine, and twelve tables, the number of facts to be learned each time is one fewer. For example, when your child begins the fours table, he will have six new facts to memorize. The fact "3×4" was already learned as "4×3" in the threes table, and similarly, 5×4 was in the fives table, 11×4 in the elevens, and he already knew 1×4, 2×4, and 10×4 before beginning to memorize tables. In this way, your child has only 39 facts to memorize, instead of all 144 in the table. That's a pretty amazing difference. (Did you follow along and do the counting up to 39 too? Try it now, and go ahead and check my work on the table on page 16.)

Table	Number of facts to learn
1	0
2	0
10	0
5	9
11	2
3	7
4	6
6	5
7	4
8	3
9	2
12	1
Total	**39**

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

The above multiplication table shows how few facts remain for your child to memorize. As you help your child learn the various parts of the table, you can:

(*a*) Start with 5s, because your child knows how to recite these and only needs to learn them out of order.

(*b*) Next work on 11s, because there are only two facts left to learn.

(*c*) Work on the 3s next—only seven facts to memorize.

(*d*) Go to the 4s, with six facts, and there is one fewer fact to learn for each of the tables 6, 7, 8, 9, and 12.

Games to Play with a Mixture of Tables

Once your child is well on the way to learning several tables, there are additional games that you can play.

"Concentration"

You can use the cards you made for learning single tables to play the "Concentration" game described above with any combination of times tables. Some pairs yield the same product (for example, $6 \times 3 = 9 \times 2$), so you will need to include two cards for 18. With your child, decide whether the 6×3 card can be paired with the 9×2 card for a legitimate winning pair. If so, make sure that you have an even number of cards for numbers like 36 that have three factor pairs: 4×9, 6×6, and 3×12.

"Four in a Row"

"Four in a Row" is a game of strategy as well as multiplication, which makes it fun and interesting for people of different math levels to play together. It can be a fair competition because many children possess the good spatial sense that is a vital part of the game. If your child is unsure of her facts through 9×9, let her refer to a multiplication table while playing. Remember that for you, winning means helping her learn those facts. I learned this game many years ago. Recently, a slightly different version has appeared in some textbooks. If your child knows that version and wants to play it, follow her rules.

The object of this game for two players (or three—sometimes playing with a partner can take the pressure off a child) is to place four of your playing pieces in a row, horizontally, vertically, or diagonally. If you do not have suitable playing pieces from another game, one person can play with pennies and the other with paper clips. Unlike most games, the person who goes second has an advantage, so you will go first (at least at first), putting one of your paper clips on one of the factors in the strip at the bottom of the game board. Your child then takes a turn, placing one of her pennies on a factor in that strip. (She can even choose the same one as you did.) She then places a penny on the product of the two factors, beginning her chance to create a row of four.

You can find a full-size copy of the "Four in a Row" game board to print out at this book's More4U web page.

more4U

"I'll start with a 5."

"I'll do 7 . . . and 5 x 7 = 35."

"I'm moving my paper clip from 5 to 3 . . . and 3 x 7 = 21."

Now it's your turn: You move your paper clip that is covering a factor to another factor, multiply that new number by the factor covered by your child's penny on the factor strip, and place one of your paper clips on the game board. Each player may move only his or her own factor piece.

This game is excellent for reinforcing multiplication facts. A child who wants to place a penny on the box containing 15 must look for an opportunity to pair 3×5. It is when the board gets more crowded that the greatest repetition of multiplication facts occurs. If your child doesn't want you to put a paper clip on the box containing 56 (for example), she must keep reminding herself that she cannot help you by placing her penny on a 7 or an 8.

A multiplication card game

Many children are very fond of the card game with the unfortunate name of "War." I think that part of the appeal for children is that the game can go on forever with the cards changing hands back and forth, and with no winner before it's time to quit.

In the usual game, two players each have half of a shuffled regular deck in a face-down stack. They each draw a card from their piles, the values are compared, and the one with the higher card (suits are irrelevant here) wins both cards, adding them to the bottom of his stack. If the cards are the same value, three cards are dealt face-down, and one turned over to decide who wins all eight of those just played.

Here's a version that practices multiplication. Each player draws two cards at once, and uses the product to decide who has the higher amount. The ace counts as "one," and face cards (jack, queen, and king) are counted as ten. Let your child make the rules for what to do when the products match. The point is to play and practice, so the precise rules do not matter.

Helping with the Memorization Process

The multiplication tables can look challenging to a child, so remember the following when helping him to memorize them:

- Don't rush your child. Work on one table at a time. Begin by learning the table in order, adding on, and then work on learning it out of order. If a fact stumps him, ask it at different times of the day, over many days, until he firmly remembers it. Don't move ahead until a table is learned thoroughly. Otherwise your child is likely to feel overwhelmed and to give up. Review tables that are "done" by asking those older facts among the new ones.

- As with all memorization, it is often helpful to have your child begin by looking at a sheet with the facts and testing you. While he's looking, he's studying, and most children find it fun to be the one doing the testing for a change.
- When you begin the serious memorization work, mark it on the calendar. Make a note each time you move on to a new table. This helps to make the progress visible and the work not seem endless. If you did the preliminary work I suggested above and did not begin before your child was ready, it is unlikely that it will take more than eight weeks for your child to know them thoroughly. Once they are learned, the child can use them and notice them in many different situations, so they are not likely to be forgotten.

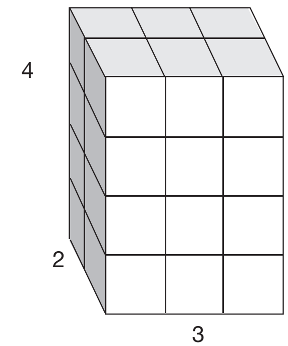
Going Even Further

Understanding a few more aspects of multiplication will help your child as he works with it in a number of contexts. In the last sections of this chapter, I'll cover several more topics you can introduce to your child, including the associative property, prime and square numbers, and factorization.

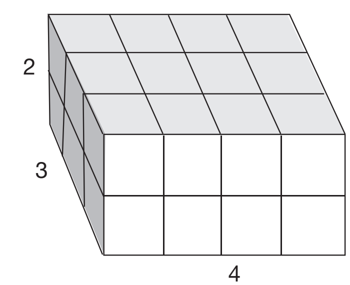
Another important property of multiplication

I already mentioned the commutative property of multiplication: that the order in which we multiply does not affect the answer. Another important property, and one that will become very useful when we multiply large numbers, is that numbers that are to be multiplied can be grouped in any way one chooses. This is called the **associative property** of multiplication—it does not matter which numbers associate together. At right there is an illustration of $2 \times 3 \times 4$ to help you see that this is true: No matter which combination we multiply first, we end up with the same rectangular prism that is $2 \times 3 \times 4$.

You probably make use of this property often. You may have noticed that addition is also associative, allowing us to add numbers by grouping together whichever we choose.



A 2 x 3 base stacked 4 high can be described as (2 x 3) x 4.



Two 3 x 4 blocks illustrate 2 x (3 x 4).

Notice that these examples are two different views of the same block.

Some special numbers: primes and squares

You may remember the term **prime number** for the numbers 2, 3, 5, 7, 11, 13, 17, 19, 23 and so on. The only way to form each of these numbers (when multiplying whole numbers) is by using 1 and themselves. Those numbers that have more than two factors are called **composite**. Some examples of composite numbers are as follows:

4, which has the three factors 1, 2, and 4
6, with the four factors 1, 2, 3, and 6

The number 1, which has only itself as a factor, is neither prime nor composite. Much elementary school time is spent finding the prime factors of composite numbers, but the reason why often gets lost. Prime factors are important for two reasons:

1. There is only one way to form a number from its prime factors

(not counting the order in which we write the factors). The factorization is unique and completely defines the number. Every composite number has a prime factorization. For example, $60 = 2 \times 2 \times 3 \times 5$. There is no other set of prime numbers that will multiply to 60. The uniqueness of the prime factorization is so important to a large amount of mathematics that it has a special name: "The Fundamental Theorem of Arithmetic."

2. Every factor of a composite number is one of the prime factors, a combination of the prime factors, or the number 1.

No other factors can sneak their way in. So the factors of 60 are as follows:

1	**2**	**3**	**5**
4 $= 2 \times 2$	**6** $= 2 \times 3$	**10** $= 2 \times 5$	**15** $= 3 \times 5$
12 $= 2 \times 2 \times 3$	**20** $= 2 \times 2 \times 5$	**30** $= 2 \times 3 \times 5$	**60** $= 2 \times 2 \times 3 \times 5$

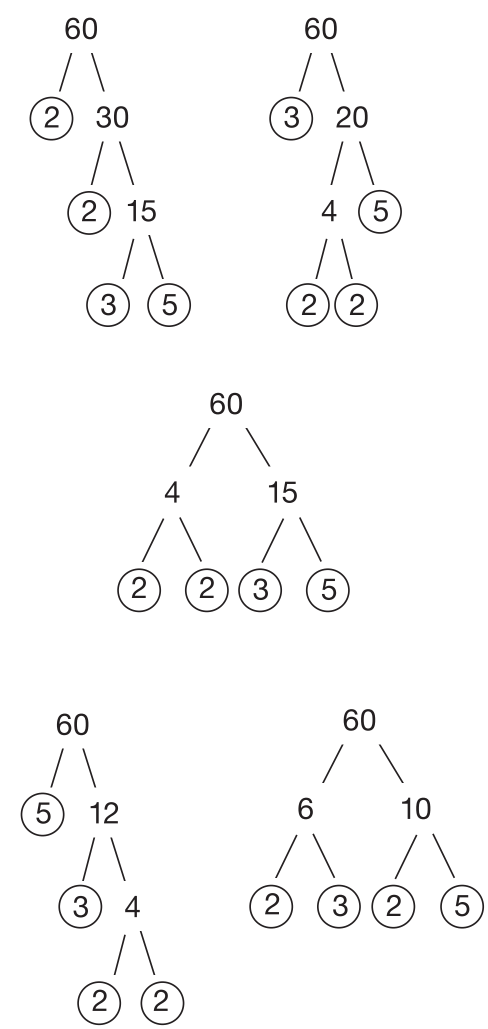
Once we have the prime factors of any composite number, we just make combinations of them to find the composite's non-prime factors—we don't have to look at any other numbers. This can save us a lot of work when performing division or looking for multiples of numbers. It will be quite useful in your child's work with fractions, because it helps us to find common denominators, least common multiples, and greatest common factors. We will talk about these more in chapter 8.

But now, when your child is first learning multiplication tables, prime numbers can be illustrated in an interesting way: by making all the rectangles that can be formed with an area for each of the numbers 1 through 40 (or however high you and your child decide to go). This activity has the added advantage of preparing for division. The rectangles for 15, for example, have sides 1 & 15 and 3 & 5, demonstrating that $15 \div 1 = 15$; $15 \div 15 = 1$; $15 \div 3 = 5$; and $15 \div 5 = 3$. Use the square grid paper at the More4U site to cut out rectangles with areas of 1, 2, 3, 4, and so on. You will need to cut and tape together several strips from a page to make a long rectangle like 1×29.

Notice that while you can make several different rectangles with some areas—for example, 6 is the area of both 1×6 and 2×3 rectangles—there is only one possible rectangle for each of the prime numbers. (There is also only one rectangle with area 1, but one is not a prime.)

Your child will probably be taught a method of finding the prime factorization of a number that is called making a "factor tree." It is made by finding any two numbers that multiply to make the number we are factoring, then factoring those numbers and continuing until all branches of the tree end with prime numbers. It doesn't matter what numbers your child chooses to start the tree: As long as the arithmetic is correct, he will always end up with the same prime factors. At right, you'll see several trees that can be made to find the prime factors of 60. Notice that each tree starts with a different factor pair but ends up with the same list of prime factors. Each factor tree yielded the prime factorization $2 \times 2 \times 3 \times 5$, although some were in a different order.

Factor Trees



No matter how you begin the tree, you will always find the same primes.

You can access the 1–100 chart at this book's More4U page.

more4U

To get a visual sense of the fundamental theorem of arithmetic, and to see that all numbers that are not primes are the products of primes, use the 1–100 chart found in More4U.

Besides the chart, you will also need five transparent or tracing paper sheets and five different-colored markers or pencils. First, print out one copy of the 1–100 grid. Place a transparent sheet over it, clipping it in place to hold it steady. On one transparent sheet, have your child color all the boxes that are multiples of 2. If a pattern is used, rather than solid coloring, the final results will be easier to see. Label that sheet "multiples of 2" and put it aside. Then do the same for the remaining prime numbers that are less than 12: 3, 5, 7, and 11. You might want to print out an extra copy of the chart and color a few yourself.

Before going on, notice the patterns formed by each of the multiples and see if you and your child can figure out why some were easy to color in (down entire columns) and some not. Can you see how each number's relationship to 10 affected the pattern? To see the relationship of the factors, take two of the colored multiple sheets, for example, "multiples of 2" and "multiples of 3," and place them over the 1–100 chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

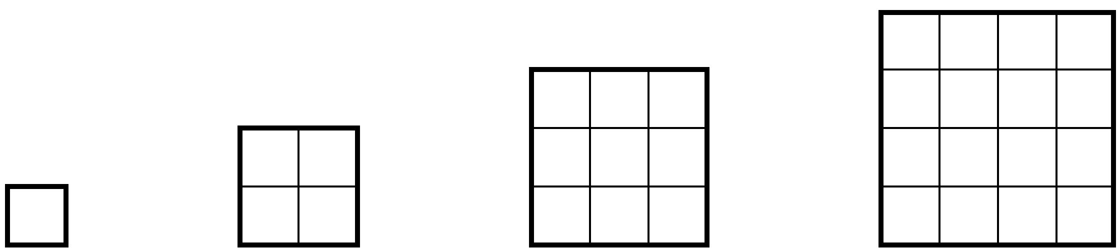
A 1–100 chart with the multiples of 3 colored in

At what numbers do both patterns show? Do you see that every multiple of 6 is also a multiple of 2 and 3? Once 6 is a factor of a number, 2 and 3 are automatically factors too. Make other combinations using pairs of sheets (or even three of them) to see this work for other numbers.

The square numbers

If you made all the possible rectangles for each number from 1 to 40, you saw that for some of the numbers, one of the rectangles is a square. (You may remember from my previous volume in this series—or from your own school days—that squares are special kinds of rectangles.) The numbers that can be used to form squares are called, unsurprisingly, the "square numbers." The first few square numbers are 1, 4, 9, 16, and 25.

Many children find it easy to remember the square numbers and can use them as stepping-stones to other products. For example, a child who knows that $6 \times 6 = 36$ can use that to find 7×6 by adding one more group of 6 to 36. Children who understand multiplication as groups of numbers can use one fact to find another, making the square numbers very useful. We will talk more about this in chapter 3 when we discuss the area and perimeter of rectangles, so save all these rectangles for use then.



$1 \times 1 = \mathbf{1}$ $2 \times 2 = \mathbf{4}$ $3 \times 3 = \mathbf{9}$ $4 \times 4 = \mathbf{16}$

Important ideas from chapter 1

Everyone uses multiplication regularly. Become aware of the situations in which you use it and share them with your child.

When we memorize basic multiplication facts, we are not actually multiplying, but instead are recalling a rote fact.

Help your child master one table at a time, and if necessary, just one fact at a time.

A child can learn some facts and then use those to find others. For example: "$6 \times 6 = 36$, so 7×6 is one more 6, or 6 more than 36, so it's 42!" This process demonstrates an understanding of the meaning of multiplication and is an important step in learning the tables.

Knowing the multiplication facts is an important step toward understanding many other parts of math. Take whatever time is necessary to help your child to memorize them completely. They will become the tool for understanding nearly all the rest of third- through fifth-grade math.