# Introduction

## A Brief History of the Common Core

Mathematics standards are not new. In fact, they have been around for more than 25 years. In 1989, the National Council of Teachers of Mathematics introduced *The Curriculum and Evaluation Standards for School Mathematics*, followed by the updated set of standards, *Principles and Standards for School Mathematics*, in 2000. Both of these documents provided a vision for K–12 mathematics and were the foundation for most of the states' grade-level standards.

In 2008, the National Governors Association and the Council of Chief State Schools Officers met to discuss the Common Core State Standards Initiative with the purpose of developing a set of common standards across states with the focus of balancing the quality of mathematics instruction and learning. Following that meeting, the process of writing the Common Core Standards began, led by William McCallum, Phil Daro, and Jason Zimba with a writing team including mathematicians, mathematics educators, mathematics education researchers, and classroom teachers. The process provided an open invitation for feedback from the National Council of Teachers of Mathematics, state mathematics associations, mathematics educators, and the general public. This feedback was considered and much of it was incorporated into the final document released in June 2010. Following the release of the standards, individual states reviewed and adopted the Common Core State Standards for Mathematics.

## The Common Core State Standards for Mathematics

"The Common Core State Standards are a clear set of shared goals and expectations for the knowledge and skills students need in English language and mathematics at each grade level so they can be prepared to succeed in college, career, and life" (www.corestandards.org/about-the-standards/frequently-asked questions/#faq-2303).

The Common Core State Standards for Mathematics (CCSSM) include two critical components for learning mathematics: (1) the Content Standards and (2) the Standards for Mathematical Practice. The first component, the Content Standards, explicitly outlines the mathematics students should know and be able to do at each grade level. The Content Standards of the Common Core are fewer in number than most previous state standards. At the same time, the expectation is that students will develop *deeper* understanding of the content, so less time is spent on reteaching from year to year. Additionally, the standards were carefully constructed to show connections among ideas at a grade level as well as vertical progressions across grades. For example, the standards in Grade 6 develop from the mathematical work students completed in previous grades. It is important for teachers to be knowledgeable of the standards not only at the level they are teaching but also at the preceding and following grade levels.

The second component of the CCSSM are the Standards for Mathematical Practice. These standards describe the habits of mind that students should develop as they do mathematics. These eight standards are the same across all grades levels, K–12. When planning lessons, teachers should consider how students will use the practices in learning and doing mathematics.

The Common Core Standards are *not* a curriculum. Decisions about mathematics programs, textbooks, materials, sequencing topics and units, and instructional frameworks are decisions to be made by local school districts. The standards *do not* tell teachers how to teach but rather describe what students need to know and be able to do. Schools and teachers know best how to help students reach both the Content and the Practice Standards.

The Common Core Standards *do not* dictate specific assessments. Many states will be using assessments developed by the PARCC (Partnership for Assessment of Readiness for College and Careers) or SBAC (Smarter Balanced Assessment Consortium). Some states will develop and use their own assessments. For additional information, check out http://www.corestandards.org.

## Instructional Shifts

The standards do not call for a particular model or philosophy. They are based on the best of existing standards. What makes them different is the call for specific instructional shifts: *focus, coherence, and rigor.*

**Focus:** The Content Standards call for greater focus on fewer topics. An examination of the mathematics standards of high-performing countries indicate that fewer, more focused topics are taught at a grade level to allow students to deepen understanding of the mathematics and gain a stronger foundation for ongoing study of mathematics. Within the standards, the major mathematical work of

each grade level has been identified (www.corestandards.org). That means that not all of the content within a grade is emphasized equally among the Content Standards. The list of Content Standards for a grade is not linear or a checklist. Some clusters require greater emphasis than others. Some clusters may need more time for students to master with depth of understanding. The major work of Grades 6–8 includes ratios and proportional relationships, expressions and equations, arithmetic of rational numbers, and linear algebra and linear functions. This means that the majority of instructional time in Grades 6–8 (65%–85%) must be spent on these mathematical topics. This does not mean that other standards should be skipped. Rather, the supporting standards should be taught to connect mathematical ideas among the essential standards. The additional standards provide students with experiences that will be foundational to work in future grades. Neglecting material will leave gaps in student skill and understanding.

**Coherence:** Mathematics must be thought of as a coherent body of knowledge made up of topics that are all connected and build on each other. The call for coherence in the Content Standards ensures that there are carefully constructed progressions from grade to grade so students build new understanding on the foundations built in previous years. Each standard is not a new topic but an extension of previous learning. In addition to the progressions across grade levels, the standards incorporate specific connections within a grade level.

**Rigor:** The third instructional shift, *rigor*, refers to how we support students in developing deep understanding of each standard. Understanding does not develop by assigning more worksheets or with more difficult examples and problems. Rather, it calls for instructional practices that balance conceptual understanding, procedural skills, and applying mathematical ideas to a variety of contexts.

The following descriptions of each component of rigor are from www.corestandards.org.

> *Conceptual understanding:* The standards call for conceptual understanding of key Grade 6 concepts such as ratios and proportional relationships and early expressions and equations. In Grade 7, the major concepts are ratios and proportional relationships and arithmetic of rational numbers. Grade 8 calls for linear algebra and linear functions. Students must be able to access concepts from a number of perspectives in order to see mathematics as more than a set of rules or procedures.
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> *Procedural skills and fluency:* The standards call for speed and accuracy in calculation. Students must practice core skills such as basic multidigit division and multidigit decimal operations in order to have access to more complex concepts and procedures. Fluency is built upon conceptual understanding, and with students, through the development of ideas using representations, models, and symbols.
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> *Applications:* The standards call for students to use mathematics in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

## Major Work of Grades 6–8

At least 65% to 85% of instructional time should focus on the major work for each grade level. Areas of major work include the following:

**Grade 6:** Understand ratio concepts and use ratio reasoning to solve problems. Apply and extend previous understandings of multiplication and division to divide fractions by fractions. Apply and extend previous understandings of numbers to the system of rational numbers. Apply and extend previous understandings of arithmetic to algebraic expressions. Reason about and solve one-variable equations and inequalities. Represent and analyze quantitative relationships between dependent and independent variables.

**Grade 7:** Analyze proportional relationships and use them to solve real-world and mathematical problems. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. Use properties of operations to generate equivalent expressions. Solve real-life and mathematical problems using numerical and algebraic expressions.

**Grade 8:** Work with radicals and integer exponents. Understand the connections between proportional relationships, lines, and linear equations. Analyze and solve linear equations and pairs of simultaneous linear equations. Define, evaluate, and compare functions. Use functions to model relationships between quantities. Understand congruence and similarity using physical models, transparencies, or geometric software.

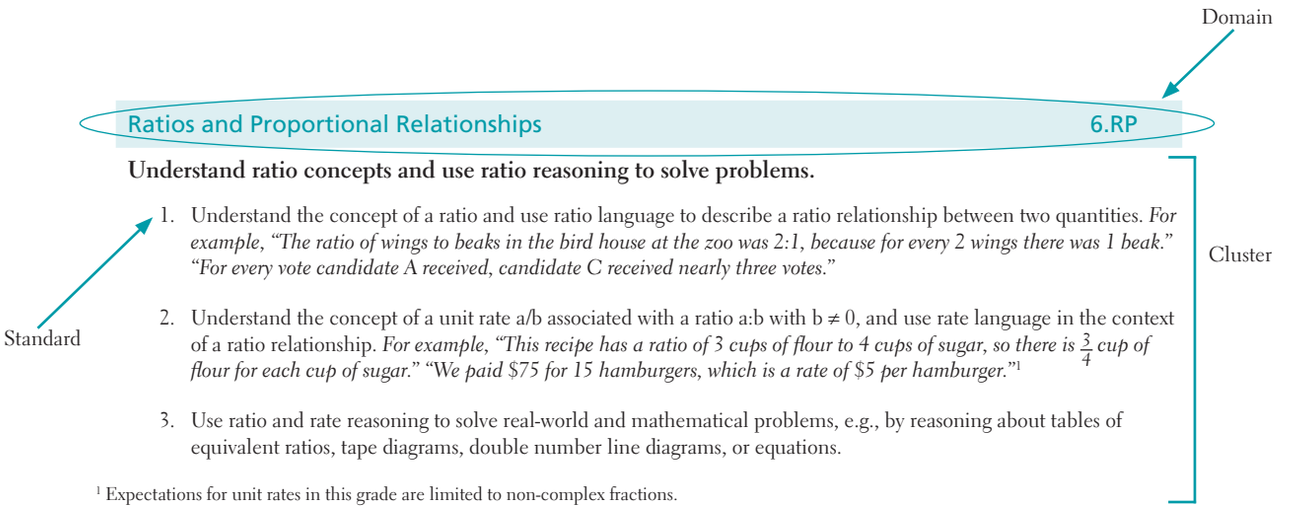
## Common Core Word Wall

The language of the Common Core differs from traditional standards. Familiarity with the terms *standards*, *clusters*, and *domains* is critical.

**Standards** define what students should understand and be able to do.

**Clusters** summarize groups of related standards. Be aware that standards from different clusters may sometimes be closely related as mathematics is a connected subject.

**Domains** are larger groups of related standards. Be aware that standards from different domains may sometimes be closely related.




*Source:* Common Core State Standards for Mathematics (www.corestandards.org).

## The Common Core Standards for Mathematical Practice

The Common Core Standards for Mathematical Practice describe eight ideas teachers must incorporate in classroom instruction to develop a depth of understanding of critical mathematical concepts in their students. The Mathematical Practices are not intended to be taught in isolation but should be integrated into daily lessons. Some of the lessons may focus on developing one or two of these Standards, and others may incorporate all eight Standards. The eight Standards are not explicitly taught but exemplify the type of mathematical thinking and doing students should practice as they develop mathematical understanding.

Throughout the following chapters, examples of the Mathematical Practices intended to be used are included in each cluster. The listed Practices are not meant to limit lessons by using only those Practices but are examples of key practices that can be included in lessons around that particular cluster. It is likely that teachers will use all of the Practices throughout the cluster and domain.

The eight Standards for Mathematical Practice (SFMP), briefly explained below, are essential for student success. If students are actively engaged in using these eight Practices, they are learning rigorous, meaningful mathematics.

## SFMP 1. Make sense of problems and persevere in solving them.

Grades 6–8 students should work to understand what a problem is asking, choose a strategy to find a solution, and check the answer to make sure it makes sense.

## SFMP 2. Reason abstractly and quantitatively.

Grades 6–8 students must make sense of quantities and their relationships in problem situations. At this level, students can model problem solutions.

## SFMP 3. Construct viable arguments and critique the reasoning of others.

Grades 6–8 students often use stated assumptions, definitions, and established results in constructing arguments. At this level, students must justify and communicate their conclusions as well as listen to other students' explanations.

## SFMP 4. Model with mathematics.

Grades 6–8 students should use various representations, models, and symbols to connect conceptual understanding to skills and application. Students should use the mathematics they know to solve problems in everyday life, society, and the workplace. At this level, students may write an equation or may connect representations and explain the connections.

## SFMP 5. Use appropriate tools strategically.

Grades 6–8 students consider the available tools when solving a mathematics problem. At times, students may choose to solve problems with mental calculations, with paper and pencil, or with other technology.

## SFMP 6. Attend to precision.

Grades 6–8 students communicate precisely with others. At this level, students explain their knowledge of mathematical symbols that explicitly connect to using the correct mathematical vocabulary.

## SFMP 7. Look for and make use of structure.

Grades 6–8 students look closely to discern patterns and structure.

## SFMP 8. Look for and express regularity in repeated reasoning.

Grades 6–8 students notice if calculations are repeated and make generalizations. At this level, students discover shortcuts through making the generalizations and understanding why they work.

## Effective Teaching Practices

Quality mathematics teaching is a critical key for student success. In *Principles to Actions* (2014), the National Council of Teachers of Mathematics outline eight valuable practices every teacher *must* incorporate to guarantee student achievement. These eight research-informed practices, briefly explained below, provide a foundation for effective common core mathematics teaching and learning.

### 1. Establish mathematics goals to focus learning.

Establishing learning goals sets the stage and helps guide instructional decisions. Teachers must keep in mind what is to be learned, why the goal is important, where students need to go (the trajectory), and how learning can be extended. Students must clearly understand the purpose of the lesson.

### 2. Implement tasks that promote reasoning and problem solving.

Implementing tasks that promote reasoning and problem solving provides opportunities for students to engage in exploration and encourages students to use procedures in ways that are connected to concepts and understanding. The tasks teachers choose should be built on current student understandings and have various entry points with multiple ways for the problems to be solved.

### 3. Use and connect mathematical representations.

Using and connecting representations leads students to deeper understanding. Different representations should be introduced, discussed, and connected to support students' abilities to justify and explain their thinking and reasoning.

### 4. Facilitate meaningful mathematical discourse.

Facilitating meaningful discourse provides students with opportunities to share ideas, clarify understandings, and develop convincing arguments. Talking and sharing aloud can advance the mathematical thinking of the whole class.

### 5. Pose purposeful questions.

Posing purposeful questions reveals students' current understanding of a concept and encourages students to explain, elaborate, and clarify thinking. Asking good questions makes the learning of mathematics more visible and accessible for student examination.

### 6. Build procedural fluency from conceptual understanding.

Building procedural fluency from conceptual understanding allows students to flexibly choose from a variety of methods to solve contextual and mathematical problems.

### 7. Support productive struggle in learning mathematics.

Supporting productive struggle in learning mathematics is significant and essential to learning mathematics with understanding. Productive struggle allows students to grapple with ideas and relationships.

### 8. Elicit and use evidence of student thinking.

Eliciting and using evidence of student thinking helps teachers access learning progress and can be used to make instructional decisions during the lessons as well as help to prepare what will occur in the next lesson.

## How to Use This Book

The overall goal of this book is to help teachers more deeply understand the mathematical meaning of clusters and standards within the domains of Grades 6–8. The intent of the resource is to use it as your personal toolkit for teaching the mathematics standards. Blank space on numerous pages has been left for you to take notes, add ideas, and reference other resources that may be helpful.

Each part of the book explains one domain and begins with an overview of how the domain progresses across Grades 6 through 8. A list of helpful materials, reproducibles, and key vocabulary from the domain is included in the overview as well.

Every domain is tracked across sixth, seventh, and eighth grades with a page for each cluster and the standards within the cluster. A description of the cluster and how the Standards for Mathematical Practice can be incorporated into your teaching of the cluster follows. Because the standards are intentionally designed to connect within and across domains and grade levels, a list of related standards is included in the cluster overview. As you prepare work on a cluster, we suggest you look at these standards to have a better idea of the mathematics students learned in previous grades and where they are going in the future. A list of all of the Standards is found in the Quick Reference Guide at the beginning of the book.

Each standard within a cluster is explained with a section called *What the TEACHER does* followed by a description of *What the STUDENTS do*. It is important to note that most standards will take several days, and you should be connecting conceptual understandings across standards and domains as you teach for understanding.

Addressing student misconceptions and common errors in developing student understanding of a concept concludes the contents for each standard.

At the end of each grade-level domain, you will find a sample planning page based on one standard for that domain. Also included are planning page templates for each cluster within the domain for you to duplicate and use for planning.

In the resource section, you will find reproducible key materials. These are designed to be samples, and we encourage you to use them or redesign them to best meet the needs of your students. A list of our favorite resource books and high-quality online resources that are particularly useful to developing mathematical ideas in Grades 6–8 is also included.

We believe that this can become your common core bible. Read it and mark it with questions, comments, and ideas. We hope that this resource will help you use these standards and good teaching practices to lay the essential foundation that will ensure your students success not only in your grade but in all of their future study of mathematics.

## Reflection Questions

**1.** How are the three instructional shifts called for by the Common Core similar to your current instructional practice?

- What is conceptual understanding?
- How is it different from procedural skills?
- What do you need to consider to teach for conceptual understanding?
- How can you connect conceptual understanding to help students develop procedural skills?

**2.** The Standards for Mathematical Practice describe the habits of mind that students need for thinking about and doing mathematics. While not every practice will be in every lesson, select one standard at your grade level and consider some ways you can incorporate these practices in a lesson for that standard.

- How will these practices provide you with information about student understanding?
- How will this help you to better assess students?
- How will this information help you in planning lessons?

**3.** The Effective Teaching Practices describe specific actions that teachers must consider in planning and implementing lessons and assessing student performance.

- How are the practices connected? Work with colleagues to plan a lesson that employs all of these practices.
- How can you modify a traditional task so that it promotes reasoning and problem solving?
- What representations will help students more deeply understand the concept?
- How will you connect conceptual understanding to build procedural fluency?
- What kinds of information will you look for to help inform your instruction?

(For more information on the Effective Teaching Practices, go to www.nctm.org.)