Thinking about Triangles

Grades 3–4

Goals

- Use geoboards to investigate properties of triangles
- Transcribe geoboard designs onto geodot paper
- Make and test conjectures about properties of triangles
- Draw conclusions on the basis of experimentation with a variety of examples

Prior Knowledge

Students should know the definition of *polygon* (see "Important Geometric Terms") and be able to recognize and draw triangles, to recognize right angles, and to compare the sizes of angles (e.g., discover that one is greater than another). If students are using geoboards for the first time, a period of free exploration should be allowed so they can become familiar with this tool.

Materials and Equipment

- A five-pin-by-five-pin geoboard with one rubber band for each student
- A copy of the blackline master "Geodot Paper for Geoboards" for each student
- Paper, pencils, and crayons for recording and writing journal reflections

Learning Environment

For this activity, students could work in small groups to encourage mathematical discourse while they are making their shapes and testing their conjectures. Depending on the group of students and the question you are exploring, however, you may wish to have the students work alone at first to make a conjecture and begin testing it. After they have done some exploration, they can come together to discuss their findings in groups of two to four and then draw and justify conclusions together. In any event, it is best if the students have their own geoboards to be able to construct their own ideas about the properties of shapes.

Important Geometric Terms

Right angle, *acute angle*, *obtuse angle*, *equilateral triangle*, *isosceles triangle*, *scalene triangle*

Polygon: A closed plane figure formed by three or more line segments called *sides*. Each side intersects exactly two other sides, but only at

This activity has been adapted from Walker, Reak, and Stewart (1995a), *Twenty Thinking Questions for Geoboards.*

Identify, compare, and analyze attributes of [two-dimensional] … shapes and develop vocabulary to describe the attributes

Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions

Geoboards allow students to explore their conjectures quickly and easily; however, "Geodot Paper for Geoboards" can be used as an alternative if necessary. Students can also use the Geoboard applet on the CD-ROM.

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their end points. Sides that have common end points are not part of the same line.

Activity

Engage

Ask the students to search the classroom to find triangular objects and list them. Ask such questions as the following:

- What do these shapes have in common?
- Where do you think the word *triangle* comes from?
- What other words do you know that start with *tri?* (tricycle, tripod)
- How are they similar to the word *triangle?* How are they different?

Explore

The explorations are a series of questions for students to conjecture about, explore using their geoboards, and discuss in small groups before finally coming to a class consensus through guided discourse. Each question is expected to take a full one-hour class period. Encourage students' discussion in their groups and pose questions rather than give answers when observing the groups in action. For example, rather than tell a group that their figure is not a polygon, ask them why they think it is a polygon and then together discuss the definition of a polygon. You might even show them examples of polygons and examples of nonpolygons and ask them in which category their figure belongs.

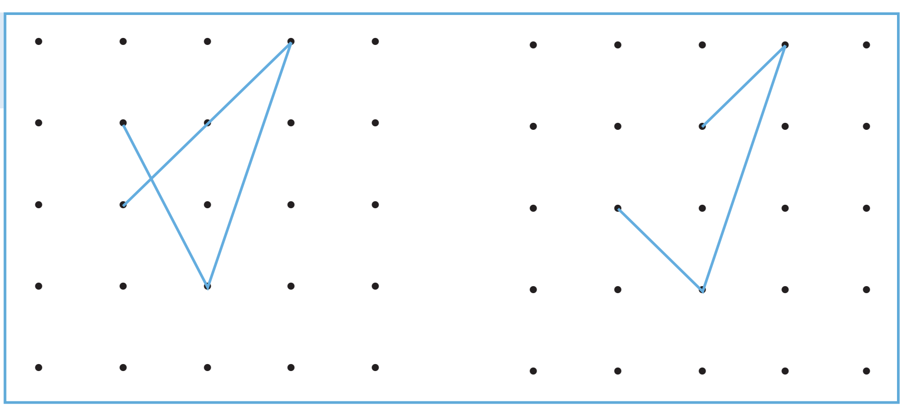
Question 1: Is it possible to make a three-sided polygon that is not a triangle?

The students first make a conjecture (yes or no) and record and tally all the students' conjectures. They then use their geoboards to explore possibilities and come to a consensus in their groups. Groups of up to four work well for this task.

If the students construct shapes like those in figure 1.2, refer them to the definition of a polygon and ask, "Is your figure a closed figure with no lines crossing?" Conversely, if they make a triangle and do not identify it as such, refer them to the origin of the word triangle (three angles). Ask, "Does your shape have three angles?"

As a closing discussion, make a class list of all the properties of triangles. The students should mention three angles and three sides. They

Fig. **1.2.**
Examples of nonpolygons



may also state that a triangle is a polygon. Alternatively, have the students write a journal reflection for assessment.

Question 2: Is it possible for a triangle to have two right angles?

Again the students make individual conjectures and tally the class's responses. They then test their conjectures on the geoboard and come to a consensus in small groups. Follow up with a whole-class discussion. If the students construct a figure that has two right angles but is not a triangle (see fig. 1.3), ask them if their figure has all the properties of a triangle.

To test for right angles, the students can use the corner of an index card or take any rectangular piece of paper and fold it in half and then in half again. The meeting of the two folds forms a right angle.

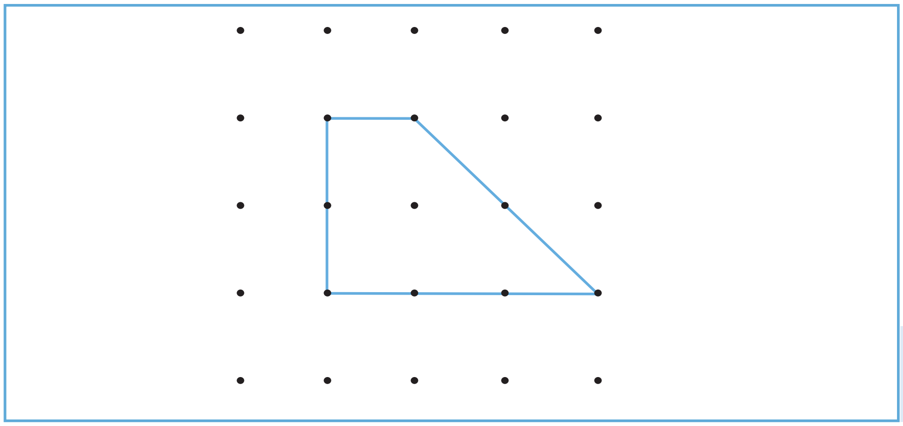


Fig. **1.3.**
An example of a nontriangle

Question 3: How many different right triangles can be made on the geoboard?

Have the students make and tally conjectures, as before. In introducing this question, discuss what *different* means. For this task, if a triangle can be flipped or turned to match one that has already been made, then it is not "different."

This is an intuitive introduction to the concept of congruence (having the same size and shape). To test for congruence, the students may need to cut out triangles or use two transparencies to overlay triangles.

The students should record all the triangles on geodot paper and explain how they know they are right triangles. If the students find only a few right triangles, encourage them to search for more. Ask them, "Are you sure you found all the right triangles? What strategy did you use to make sure you have found them all?"

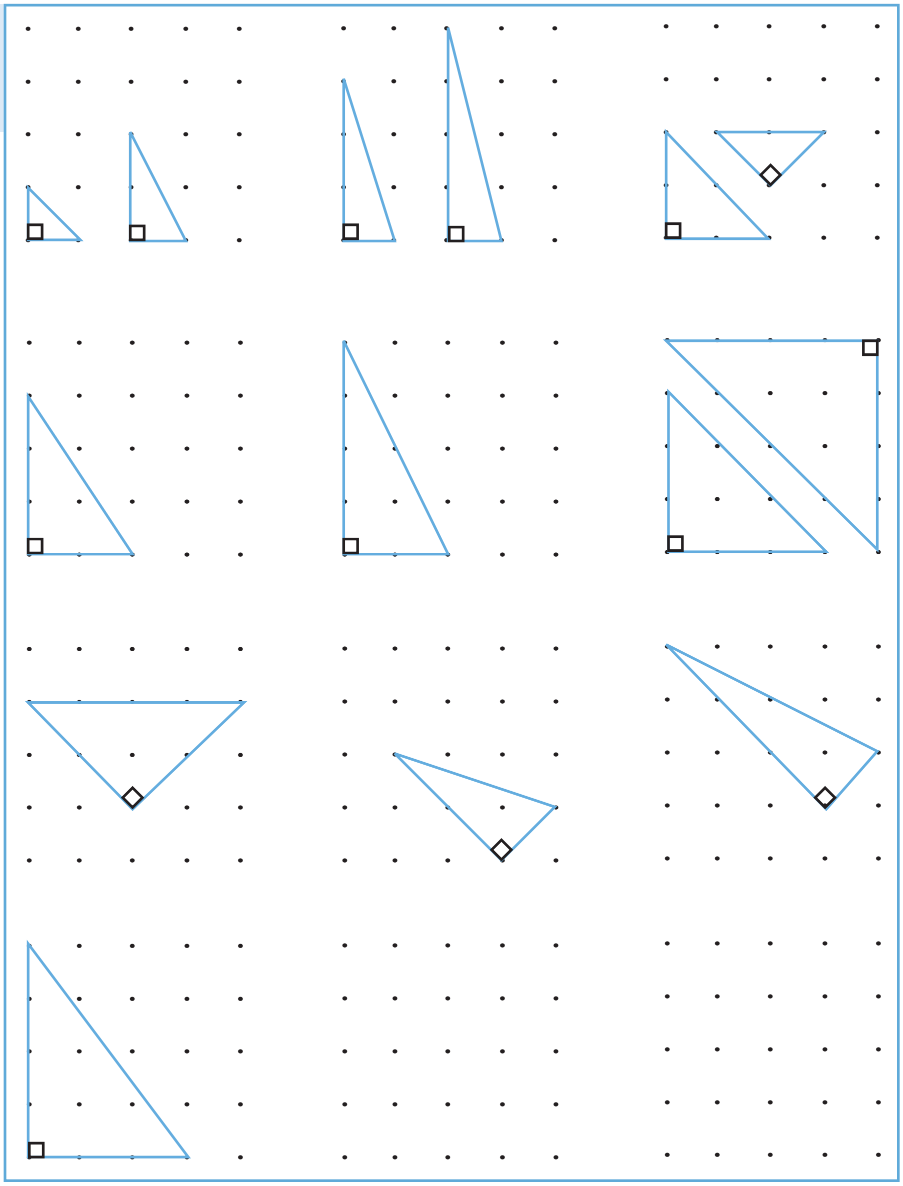
Teachers are advised to attempt this task before assigning it to their students so that they can devise a strategy and experience the discovery and thinking processes that their students will use. The fourteen right triangles that can be constructed on the five-pin-by-five-pin geoboard are shown in figure 1.4. If the students are having difficulty coming up with a strategy to find them all, model your approach. For example, "I started with a right triangle that had a base of one unit and a height of one unit. I kept the base the same and tried to find how many more right triangles I could make. Then I made one with a base of two units.…"

Question 4: How many different types of triangles can you find?

Have the students make and tally conjectures, as before. Thus far, this activity has focused on right triangles. Model examples of one or two other types (e.g., an isosceles triangle or a scalene triangle). Encourage the students to think about the sizes of the angles and the

Fig. **1.4.**

The fourteen right triangles that can be made on a five-pin-by-five-pin geoboard

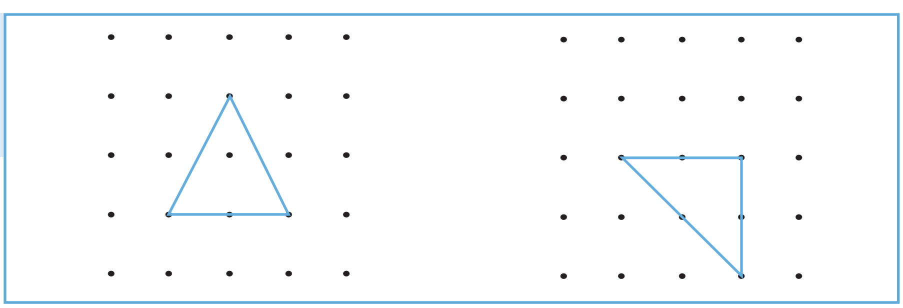


lengths of the sides of other triangles they can create. Have them record their triangles on geodot paper.

It is not possible to make an equilateral triangle on the geoboard. Some students may claim that the triangles in figure 1.5 are equilateral because the lengths of the sides look equal or because the band on the triangle on the right connects two pegs with one peg in between. Encourage them to measure the sides to prove that the triangles are not equilateral. Discuss the properties of equilateral triangles, and give examples.

Fig. **1.5.**

Examples of students' unsuccessful attempts to construct equilateral triangles on the geoboard



In their groups, the students can share the triangles they recorded on geodot paper, put into piles those that are the same, and then label each pile with a defining characteristic. Restrictions can be placed on the number of piles (e.g., there must be at least three piles) and the number of triangles within a pile (each pile must have at least three triangles in it). The students could create posters with triangles displayed by categories and make presentations to the class explaining the strategies they used to determine the classifications. They should be prepared to justify the labels and defend the placement of the triangles. Follow the presentations with a class discussion that introduces the terms *acute*, *obtuse*, *scalene*, and *isosceles.* Do not introduce these terms until after the students have made their presentations. It is very important that the students think up their own categories to describe the triangles. The actual geometric terms will evolve naturally from their classifications.

Assess

During each of the explorations, the teacher should circulate among the groups, observing and attending to students' conjectures and the reasoning behind them, students' discussions of congruence and of geometric terms, and their classifications for different triangles.

This activity lends itself nicely to written journal reflections from which teachers can determine what individual students have learned. The following can be used as prompts for reflections about each of the questions explored:

- Question 1: What have you learned about a triangle from this investigation?
- Question 2: If you could make a triangle that was as large as you wanted, would you be able to make one that had two right angles? Explain your thinking. (Examples of students' responses are shown in fig. 1.6.)
- Question 3: Write everything you know that is true about all right triangles.
- Question 4: Write in your own words the definitions for the new geometric terms we have found (*isosceles*, *scalene*, *acute*, and *obtuse*).
- Summary question: Finish each sentence with as many different answers as possible:

  All triangles have …

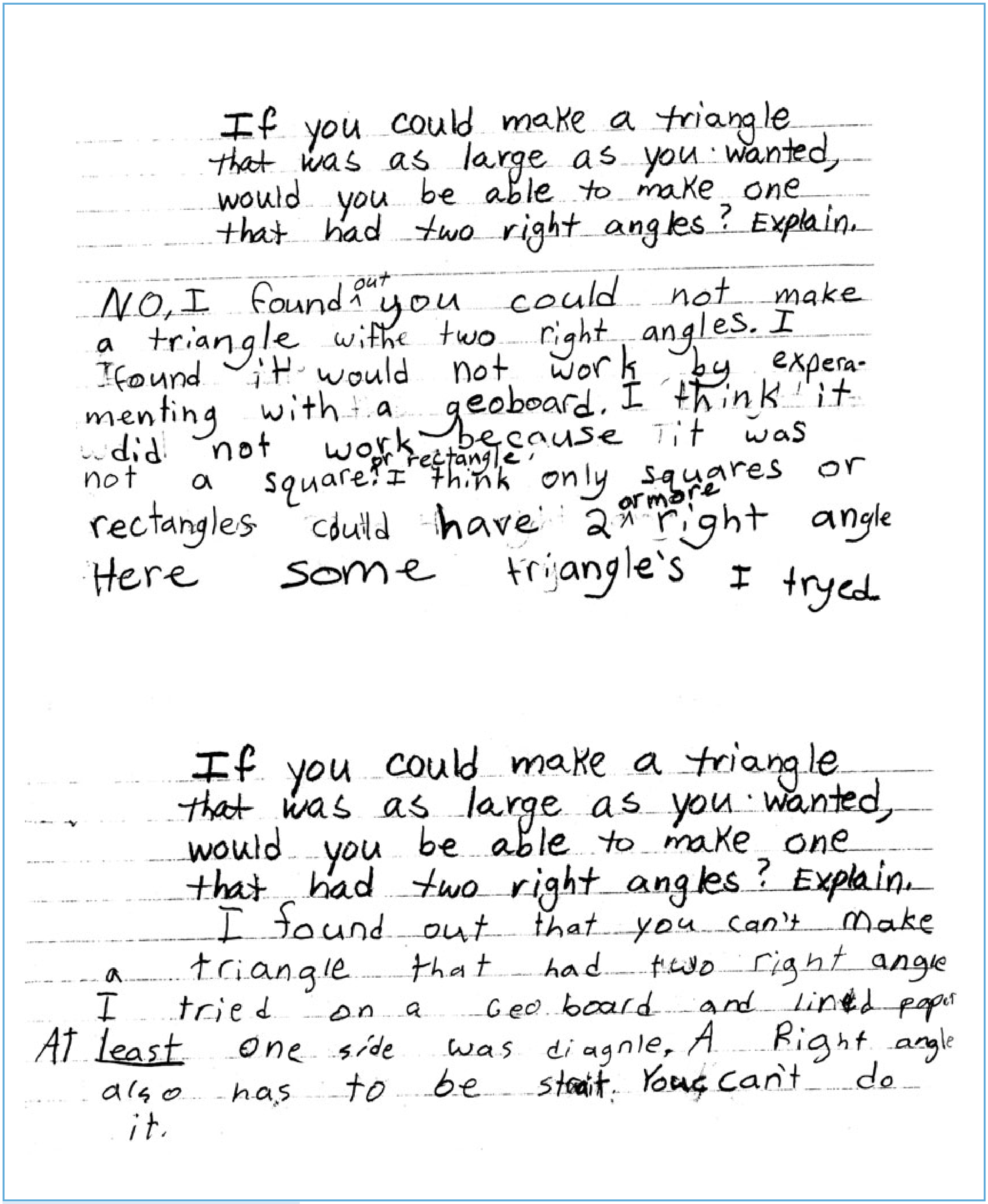
  Some triangles have …

Extend

Using straws of different lengths or The Geometer's Sketchpad (Jackiw 1991), students can explore the following questions:

- Can a triangle be made with segments measuring five, six, and seven units? Can more than one triangle be made? Why or why not?
- If you are given any three lengths, can you always make a triangle? Why or why not?
- Using several different sets of three lengths, try to make triangles. Can you make up a rule about the lengths of the sides of triangles?

In answering these questions, the students derive the theorem that the sum of the lengths of any two sides of a triangle must always be greater



If you could make a triangle that was as large as you wanted, would you be able to make one that had two right angles? Explain.

NO, I found out you could not make a triangle withe two right angles. I Ifound it would not work by experamenting with a geoboard. I think it did not work because it was not a square or rectangle, I think only squares or rectangles could have 2 or more right angle Here some triangle's I tryed

If you could make a triangle that was as large as you wanted, would you be able to make one that had two right angles? Explain.

I found out that you can't make a triangle that had two right angle I tried on a Geo board and lined paper At least one side was diagnle, A Right angle also has to be stait. You can't do it.

Fig. **1.6.**

Students' reflections on their attempts to create a triangle with two right angles than the length of the third side. Can a triangle be made with three sides of any length? A triangle can be made with sides of lengths five, six, and seven units, since $5 + 6 > 7$, $6 + 7 > 5$, and $5 + 7 > 6$. However, a triangle cannot be constructed with sides of lengths one, two, and three units, since $1 + 2$ is not greater than 3. The students are learning the importance of a counterexample to disprove an idea here. If they find one example that does not work, then the conjecture is invalid. Note that this investigation can also take place by putting together geostrips to test conjectures instead of using the computer.

Similar questions can be used to explore squares and other rectangles:

- How many different squares can you make on your geoboard?
- How many different rectangles can you make on your geoboard?
- How do you know that you have found them all?
- How do you know they are all different?
- How are a rectangle and square alike? How are they different?

In grades 4 and 5, students can proceed to investigate parallelograms and come up with the definition of a rectangle. To help them examine their definition, ask the following questions:

- Could a square be considered a special type of rectangle? In what way?
- Could a square be considered a special type of parallelogram?

A Venn diagram can then be developed to show the relationship among the three types of quadrilaterals (see fig. 1.7).

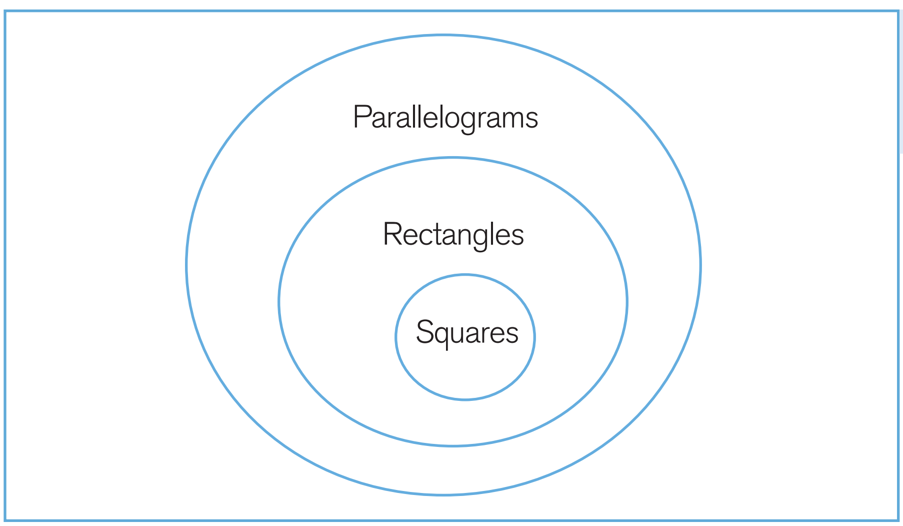


Fig. **1.7.**

A Venn diagram showing the relationship among three types of quadrilaterals