

Planning for Student Learning of Mathematics in Grades 6–8

Mathematics is a conceptual domain. It is not, as many people think, a list of facts and methods to be remembered.

—Jo Boaler

The first critical question of a PLC is, What do we expect all students to know and be able to do? (DuFour et al., 2016). As your collaborative team successfully answers this question for each unit of study, members build a common understanding of the mathematics students learn at your grade level. What is the mathematics story that unfolds as student learning progresses from one mathematics unit to the next? How do the units fit together and build on one another within and across sixth, seventh, and eighth grades?

Guaranteed and Viable Curriculum

Your sixth-, seventh-, or eighth-grade team effectively backward plans the year by grouping essential

mathematics standards into units to create the guaranteed and viable mathematics curriculum students must learn. The order in which you teach the units provides the framework for your grade-level mathematics story. Within each unit, your daily lessons create the beginning, middle, and end for that part of the story.

Thus, evidence of your team’s guaranteed and viable curriculum includes (1) semester- and yearlong pacing plans (proficiency maps or pacing guides), (2) unit plans, and (3) daily lessons. The graphic in figure 1.1 illustrates these three areas of team planning for a mathematics guaranteed and viable curriculum. The thick black line down the middle shows the end of semester 1 and the start of semester 2 for your team.

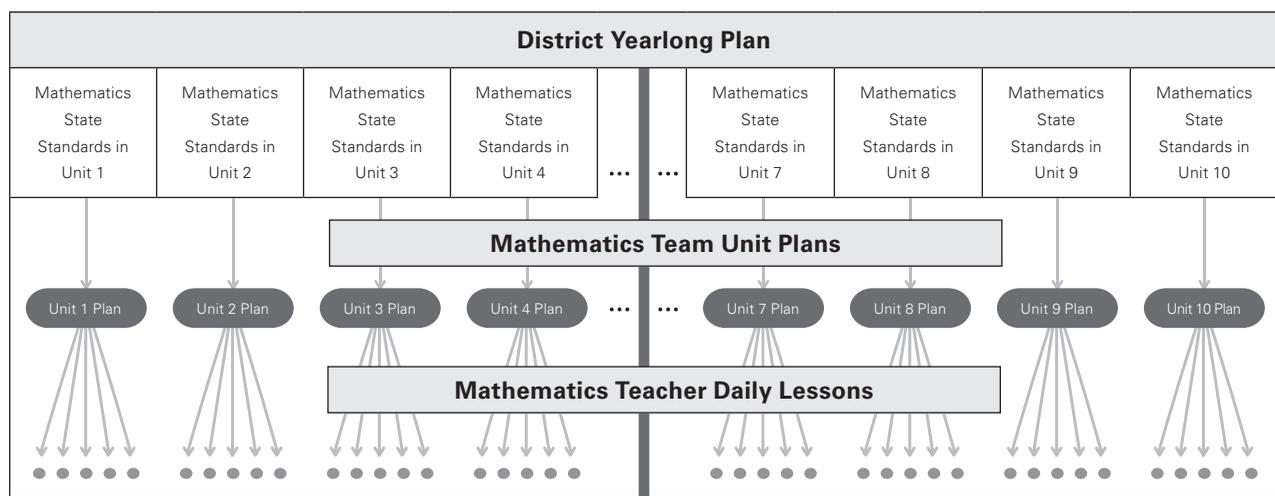


Figure 1.1: Mathematics guaranteed and viable curriculum plan.

Together, the mathematics units of study tell the story of the grade-level standards teachers expect students to learn throughout the year and from one year to the next.

As figure 1.1 (page 9) shows, a district yearlong pacing guide or proficiency map (showing a time line for student proficiency with each mathematics standard) first defines your grade-level team’s guaranteed and viable curriculum. Your team then determines a time frame appropriate for each mathematics unit of study, typically two to four weeks at the middle school level. This process eliminates the potential risk of running out of time and skipping units on essential standards that fall at the end of the year.

If your collaborative team does not have a year-long plan with standards in clearly defined units, see appendix A (page 109), “Create a Proficiency Map,” for additional support. Helping each teacher on your team become comfortable with the progression of mathematics units throughout the school year will support your students’ understanding of the mathematics story arc for various standards.

Mathematics Unit Planner

Once your team determines the mathematics units for your grade level (detailing the standards and time line for each unit) for the year, the team next plans for student learning on a *unit-by-unit* basis (see figure 1.2; Kanold & Schuhl, 2020).

The Mathematics Unit Planner in figure 1.2 provides a template your team can use as it develops a shared understanding of what students are expected to learn in each unit of study. The numbered sections in the Mathematics Unit Planner represent the seven elements of unit planning. Throughout this book, you will see numbered headings that correspond with these seven areas. (Find completed examples of unit planners for a sixth-grade ratios and rates unit in figure 3.11 [pages 56–57], a seventh-grade proportional reasoning unit in figure 4.10 [pages 78–79], and an eighth-grade linear functions and equations unit in figure 5.10 [pages 103–104].)

Once the elements of the Mathematics Unit Planner (figure 1.2) are complete, your team can use the information to plan common assessments and design daily lessons (see *Mathematics Assessment and Intervention in a PLC at Work* [Kanold, Schuhl, Larson, Barnes, Kanold-McIntyre, & Toncheff, 2018] and *Mathematics*

Instruction and Tasks in a PLC at Work [Kanold, Kanold-McIntyre, Larson, Barnes, Schuhl, & Toncheff, 2018]). Additionally, you and your collaborative team can reference the planner for each successive unit in the year and from one year to the next as your team continues to deepen its understanding of the required student learning.

In *Principles to Actions*, researchers for the National Council of Teachers of Mathematics (NCTM, 2014) note, “Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions” (p. 12). Therefore, as your mathematics department and grade-level teams develop unit plans for the year, be sure to make sense of the mathematical content trajectories (progressions) students are learning across the grades 6–8 band.

Mathematics Concepts and Skills for Grades 6–8

Students in grades 6–8 deepen their understanding of number, place value, fractions, operations, and geometry and measurement learned in grades preK–5. Throughout grades 6–8, teachers expect students to grow their mathematical understanding of number, to include positive and negative rational numbers, and to learn how to use numbers to make sense of variables in expressions and equations. Students also extend their conceptual knowledge and skills related to fractions to develop an understanding of ratios and proportional reasoning. In eighth grade, students use various representations to grow their understanding of linear functions and equations and systems of equations. In geometry, students more critically reason about shapes in two and three dimensions as an extension of work done in grades 3–5. Teachers also introduce the concepts of statistical reasoning and probability.

Table 1.1 (page 12) shows some of the key mathematics concepts teachers expect students to learn in grades 6–8, both by grade and as a vertical trajectory NCTM’s (2006) *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics* first defines.

Sixth graders complete number operations using algorithms they started learning in fourth and fifth grades, with a focus on division in sixth. Students in sixth and seventh grades apply operations to multidigit

Unit: _____					
Start Date: _____	End Date: _____ Total Number of Days: _____				
Unit Planning					
1 Essential Learning Standards	List the essential learning standards for this unit.				
3 Prior Knowledge	List standards from a previous unit or grade students will access in this unit.				
4 Vocabulary and Notations	List the mathematical academic vocabulary and notations for this unit.				
5 Possible Resources or Activities	List the possible resources or activities to use when teaching the essential learning standards.				
6 Tools and Technology	List the essential tools, manipulatives, and technology needed for this unit.				
7 Reflection and Notes	After the unit, reflect and list what to do again, revise, or change.				
Unit Calendar					
	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1					
Week 2					
Week 3					
Week 4					
Week 5					

Source: Adapted from Kanold & Schuhl, 2020, p. 30.

Figure 1.2: Mathematics Unit Planner.

Visit go.SolutionTree.com/MathematicsatWork for a free reproducible version of this figure.

Table 1.1: Key Mathematics Concepts and Skills for Grades 6–8

	Grade 6	Grade 7	Grade 8
Number Systems, Expressions, and Equations	<ul style="list-style-type: none"> • Understand and develop fluency with division of whole numbers and fractions. • Fluently compute with multidigit numbers. • Understand the system of rational numbers, including positive and negative numbers. • Understand the absolute value of rational numbers. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent and independent variables. 	<ul style="list-style-type: none"> • Understand fractions and demonstrate the ability to add, subtract, multiply, and divide rational numbers. • Convert rational numbers to decimals. • Use properties of operations to generate equivalent expressions. • Solve multistep real-world and mathematical problems using numerical and algebraic expressions and equations. 	<ul style="list-style-type: none"> • Understand the existence of irrational numbers that can be approximated using rational numbers. • Work with radicals and integer exponents. • Solve linear equations with one variable. • Analyze and solve pairs of simultaneous linear equations.
Ratios, Proportional Reasoning, and Functions	<ul style="list-style-type: none"> • Understand ratios and use ratio reasoning to solve problems including tables of equivalent ratios, unit rates, percentages, and measurement conversions. 	<ul style="list-style-type: none"> • Compute unit rates associated with ratios of fractions. • Recognize and represent proportional relationships between quantities using tables, equations, and graphs. • Solve multistep ratio and percent problems. 	<ul style="list-style-type: none"> • Define, evaluate, and compare functions. • Use functions to model relationships between quantities. • Understand the connections between proportional relationships, lines, and linear equations. • Graph proportional relationships, interpreting the unit rate as slope.
Geometry	<ul style="list-style-type: none"> • Find the area of right triangles, other triangles, special quadrilaterals, and polygons. • Find the volume of right rectangular prisms with fractional edge lengths. • Represent three-dimensional figures using nets. • Draw polygons on a coordinate plane. 	<ul style="list-style-type: none"> • Solve problems involving scale drawings of geometric figures. • Draw and construct geometric figures. • Know and apply the formulas for area and circumference of circles. • Use angle relationships to solve problems. • Solve for area, volume, and surface area of two- and three-dimensional shapes composed of triangles, quadrilaterals, polygons, cubes, and right prisms. 	<ul style="list-style-type: none"> • Understand transformations and their effects on congruence and similarity. • Understand angles formed by parallel lines cut by a transversal. • Understand and apply the Pythagorean theorem. • Solve real-world and mathematical problems involving volumes of cylinders, cones, and spheres.
Statistics and Probability	<ul style="list-style-type: none"> • Develop an understanding of statistical variability. • Display numerical data in plots on a number line, dot plots, histograms, and box plots. • Summarize numerical data sets in relationship to their contexts. 	<ul style="list-style-type: none"> • Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate chance processes and develop, use, and evaluate probability models. • Find the probability of compound events. 	<ul style="list-style-type: none"> • Construct and interpret scatterplots. • Investigate patterns of association in bivariate data.

Source: Adapted from Kanold, Briars, Asturias, Foster, & Gale, 2013; NCTM, 2006.

numbers and positive and negative rational numbers. They use the operations and understanding of number to write and simplify expressions. Throughout grades 6–8, students develop an *understanding of number values* and use that knowledge to build equations and inequalities and solve them using the four operations.

Students develop an understanding of *ratio and proportional reasoning* throughout grades 6–8 as they make sense of ratios and rates; generate equivalent ratios and rates; compare them; and ultimately solve problems, graph proportional relationships, and generate proportional values using tables. In eighth grade, students apply this knowledge to an understanding of slope as a constant rate of change and graph linear equations. Grade-level teams expect students to simultaneously develop an understanding of percentages and work with values as decimals, fractions, or percentages.

In *geometry*, students extend their work with areas of triangles and rectangles and volumes of rectangular prisms to then find the areas, surface areas, and volumes of shapes they can decompose into known shapes. They create nets for three-dimensional figures and, in eighth grade, learn about the Pythagorean theorem, angles parallel lines form when cut by a transversal, and transformations related to congruence and similarity.

Statistics and probability in grades 6–8 consist of displaying and analyzing information in graphs as well as understanding measures of center and spread. In seventh grade, students compare and contrast two different sets of data. Seventh grade also includes an initial study of probability. In eighth grade, students explore bivariate data and scatterplots.

Your team may want to explore mathematics learning progressions as defined in your state standards or reference online mathematics learning progression documents, such as those of the University of Arizona (Common Core Standards Writing Team, n.d.; available at www.math.arizona.edu/~ime/progressions) or Achieve the Core's (n.d.) coherence map (available at <https://achievethecore.org/page/1118/coherence-map>). Your team may also want to engage in a book study, perhaps referencing NCTM resources related to understanding the essential content and skills needed for mathematics in grades 6–8, in order to learn various content and strategies applicable to each grade's essential learning standards.

With so much mathematics content to learn, your mathematics department's and team's collaborative unit planning helps ensure a guaranteed and viable mathematics curriculum at your grade level and across grades 6–8. Planning the units together to more deeply learn your own grade-level content and its importance in the grades 6–8 trajectory builds teacher team self-efficacy.

Connections Between Mathematics Content and Unit Planning

For each unit at your grade level, you support your team's progress toward better understanding the standards that support the guaranteed and viable mathematics curriculum. Together, you and your team use the Mathematics Unit Planner template in figure 1.2 (page 11) to record answers to the following questions.

- What exactly do students need to know and be able to do in this unit?
- Which mathematics standards should we commonly assess? When?
- How does the mathematics learning in this unit connect to the standards students must learn in previous or future units?
- Which academic mathematics vocabulary and notations must students learn to read, write, and speak to be proficient in the unit standards?
- What are examples of higher- and lower-level-cognitive-demand mathematical tasks students should demonstrate proficiency with if they have learned the standards?
- Which mathematical tools or technology should students learn or utilize to demonstrate an understanding of the unit standards?

Answering these questions as a team creates more equitable student learning experiences from one teacher to the next. Additionally, developing teacher efficacy strengthens your instructional practices. Consequently, student learning improves because your entire team is working to ensure each student *learns* the organized mathematics content from one unit to the next.

Chapter 2 (page 15) provides tools and protocols that help your sixth-, seventh-, or eighth-grade mathematics team unpack unit standards and learn how to intentionally address each unit-planning element as your mathematics story arc develops for the school year.