Grades 6–8

Navigating *through* Geometry

Chapter 3

Transformations and Symmetry

Important Mathematical Ideas

Motion plays a significant role in our daily life. Students can relate motions—reflections, translations, and rotations—to their everyday experiences of riding a bike, running on the playground, or going to a department store. Hands-on explorations of motion are appealing to middle-grades students who are making the transition from generalizing from inductive, concrete experiences to making more deductive, abstract generalizations.

Objects that are translated, reflected, or rotated preserve their size and shape, although their location and orientation may change. This chapter focuses on ways transformations help students understand similarity and symmetry and help them classify polygons, thus extending the work begun in chapter 1 on recognizing the properties and characteristics of shapes.

Principles and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM] 2000) indicates that middle-grades students should have experiences that develop the skills and understandings that allow them to successfully—

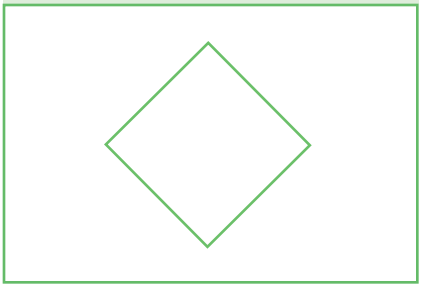
- describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling;
- examine the congruence, similarity, and line or rotational symmetry of objects using transformations. (P. 232)

Students can trace one triangle on tracing paper and physically translate, reflect, or rotate it to match another triangle. This inductive experiment to determine coincidence has a parallel deductive proof that the two triangles are congruent.

"Transformational geometry offers another lens through which to investigate and interpret geometric objects." (NCTM 2000, p. 235)

Fig. **3.1.**

Some students may call a square in this orientation a "diamond."



What Might Students Already Know about These Ideas?

Students' prior experiences with transformations may have included reflections, translations, and rotations (flips, slides, and turns). They may have determined congruence by placing one figure on top of another and extended that strategy to turning and reorienting figures to demonstrate congruence. They are probably able to identify a line of symmetry and find the lines of symmetry in figures.

The orientation of a figure in a plane may, however, distract some students from identifying the shape. Younger students are especially distracted by a geometric figure that is not oriented so that its base is "at the bottom" and parallel to the edge of the page. Even some middle-grades students may fail to recognize a square when it is displayed as shown in figure 3.1. Students frequently refer to this shape as a "diamond."

Students who have had limited experience with transformations and other spatial-reasoning tasks often identify figures according to their appearance. They are influenced by irrelevant attributes of a figure, such as its orientation, and are unlikely to recognize a rotated figure. With more experience, students analyze a figure by examining its parts and the relationships among them.

The following activity allows teachers to preassess students' ideas about transformations. It focuses on reflections in the plane, reorientations that do not change any of the properties of the figure. Transformations that preserve congruence are called *rigid motions*.

Reflection of Images

Goals

To assess students'—

- understanding that reflections preserve the properties of figures;
- ability to draw reflections of figures.

Materials and Equipment

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- A copy of the blackline master "Reflection of Images" for each student
- Rulers
- Tracing paper

Activity

Students draw the image of each of four figures after the figures have been reflected over the *y*-axis of a coordinate grid, and then they draw the image of each figure after it has been reflected over the *x*-axis.

Discussion

The students might verify their answers by tracing the figure and then turning the paper over to show the reflected image of the original. If they say that the figures are the "same," ask them to explain in what ways the figures are the same. They should mention the lengths of the sides and the measures of the angles. Next, ask them why these properties remain unchanged. They should recognize that a reflection merely reorients an object without changing anything else about it. If the students fail to notice that the shape and size have not changed, prompt them to reflect the figure over one of the axes and observe the results. Once again, focus the students' attention on the figure's side lengths and angles. The degree to which students are troubled by the orientation of a figure can help you distinguish students with limited prior experiences from those who are familiar with transformations. For students with some background in transformations, orientation will be less troubling, and they should be able to use appropriate vocabulary to describe transformations.

Selected Instructional Activities

Tracing tasks give students experience in determining the image of a figure under a transformation. Exploring relationships between the preimage (original figure) and the image in translations, reflections, and rotations is the goal of the following activity (adapted from the Reconceptualizing Mathematics Project n.d.). The students may be more familiar with the terms *slide*, *flip*, and *turn* than with *translation*, *reflection*, and *rotation*. Allow the students to use the familiar terms to describe their actions, but also introduce the geometric vocabulary for such motions.

The CD-ROM includes an interactive applet, Transformation Tools, that can help students visualize transformations.

"Transformations can become an object of study in their own right. Teachers can ask students to visualize and describe the relationship among lines of reflection, centers of rotation, and the positions of preimages and images."
(NCTM 2000, p. 236)


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Translations, Reflections, and Rotations

Goals

- Explore relationships between the preimage and the image in rigid motions
- Develop appropriate language to describe rigid motions
- Perform three rigid transformations: reflections, translations, and rotations

Materials and Equipment

- A copy of the blackline master "Translations, Reflections, and Rotations" for each student
- Tracing paper (waxed paper or any paper that is easy to see through)
- Rulers

Activity

The students perform a translation, a reflection, and a rotation according to the instructions on the activity sheet. They should be encouraged to execute each transformation carefully so that the characteristics of each figure are preserved. The students should repeat each transformation with a shape of their choice to give them practice in performing transformations.

Translation

Translations are described by answering the questions, How far? and In what direction? Translations are represented by a translation arrow, or *vector*, that indicates how far to translate and in what direction. The students are instructed to use a vector to complete the translation.

Reflection

The line that is the axis of the reflection is sometimes called the *line of reflection* or the *mirror line*. Corresponding points on the reflection (*image*) and the initial figure (*preimage*) are the same distance from the line of reflection. The students are instructed to specify a line of reflection over which they flip the shape they designate.

Rotation

To describe a rotation, it is necessary to specify the point on which the rotation is centered, called the *center of rotation*, and an angle that shows the size and direction (clockwise or counterclockwise) of the rotation. The students are instructed to specify both a center of rotation and an angle of rotation for this transformation.

Discussion

Ask the students to observe the preimage (the original figure) and the image. The discussion should focus on the characteristics of both figures and the characteristics of the motion itself. Ask questions such as

Some people prefer to use the terms *original figure* and *image* to distinguish between the starting shape and the ending shape.

Is reflection commutative? A figure is reflected first across the *x*-axis and then across the *y*-axis. Is the resulting image congruent to, and in the same orientation as, the image formed if the figure were reflected first across the *y*-axis and then across the *x*-axis?

How did the figure move? and Can you describe the path of the motion? In observing the figures, questions such as What changed? and What stayed the same? will help the students analyze the relationship between the preimage and the image. The motion of a reflection is like a flip; the resulting image is congruent with the preimage, but the orientation has changed. A translation is a slide; it always moves in a straight line. It may be in any one direction. The resulting image is congruent with the preimage, but its position has changed. A rotation is a circular motion about a fixed point, which may or may not be a point on the original figure. The resulting figure is congruent with the preimage, but its position has changed.

From their sketches, the students should be able to tell how the orientations of the original figure and its image are related for each type of rigid motion. *Orientation* refers to the ordering of the points in a figure either clockwise or counterclockwise. If, for example, you pick three points on a figure and assign them *P*, *Q*, *R*, then the order *P*-*Q*-*R* gives a clockwise or counterclockwise orientation to the figure.

Students who have identified the basic characteristics of rigid motions can go on to more-analytical observations. They can, for instance, connect corresponding points of the original figures and their reflection images and observe that the segments that connect the corresponding points are perpendicular to the line of reflection and that the line of reflection bisects the segments connecting the corresponding points. They can draw and test many vectors that result in the same translation and observe that all the vectors have the same lengths and lie parallel to one another. The students can examine rotations and discover that the angle between the "starting ray" and the second ray determines the size of the turn and that, in order to achieve the same degree of rotation, points farther from the center of rotation have to "move farther" along an arc than points closer to the center do.

Examining Congruence and Similarity

The previous chapters have presented several activities that involve similarity and congruence. Chapter 1 discussed some properties of similar and congruent figures and introduced dilations. Chapter 2 related those concepts to coordinate geometry. You may want to review some of the ideas in the first two chapters before continuing your class's explorations with related concepts.

"Similarity: Investigations at the Middle Grades Level (Friedlander and Lappan 1987) is one of the helpful resources on the CD-ROM.

Sierpinski pyramids can be used to examine various features of similarity and congruence. See "Build a Sierpinski Pyramid" (Kelley 1999) for directions for constructing the pyramid.