CHAPTER 1

Introducing the Problem-Solving Cycle

> The PSC gives me a professional development model to use across all the middle schools in my district. When we rolled out the Common Core State Standards here, I already had a structure in place that allowed my teachers to collaboratively learn and then analyze their teaching practice related to CCSSM practices.
>
> —Joanie Funderburk,
> secondary mathematics coordinator, Cherry Creek School District

Today, more than ever, educational leaders and administrators are seeking out professional development opportunities for teachers. They are looking for PD models that can accommodate large-scale, system-level implementation of their reform directives and promote the learning of all students. Ideally, educational leaders want PD that they are confident will result in improved teaching and increases in student learning. The demand is increasing for PD models that can be implemented by teacher leaders working in their own schools, and that are flexible enough to accommodate the latest calls for reform as well as the specific needs of a particular school. Local administrators and professional development leaders are looking for models that can support initiatives such as the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), Learning Forward's *Standards for Professional Learning* (Learning Forward, 2011), and the National Council of Teachers of Mathematics's *Principles to Actions* (NCTM, 2014), as well as trends such as using data to inform practice. Models that have been carefully articulated, researched, and used successfully with a variety of teachers across schools and districts are particularly appealing. This book introduces two such models: the Problem-Solving Cycle model to support the professional growth of teachers, and the Mathematics Leadership Preparation model to support the professional growth of teacher leaders.

The PSC is a research-based model of PD that promotes teachers' learning and teaching of mathematics in line with reform principles, and in ways that are personally relevant to individual participants. This model is structured as a professional learning community that is typically school-based and led by a teacher leader at the school. It incorporates the core features of effective PD that have been identified in numerous research studies (e.g., Desimone, 2009; Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2009). That is, it focuses on subject-matter content and instructional practices, provides opportunities for teachers to participate actively and collaboratively in a professional community, and is ongoing and sustainable over time. There are many advantages of using the PSC model, such as the fact that it is highly adaptable and is focused on specific problems of practice that are of interest to the participating teachers and administrators. Additionally, it can be tailored to highlight federal, state, district, and school-based initiatives that are ever-changing and ongoing in the life of a teacher.

At this unique historical juncture, school districts across the United States are in the midst of widespread adoption of the CCSSM. For most mathematics teachers, this means that they must teach new content, using new instructional materials, while facing increased scrutiny of their classroom practices and students' learning. The intentionally flexible nature of the PSC allows teachers to engage in conversations and experiences during their PD time that directly target areas of need, as identified by the district, school, or teachers themselves. Furthermore, because the PSC is intended as a long-term model, these areas of need can be continually revisited and modified over time. For example, a school or district might decide to focus initially on one of the Standards for Mathematical Practice described in the CCSSM on the basis of a needs assessment. That focus could then shift to a different Standard for Mathematical Practice once the group determines they are ready to move on.

WHAT IS THE PSC, AND HOW DOES IT ENHANCE TEACHER KNOWLEDGE AND PRACTICE?

When teachers engage in the PSC, they become part of a professional learning community with other teachers, typically from within their own school or district. This community engages in mathematical problem solving, examining video of classroom teaching, and sharing ideas about teaching and learning mathematics. Another central component of the PSC model is a focus on teachers' own classrooms. For example, during PSC workshops the group watches and discusses video that comes entirely from participating teachers' classrooms. Additionally, in PSC workshops teachers consider aspects of context, culture, and linguistic diversity that are directly relevant to supporting the needs of their students.

Each cycle of the PSC begins with teachers working together on a selected mathematical problem. They analyze the mathematical concepts embedded in the problem, consider how students develop an understanding of those concepts, and discuss correct and incorrect ways that students might approach the problem. The objective is for teachers to take part in discussions that will help them think about important mathematical content from different perspectives, and to generate ideas about supporting their own unique groups of students in learning that content.

After teachers have discussed a selected problem together, each teacher modifies the problem and creates a lesson plan tailored to his or her own students. The teachers then teach the lesson with the modified PSC problem to at least one of their classes, and the lessons are videotaped. Teachers bring this video footage and student work to the next two PSC workshops, and together with their colleagues, they use the video and student work as a springboard for analyzing and discussing their experiences.

The PSC facilitator plays an important role by choosing appropriate video clips from the unedited footage and guiding the discussions to focus on selected themes. The video clips and associated conversations generally highlight student thinking and/or instructional moves. For example, as teachers watch a clip, the facilitator might encourage them to consider the development of students' mathematical thinking in a particular content area, strategies to support English language learners (ELLs), or what it means to apply a specific mathematical practice standard in the given context.

By electing to participate in the PSC, teachers demonstrate their commitment to broadening their knowledge base and, consequently, to becoming better able to support their students' learning of mathematics. Taking part in the PSC affords teachers the opportunity to become part of a strong collaborative and supportive learning community. As part of this community, teachers can contribute to the selection of goals for the workshops, including personal goals they have for learning mathematical content and exploring new pedagogical practices. Most important, the PSC offers a structure for helping teachers meet these goals, providing them time to think deeply about important mathematical content, the nature of students' reasoning, and effective mathematics instruction. The opportunities for deepening knowledge and improving practice that the PSC provides are increasingly critical for teachers as they strive to meet the demands of new initiatives such as the CCSSM and teacher accountability.

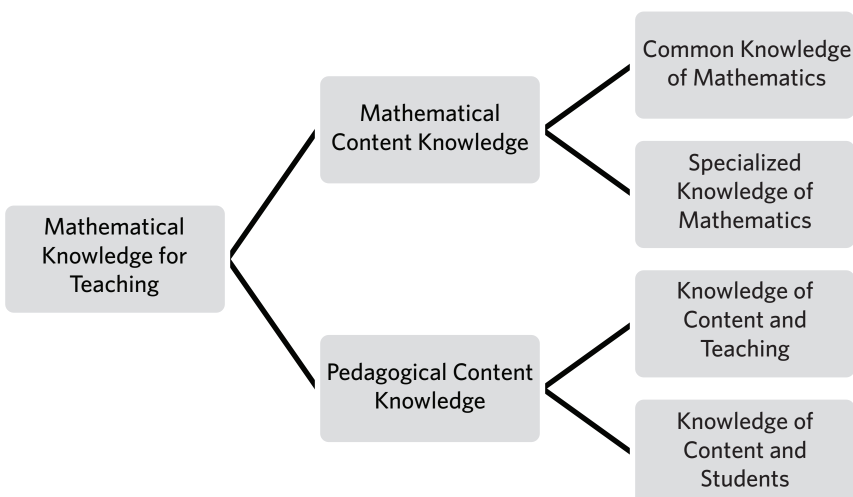
Developing Mathematical Knowledge for Teaching Through the PSC

Participating in the PSC enables teachers to develop an essential type of knowledge that has been labeled "mathematical knowledge for teaching" (MKT), which refers to the professional knowledge that mathematics teachers need to effectively carry out the mathematical work of teaching. Prominent education researcher Lee Shulman (1986) argued that teachers draw on both subject-matter content knowledge and pedagogical content knowledge when they orchestrate their classroom lessons. Deborah Ball and her colleagues (Ball, Thames, & Phelps, 2008) extended Shulman's work into the field of mathematics education and coined the now commonly used term *MKT*. As shown in Figure 1.1, MKT is multifaceted and incorporates both subject-matter and pedagogical content knowledge.

The PSC and mathematical content knowledge. The content knowledge that teachers draw on includes both "common" and "specialized" knowledge of mathematics. Common content knowledge can be defined as a basic understanding of mathematical skills, procedures, and concepts acquired by any well-educated adult. Specialized knowledge refers to a deeper, more nuanced understanding of mathematical skills, procedures, and concepts that is particularly relevant to teaching and learning. Our research indicates that PSC workshops promote teachers' specialized content knowledge when they discuss multiple solution strategies for selected problems, carefully unpack student errors, consider how to provide accurate mathematical explanations, and share developmentally appropriate mathematical representations.

The PSC and pedagogical content knowledge. Deborah Ball and colleagues (2008) propose two distinct components of pedagogical content knowledge: knowledge of content and teaching, and knowledge of content and students. In the language of MKT, both components are essential to teaching. When making instructional decisions and reflecting on their teaching, mathematics teachers draw on a sophisticated understanding of which instructional practices are most appropriate for teaching specific content, and how best to support their students' thinking as they learn that content.

Figure 1.1. The Mathematical Knowledge for Teaching



The PSC model supports the development of both components of pedagogical content knowledge by engaging teachers in focused lesson-planning activities and in critically analyzing instances of teaching and learning that occurred when they taught the PSC problem. PSC workshops promote teachers' knowledge of content and teaching when they do the following collaboratively:

- Explore the most appropriate strategies and materials for a lesson
- Modify a problem to meet the needs of their students
- Consider how to sequence content to facilitate student learning
- Examine the instructional pros and cons of different representations
- Discuss ways to improve their instructional practices the next time they teach a lesson with related mathematical content

PSC workshops foster teachers' knowledge of content and students when they engage in activities such as the following:

- Predicting how students will approach specific mathematical problems
- Anticipating student errors
- Interpreting incomplete student ideas
- Considering how to respond to the various correct and incorrect pathways that students explore

The various components of MKT are inextricably intertwined in teachers' instructional practices and are woven into each PSC workshop. In the classroom, teachers must routinely make decisions that draw on all aspects of their knowledge as they engage in the numerous, complex activities of teaching. The PSC provides a space for teachers to tap into and enhance their knowledge as they plan, implement, analyze, and reflect on specific lessons. Different knowledge strands are foregrounded in individual workshops, and in keeping with the flexible nature of the model, the needs and interests of particular groups of teachers can drive precisely which aspects of MKT are highlighted at any given time.

Improving Teaching Through the PSC

As the educational community strives to improve learning outcomes for all students, teachers are frequently asked to use instructional strategies that differ from what they experienced themselves as students. Current standards and reform efforts in mathematics education encourage teachers to plan and implement instruction that builds on students' existing knowledge and promotes the development of both skills and conceptual understanding (CCSS, 2010; NCTM, 2000). These standards encourage teachers to be aware of and use instructional moves that are sensitive to students' needs, utilize student-generated ideas, engage students in mathematical practices, and foster deep understanding of mathematical concepts.

The PSC provides a safe forum for teachers to discuss their prior teaching and learning experiences, and to generate ways to improve their instructional practices. In PSC workshops, teachers individually and collectively view, reflect on, unpack, and discuss their classroom instruction. They consider whether or not the students understood the targeted content, if there were challenges in teaching mathematical concepts and practices, or how they might have taught the lesson more effectively. Other aspects of classroom practice that teachers might discuss in PSC workshops include their experiences:

- Launching a problem
- Identifying student misunderstandings
- Asking for clarification of ambiguous explanations
- Handling student errors
- Drawing connections among mathematical representations
- Facilitating discussions that promote conceptual understanding

These types of conversations support teachers as they try to reconcile or reconstruct their teaching practices in line with new reform initiatives and meet the needs of the students in their classes.

THE PSC PROCESSES AND HOW THEY UNFOLD

Gaining Mathematical Knowledge by Collaboratively Solving Rich Problems

Rich mathematical problems are an integral component of the PSC model, laying the foundation for teachers to participate in a productive learning environment and gain valuable MKT. We are often asked what we mean by "rich mathematical problems." We define rich mathematical problems as those problems that meet the following criteria:

- Address several critical mathematical concepts and practices
- Are accessible to students with different levels of mathematical knowledge
- Contain multiple entry and exit points
- Encourage a variety of solution strategies

In each PSC cycle, teachers work with a selected problem through a series of interconnected workshops. These workshops entail solving the problem and preparing to teach it, and then analyzing the teachers' own and other group members' implementation of the problem. When teachers solve the rich PSC problem, they collaboratively explore multiple solution strategies, share ideas about the mathematical concepts and practices that are embedded in these strategies, and consider ways of adapting the problem to ensure that it is accessible to all students. In workshops subsequent to teaching their PSC lessons, teachers have opportunities to further explore the complexities of the mathematical concepts that the problem entails, and the affordances and limitations of various representations and solution strategies.

Using Video Clips and Classroom Artifacts

Another central feature of the PSC model is the use of video recordings of participating teachers' own PSC lessons and, to a lesser extent, student work produced during these lessons. Video and other classroom artifacts serve to bring the teachers' classrooms into the PD setting, making them available for study, reflection, and discussion. Teachers view short video clips from one another's lessons in order to anchor conversations and highlight particular PD goals. Video can be viewed repeatedly and from different perspectives, enabling teachers to closely examine one another's instructional practices and students' mathematical thinking and learning. Although any video of classroom instruction is likely to prove meaningful for teachers, there are strong arguments for using video from participants' own classrooms as is done in the PSC. Video from teachers' own classrooms situates their exploration of mathematics teaching and learning in a more familiar, and potentially more motivating, environment than video from unknown teachers' classrooms.

Building Professional Learning Communities

Developing and maintaining a professional learning community is a hallmark of a successful PD effort. The PSC offers teachers the opportunity to actively participate in and cultivate such a community. Throughout the PSC workshops, teachers work collaboratively to develop their mathematical understandings and to explore ways of improving their teaching. Because these PSC practices have the potential to expose teachers' limitations or weaknesses, they may be uncomfortable or threatening to participants. At the same time, when teachers do begin to share their ideas and especially video recordings of their teaching with colleagues, they have the opportunity to create an atmosphere of openness and trust that is rare in other professional learning environments. For these reasons, the development and maintenance of a productive learning community around mathematical problem solving and analyzing video is an integral component of the PSC model.

PSC facilitators must explicitly attend to developing communication norms that enable challenging yet supportive discussions about teaching and learning. They must encourage teachers to maintain a balance between respecting individual community members and critically analyzing issues in their teaching. In our experience, PSC participants overwhelmingly express the positive nature of the experience, noting that the strength of their bond as a community increases over time, which allows them to pursue relevant goals more deeply. The use of video, in particular, is touted by most PSC participants as central to their professional growth. They report that both reflecting on their own footage and seeing clips of their colleagues in action help them better understand their students' thinking and learn new teaching strategies.

PSC FACILITATORS

The PSC model is designed to be implemented by a knowledgeable facilitator who plans and leads each workshop. PSC workshops can also be conducted by cofacilitators when appropriate for a particular context. Facilitators of the PSC might be mathematics teachers, department chairs, coaches, district-level leaders, or other teacher educators with leadership capabilities, a deep understanding of mathematics, and knowledge of current state, local, and school-based initiatives. Additionally, ideal facilitators are knowledgeable about the PSC model and are enthusiastic about using it to work with teachers.

In developing the PSC model, we learned a great deal about effective strategies for leading the workshops, as well as techniques for preparing and supporting facilitators. We also identified several areas in which novice PSC facilitators are likely to need support. These include engaging teachers in productive mathematical work, leading discussions about student reasoning and instructional practices, and building a strong professional community. We drew upon this knowledge to develop our Mathematics Leadership Preparation (MLP) model and other resources for PSC facilitators.

OVERVIEW OF THE BOOK

In the chapters that follow, we provide a detailed look at the PSC and MLP models. Chapters 2–5 are focused on PSC workshops and activities for supporting teacher learning. In Chapter 2 we describe and illustrate exactly what the PSC entails and share lessons learned from our experiences using the PSC. Chapter 3 presents vignettes that illustrate typical activities in each of the three PSC workshops. Chapter 4 focuses on the importance of video in the successful implementation of the PSC model, and Chapter 5 summarizes the key findings from our research regarding the impact of the PSC on teachers and students.

Chapters 6–8 take up the MLP model, which prepares teacher leaders to facilitate the PSC. In Chapter 6 we provide a detailed description of the MLP and share insights from our experiences conducting this leadership program. Vignettes in Chapter 7 illustrate typical activities in the two components of the MLP model: the Summer Leadership Academy and Leader Support Meetings. Chapter 8 summarizes key findings from our research on teacher leaders' facilitation of PSC workshops.

In the final chapter, we provide a case study of one school district that implemented the PSC and MLP. Throughout the book, we feature the voices of many PSC participants by using quotes from interviews, examples from PSC workshops and MLP meetings, and images of changing classroom instruction. At the end of the book we include a list of published journal articles and book chapters related to the PSC and MLP models, where interested readers can find more information about our research methods, data analyses, and findings.