NAVIGATING *through* ALGEBRA

Chapter 1
Patterns

Beginning as early as three and a half years of age, children evidence strong interest in patterns. In their study of young children's mathematical behavior during free play, Ginsburg, Pappas, and Seo (forthcoming) discovered that pattern play occurred more often than any other type of activity and that their findings were true of all children, regardless of ethnicity, gender, or social class. For this reason and because pattern recognition and generalization are central components of algebraic reasoning, this chapter focuses on developing children's abilities to replicate, complete, continue, describe, generalize, and create repeating and growing patterns. Children learn to predict "what comes next" by carefully analyzing relationships between pairs of elements that "come before" in the pattern. They also learn to represent patterns in various ways and to move between those different representations.

Repeating Patterns

In the activities with repeating patterns, emphasis is placed on the cyclical nature of the repetition and the identification of the elements in the cycle. For example, in the pattern A, B, C, A, B, C, A, B, C, A, …, there are three elements, A, B, and C, in each cycle that repeats and the elements appear in the same order in each repetition. With that information, children can create three-element repeating patterns that are differ-

ent representations of the ABC pattern, for example, square, triangle, circle, square, triangle, circle, …; 1, 2, 3, 1, 2, 3, …; clap, pat, snap, clap, pat, snap, …; and do, re, mi, do, re, mi, …. They can also solve more-advanced problems like determining the number of *A*'s needed to write the first thirty elements in the pattern.

At a more complex level, children begin to investigate the relationship between an element in a pattern and its position in that pattern, and they use that information to continue the pattern and predict what comes next. Consider the repeating pattern below:

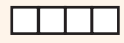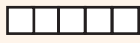|	A	B	A	B	A	B	A	…
Position	1	2	3	4	5	6	7	…

Children observe that the letter *A* is in positions 1, 3, 5, and 7, and because all the positions for *A* are odd numbers, the letter most likely to be in the ninety-ninth position is *A*.

The activities Clown Line-Up, Footprints, and What's Above? provide opportunities for students to develop their understanding of repeating color, shape, movement, and number patterns and the different ways these patterns can be represented. The activities also help students learn to relate the elements in a pattern to their positions in that pattern.

Growing Patterns

Unlike repeating patterns, growing patterns show an arithmetic change between pairs of elements in the pattern. For example, the counting sequence 1, 2, 3, 4, … is a growing pattern. The numbers in each pair differ by 1, and the numbers are in increasing order. Likewise, counting by fives creates a growing pattern, as does counting by tens. It should be noted that although counting by ones is a growing pattern, the occurrence of the ones digits in the decades is cyclic. Consider, for example, 21, 22, 23, 24, …, 31, 32, 33, 34. Recognition of both the growing pattern in the tens digits and the repeating pattern of the ones digits will help children recall and name the numbers in sequence. Although the word *growing* is used to distinguish this type of pattern from repeating patterns, growing patterns may show numbers in decreasing order or buildings in decreasing size. For example, 100, 99, 98, 97, … is a member of this family of patterns.

To help children recognize changes between pairs of elements, growing patterns are often shown using geometric elements. For example, squares are used to construct the growing pattern below:

Position	1	2	3	4

When examining this pattern, students observe that there are two squares in position 1, three squares in position 2, four squares in position 3, and five squares in position 4. From these observations, children

identify and describe the pattern ("The number of squares is one more than the position number") and determine the number of squares in, for example, position 20.

The generalization of the growing pattern—that the number of squares is 1 more than the position number—is a function. If P stands for the position number, then $P + 1$ represents the number of squares in that position. As can be seen, the analysis and description of growing patterns provide a natural connection between the important ideas of patterns and those of variable and function.

Snakes and More Snakes, Footprints, Follow the Number Roads, How Does It Grow? and Jumping Rules facilitate students' exploration of growing patterns and their replication, extension, and description. Students investigate ways to generate such patterns and how to generalize the relationship between elements and their locations in a sequence. Particular attention is given to counting and "fancy counting" (e.g., counting by twos, fives, tens) as examples of growing patterns.

Expectations for Students' Accomplishment

By the end of grade 2, students should be able to identify the set of elements that repeat in a repeating color, shape, number, letter, or action pattern; predict "what comes next" in the pattern; and continue the pattern. They should also be able to describe the change between successive elements in a pattern that grows or decreases at a constant rate. This description in words sets the stage for the representation of patterns using symbols and graphs, which is developed in higher grades. For simple repeating and growing patterns, students should be able to relate an element in the pattern to its location in that pattern.

Clown Line-Up

Prekindergarten–Kindergarten

Summary

Students sort clown pictures, identify and describe clown patterns, extend the patterns, and create their own patterns. They then translate their patterns into other forms using letters, sounds, and movements.

Sort, classify, and order objects by size, number, and other properties

Goals

- Identify, extend, and create repeating patterns
- Identify missing elements in a pattern, and give justifications for their inclusion
- Translate patterns into other forms, using letters and movements

Prior Knowledge

- Counting up to ten objects
- Identifying colors
- Sorting and classifying by color
- Identifying the *first*, *second*, *last*, and *next* elements in a pattern

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Materials

- Blackline master: "Clowns"
  Reproduce the blackline master on paper or card stock. Cut out the squares. Each student needs at least twenty-one clown pictures, seven in each of three colors. The students can color the clowns, or the clown squares can be printed on colored construction paper.
- Crayons
- Glue or paste
- Construction paper in three colors
- One twenty-four-inch strip of construction paper for each student
- Three 2 1/2 in. × 3 1/2 in. pieces of dark construction paper for each pair of students to use as "covers"

This activity can be done with other materials such as stickers, stamps, or painted sponge cutouts.

Activity

Engage

Show students this pattern with the clowns:

red, blue, blue, yellow, red, blue, blue, yellow, red, blue, blue, yellow

Point to the clowns as you identify them and their colors. Say, "Here is my repeating pattern. It is red, blue, blue, yellow, red, blue, blue, yellow, red, blue, blue, yellow. There are four clowns in each part of my pattern that repeats: one red, two blue, and one yellow."

To be sure that the students understand that at least two repetitions (i.e., a total of three sets of elements) must be seen before the pattern can be discovered, cover all but the first two pictures and ask, "Can you tell what my pattern is now?" (no) "Why not?" (There are not enough clowns to tell what comes next.) Continue revealing more of the pattern and asking the students if they can tell what comes next. Talk about how many clowns they need to see before they can identify the pattern.

Explore

Have the students create patterns with the three colors of clowns. Say, "Use the clowns to make a repeating pattern. Use at least nine clowns in your pattern. Be sure that each part of your pattern repeats." Encourage the students to share their patterns with others in their group. Give them time to make more-complex patterns. After you have checked their patterns, have the students glue the patterns to strips of construction paper and use their pattern strips to play the "cover up" game.

Students play the "cover up" game in pairs. One student is the pattern maker, and the other is the pattern finder. The pattern finder closes his or her eyes, and the pattern maker, using a construction-paper cover, covers one of the clowns in the pattern. The pattern finder then tries to figure out the color of the clown that is covered. The students change roles and play the game again.

Demonstrate the game several times, covering a different color each time, before having the students play the game. You can make the game more challenging by covering more than one clown in the pattern.

After the students have played the game, you may want to have them continue their patterns using at least twenty clowns. When they have completed their longer patterns, ask questions about the number of clowns of each color in the first eight clowns, the first ten clowns, and the first twenty clowns. Have the students imagine that their patterns continue and predict the colors of the twenty-first, twenty-fifth, and thirtieth clowns. Be sure that they provide rationales for their predictions.

Extend

To extend students' work with repeating patterns, help them recognize different representations of the same pattern. For example, show them the following two patterns, one formed from colored clowns and the other formed from letters:

> red, red, yellow, red, red, yellow, red, red, yellow
> A, A, B, A, A, B, A, A, B

Talk about how the patterns are different (colors versus letters) and how they are the same (two of a color or letter followed by one of a different color or letter; the two-one pattern is repeated). Demonstrate this movement pattern: clap, clap, jump, clap, clap, jump, clap, clap, jump. With the students, talk about how this movement pattern is different from the color and letter patterns and how it is like the others. Follow the same procedure for some of the patterns the students have made. The students may want to create their own alternative representations.

You may want to store the longer patterns and covers in your math center so that students can play the "cover up" game during free play.

Discussion

Too often the study of repeating patterns stops before students have opportunities to analyze the patterns and represent them in different ways. These steps are essential in preparing students to generalize patterns and describe the relationship between an element in the pattern and its location in the pattern by writing function rules.

Repeat Clown Line-Up or other repeating-pattern activities throughout the year. Increase the complexity of the patterns and of the questions about the elements in the patterns. Have the students create similar patterns using letters, toys, movements, or sounds.