



Introduction

A Brief History of Current State Standards

Contrary to popular belief, academic standards are not new. In fact, they have been around for more than 25 years. The first set of curriculum specific standards, *The Curriculum and Evaluation Standards for School Mathematics*, was released by the National Council of Teachers of Mathematics in 1989, followed by an updated set of standards, *Principles and Standards for School Mathematics*, in 2001. Both these documents provided a vision for K–12 mathematics by grade-level band. They also formed the foundation for most states’ grade-level standards.

In April 2009, the National Governor’s Association and the Council of Chief State School Officers met to discuss the creation of the Common Core State Standards Initiative. The purpose of such an initiative was to develop a set of common standards across states to balance the quality of mathematics instruction and learning. Following that meeting, the process of writing the Common Core standards began. The Standards Development team, led by William McCallum, Phil Daro, and Jason Zimba, included mathematicians, mathematics educators, mathematics education researchers, and classroom teachers. The process included an open invitation for feedback, not only from mathematics educators and associations including the National Council of Teachers of Mathematics but also from the general public. This feedback was considered, and much of it was incorporated into the final document, which was released in June 2010. Following the release of the standards, individual states went through their own processes for reviewing, adopting and, if necessary, ratifying the adoption of the Common Core State Standards.

Since the release of the Common Core State Standards, many of the states that adopted these standards have made some minor revisions by clarifying a standard, moving a standard to a different grade level, combining standards, and, in relatively few cases, adding or deleting a standard. A few states have totally revised their standards. The few states that did not adopt the Common Core State Standards for Mathematics (CCSSM) continue to work with the standards adopted by their state. Even so, the mathematical content remains similar across states. That is good news so that the relevancy, rigor, and notion of shared standards based on the Common Core have not been lost.

College and Career Ready Standards for Mathematics

“The Common Core State Standards are a clear set of shared goals and expectations for the knowledge and skills students need in English language arts and mathematics at each grade level so they can be prepared to succeed in college, career, and life” (Common Core State Standards Initiative, n.d.). Keep in mind that most states have maintained the core standards from the original document. At the same time, some states have clarified, combined, or moved some of the standards to a different grade level. In fewer cases, standards have been added or deleted. In this book, we have designed a tool for teachers—the index in the front of this book—to find their state standards, see how they relate to the Common Core standard, and then find the relevant content in this book that will help them better understand what instruction around any mathematical content might look and sound like in the classroom.

The CCSSM, updated state standards, revised state standards, and rewritten state standards all have a few things in common. They include two critical components of learning mathematics: the content standards and the standards for mathematical practice, sometimes referred to as *process standards*.

The first group of standards—the content standards—explicitly outlines the mathematics we want students to know and be able to do at each grade level. In all cases, the content standards are fewer in number than most previous state standards, and the expectation is that students will develop deeper understanding of that content so less time is spent on reteaching from year to year. Additionally, the standards have been constructed to show connections among ideas at a grade level as well as vertical progressions across grades. For example, you will find that the standards in Grade 4 develop from the mathematical work that students have completed in previous grades. Similarly, the standards in Grades 7 and 8 develop into work that will be completed in high school. Thus it is important for teachers to be knowledgeable of the standards not only at the level they are teaching but also at the preceding grade level and the next grade level. This continuity is present across all states’ standards.

The second group—the standards for mathematical practice, referred to as *process standards* in some states—describes the habits of mind that students should develop as they do mathematics. Some states have maintained the National Council of Teachers of Mathematics’ (NCTM) original nomenclature of process standards, but the underlying principles are the same. These standards





remain the same across all grade levels, K–12. As teachers plan mathematics lessons they should consider how students will use the practices in learning and doing mathematics.

True across all states is that the standards *are not* intended to serve as a curriculum. Decisions about mathematics programs, textbooks and materials, sequencing topics and units, and instructional frameworks are left for local and state school districts to make. They do not tell teachers how to teach. It is important to remember that they describe what students need to know and be able to do. Schools and teachers know best how to help students reach both the content and the practice standards.

Further, there is no universal body that dictates specific assessments. Some states use assessments developed by SBAC (Smarter Balanced Assessment Consortium), AIR (American Institutes for Research), or PARCC (Partnership for Assessment of Readiness for College and Careers). Others have developed and are using their own assessments. You can find other facts and information at <http://www.corestandards.org> or on your state department of education website.

Instructional Shifts

While the standards do not call for a particular instructional approach or philosophy, they are based on three specific curricular foci that require shifts in instructional approach: *focus*, *coherence*, and *rigor*.

Focus: The content standards call for greater focus on fewer topics. An examination of the mathematics standards of high-performing countries indicates that fewer, more focused topics are taught at a grade level to allow students to deepen understanding of the mathematics and gain a stronger foundation for ongoing study of mathematics. At the high school level, the standards are organized into six *conceptual categories* (number and quantity, algebra, functions, modeling, geometry, and statistics and probability). The major mathematical work within each of those conceptual categories is described in the standards. However, the standards are organized by states into courses; the list of content standards for a particular course is not linear, nor is it a checklist. Instead, emphasis should be placed on achieving the major themes of the standards.

Coherence: The content standards also emphasize the view of mathematics as a coherent body of knowledge made up of topics that are all connected and build on each other. The call for coherence in the content standards ensures that there are carefully constructed progressions across the grades so students build new understanding on the foundations built in previous years. At the high school levels, each standard must be considered as a part of a larger web of understandings that students are developing within and across courses. This web exists both within conceptual categories and across conceptual categories so that students develop a coherent understanding of mathematics.

Rigor: The third instructional shift, *rigor*, refers to how we support students in developing deep understanding of each standard. Rigor is not built by merely assigning more worksheets of more difficult examples and problems. Rather, rigor calls for instructional practices that balance conceptual understanding, procedural skills, and applying mathematical ideas to a variety of contexts. The following descriptions of each component of rigor are from www.corestandards.org.

Conceptual Understanding: “The standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives in order to see math as more than a set of mnemonics or discrete procedures.” At the high school level, this suggests focus on underlying properties and principles rather than shortcuts such as “FOIL” for multiplying binomials.

Procedural Skills and Fluency: “The standards call for speed and accuracy in calculation. Students must practice core functions, such as single-digit multiplication, in order to have access to more complex concepts and procedures. Fluency must be addressed in the classroom or through supporting materials, as some students might require more practice than others.” At the high school level, many teachers observe that their students may not have developed the necessary speed and accuracy with basic computations from earlier grades. However, requiring remediation of those skills prior to addressing high school material will not allow sufficient time to achieve the high school standards. Instead, remediation should be embedded in instructions, or other supports—such as calculators that allow students to progress with high school level material—should be provided. Additionally, throughout the high school grades, attention should be given to developing basic procedures with each course that are needed for later work.

Applications: “The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.” At the high school level, this is particularly addressed through the modeling conceptual category. While this conceptual category does not include specific content standards, standards throughout the other conceptual categories are designated as addressing modeling.





Major Work of High School

The content standards for high school are broken into six conceptual categories or strands of mathematics. These conceptual categories describe the mathematics that students should learn across the high school grades, whatever course organization is used by a state or district. A brief summary of each conceptual category follows.

Number and Quantity includes work in four domains. The work from previous grades is extended to two new number systems: *The Real Number System* and *The Complex Number System*. Students also focus on *Quantities*—numbers with units based on measurement—and how they can be used to describe real-world situations. Finally, students learn to represent and model with *Vector and Matrix Quantities*, as well as perform operations with them.

Algebra builds students' use of algebraic symbols to describe mathematical and real-world contexts, with an emphasis on developing both conceptual and procedural fluency. *Seeing Structure in Expressions* focuses on meaningfully interpreting and creating equivalent expressions. *Arithmetic with Polynomials and Rational Expressions* includes seeing the polynomials as a mathematical system, understanding the relationship between zeros and factors of polynomials, using polynomial identities, and working with rational expressions, which are quotients of polynomials.

Functions describe relationships between quantities, where one quantity determines another. They are foundational to analyzing both mathematical and real-world situations. *Interpreting Functions* involves being able to understand functions and their symbolization, to interpret what a function tells us about a situation it describes, and to use different representations of a function to better understand its features. *Building Functions* involves creating a function to describe a relationship between two quantities, as well as to build new functions based on existing functions through various transformations and finding inverses. Particular classes of functions to be explored include *Linear, Quadratic, and Exponential Models* and *Trigonometric Functions*.

Modeling involves using mathematics (and statistics) to better understand problems that arise in everyday life or the workplace. Rather than having its own standards, attention to this conceptual category is to be integrated within the other conceptual categories. While Modeling is a Standard for Mathematical Practice that carries across all grades, K–12, Modeling is singled out at the high school level as a curricular goal in and of itself, implying that it should be more explicitly and intensively addressed.

Geometry is very useful in understanding both real-life situations and other areas of mathematics. In contrast to the typical approach in high school, geometry in the CCSSM is developed through an emphasis on transformations. In *Congruence*, students explore transformations and rigid motions that preserve congruence as a backdrop to developing more formal geometric proofs and constructions. *Similarity, Right Triangles, and Trigonometry* includes understanding similarity transformations and applying them to proofs involving similarity and to trigonometry. *Circles* focuses on theorems about circles, as well as measurements of circles. *Expressing Geometric Properties with Equations* uses algebra notation to describe conic sections and prove simple theorems. *Geometry Measurement and Dimension* particularly focuses on three-dimensional objects, and *Modeling with Geometry* explores the use of geometric models to explore real-world situations.

Statistics and Probability involves developing the understanding needed to make real-world decisions based on data. *Interpreting Categorical and Quantitative Data* involves analyzing data involving either count or measurement variables, including relationships between two such variables. *Making Inferences and Justifying Conclusions* focuses on understanding the importance of randomization in statistical experiments and drawing valid conclusions based on a statistical study. *Conditional Probability and the Rules of Probability* focuses on understanding and using independence, conditional probability, and the rules of probability. *Using Probability to Make Decisions* involves calculating and using expected values and other probabilistic models to evaluate real-world situations.

Note that these conceptual categories are not distinct but rather interact and overlap in many ways. For example, algebraic notation is used across all the conceptual categories, particularly functions, and geometry representations can be useful in visualizing ideas within other conceptual categories. Moreover, these content standards cannot be fully achieved without integrating attention to the standards for mathematical practice or process standards; the practices provide meaning and motivation for the content that is being studied.

Terminology Used in This Book

The language of the Common Core and other standards differs from traditional standards. Familiarity with the structure, descriptive terminology, and the functions of each section will help you make the best use of your standards. While what follows describes the Common Core Standards' structure, most states have utilized the same description in their updated standards. You will need to check your individual state standards to find any changes to these descriptors. Numbering and identification systems for your state standards may also be slightly different; though the index in the front of this book will help you correlate your standards with the original numbering system of the Common Core Standards.



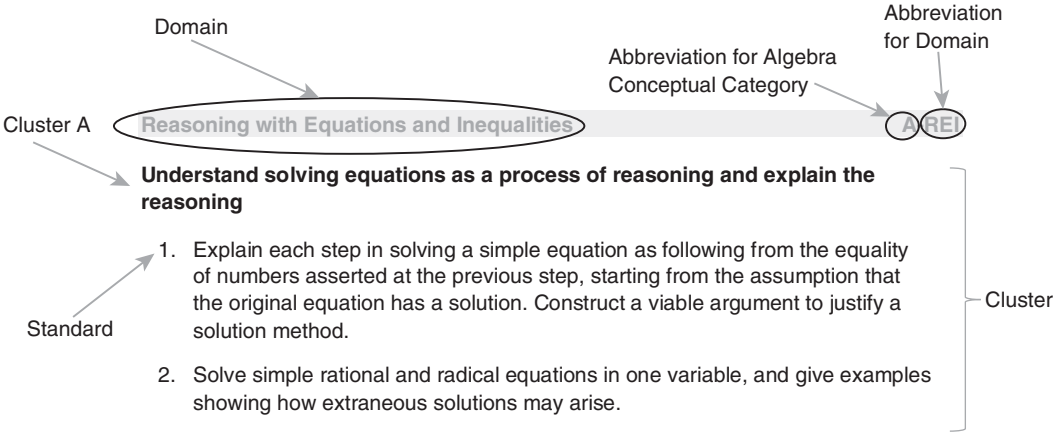
Standards define what student should understand and be able to do.

Clusters summarize groups of related standards. Be aware that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Be aware that standards from different domains may sometimes be closely related.

Conceptual categories define broad areas of high school mathematics.

In referring to Standard 1, this book uses the code “A.REI.A.1.” “A” refers to the Algebra conceptual category, “REI” refers to the Reasoning with Equations and Inequalities domain, “A” refers to the first cluster within the domain, and “1” refers to the standard number. When a standard includes parts, these are referred to as “a,” “b,” and so forth.



Source: Common Core State Standards for Mathematics (www.corestandards.org).

The Common Core Standards for Mathematical Practice (Processes)

The Common Core Standards for Mathematical Practice (Processes) describe the habits of mind teachers must incorporate in classroom instruction to help students develop a depth of understanding of critical mathematical concepts. The mathematical practices are not intended to be taught in isolation but should be integrated into instruction on a daily basis. These standards exemplify the type of mathematical thinking in which students should be engaged as they are developing mathematical understanding. The mathematical practices are not meant to be explicitly taught in isolation from important content; teachers, however, may want to highlight how they were used in a particular problem or context. While some lessons may particularly incorporate focus on one or two of the mathematic practices, other lessons may incorporate a number of these standards.

Throughout the following chapters, examples of the mathematical practices intended to be used are included in each cluster. The listed practices are not meant to limit lessons by using only those practices but provide examples of the key practices that can be included in lessons around that particular cluster. It is likely that teachers will address all the practices at some point throughout each cluster and domain.

The eight practices, briefly explained here, are essential for student success. If students are actively engaged in using these eight practices, they are learning rigorous, meaningful mathematics. Keep in mind that your state may have combined some of these or reverted to the five processes from NCTM’s *Principles and Standards for School Mathematics* (2000). Whether you are dealing with the practice standards or the process standards, you will find the foundational practices (problem solving, communication, connections, representations, and reasoning and proof) to be very similar across all these standards.

SFMP 1. Make sense of problems and persevere in solving them.

High school students should work to understand what a problem is asking, choose a strategy to find a solution, and check the answer to make sure it makes sense. When unable to immediately identify a strategy that will work or when their selected strategy does not



work as intended, they must learn to persist in trying a range of potential approaches, looking for how the current problem may relate to previous work they have done. Solving problems is the essence of mathematical work.

SFMP 2. Reason abstractly and quantitatively.

High school students need to make sense of quantities and their relationships in problem situations. Particular attention to units associated with quantities is essential to understanding how the quantities are related. For example, a linear function relating the distance traveled to time needs to specify the units, such as feet and seconds. The slope of that function then gives the rate of change (velocity) given in feet per second. Students should consistently consider the reasonableness of their answer within the context of the problem; if a new car is found to cost \$10, the model used to find that answer should be questioned.

SFMP 3. Construct viable arguments and critique the reasoning of others.

High school students are increasingly expected to make formal mathematical arguments based on stated assumptions or properties, well-defined definitions, and previously established results. While this has traditionally focused on *writing proofs* related to geometry, careful attention to the reasoning underlying mathematical procedures or statements should be incorporated across the curriculum, and students should be increasingly expected to make formal arguments as they progress through the high school grades. Moreover, experience with critiquing the arguments produced by classmates is essential to their mathematical development. Reasoning undergirds deep conceptual understanding.

SFMP 4. Model with mathematics.

High school students need to learn to use mathematics to address problems in everyday life, society, and the workplace. This should occur at a range of levels, from more specific application of mathematical ideas to full-scale mathematical modeling, as described in Section 2.

SFMP 5. Use appropriate tools strategically.

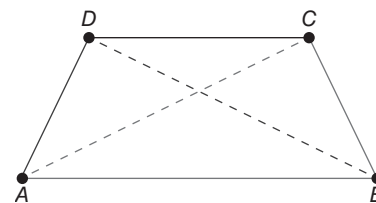
High school students need to be comfortable in using applicable tools when solving a mathematics problem. A range of technological tools should be available, including graphing calculators and software, computer algebra systems, spreadsheets, dynamic geometry software, and statistical software. Although some view them as less necessary for older students, physical manipulatives, such as algebra tiles and geometric models, can be very useful for many, if not all, students and should be incorporated into the classroom. In addition, students should be comfortable using paper-and-pencil representations, such as tables, graphs, and other visual representations.

SFMP 6. Attend to precision.

High school students need to learn to communicate effectively with others, using precise vocabulary. They should use symbols to represent their thinking, clearly describing the meaning of those symbols. As stated in the Common Core State Standards, “By the time they reach high school they have learned to examine claims and make explicit use of definitions.” They also need to be able to express the precision of the answers they give in real-world contexts, based on the accuracy of the given information; using all the digits on the calculator display of an answer is very likely not appropriate.

SFMP 7. Look for and make use of structure.

High school students examine mathematical situations to detect a pattern or structure that may provide further insight. For example, as stated in the Common Core Standards, “They can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .” Likewise, drawing an auxiliary line in a geometric figure may help them to better understand the structure behind a situation. For example, drawing in the diagonals of an isosceles trapezoid creates two overlapping triangles ABC and BCA as shown in the following figure; if we can demonstrate their congruence, other properties of the figure will become apparent.





SFMP 8. Look for and express regularity in repeated reasoning.

High school students notice patterns in calculations that are performed in order to form generalizations, as well as shortcuts, for calculation. For example, as shown in the Common Core Standards, high school students may note that when multiplying expressions of the form $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, $(x - 1)(x^3 + x^2 + x + 1)$, and so forth, all but the leading and constant terms cancel; this could lead to a general formula for geometric series.

Effective Mathematical Teaching Practices

Quality mathematics teaching is a critical key for student success. In *Principles to Actions* (2014), the National Council of Teachers of Mathematics outlined eight effective mathematical teaching practices that teachers should incorporate to help their students achieve both the content standards and the standards for mathematical practice. These eight research-informed practices are briefly explained as follows.

1. Establish mathematics goals to focus learning.

Effective mathematics teaching begins with the establishment of clear goals of what mathematics students should know and be able to do. These goals are grounded in the trajectory of the standards being addressed in a lesson, series of lessons, and an overall unit. Teachers must keep in mind what is to be learned, why the goal is important, and where students need to go, as well as how learning can be extended. Students must clearly understand the purpose of the lesson beyond repeating the standard.

2. Implement tasks that promote reasoning and problem solving.

Implementing tasks that promote reasoning and problem solving provides opportunities for students to explore mathematics in ways that build on their existing understanding. Rather than providing straightforward solution strategies, tasks should provide students with multiple entry points to their solution. Students should be encouraged to use a variety of approaches and strategies while maintaining a high level of student engagement and cognitive demand.

3. Use and connect mathematical representations.

Teachers should support students in developing facility with a range of mathematical representations and to then select which they find most useful in a given situation. Moreover, they should help students develop connections among representations of a problem and to focus on the essential features of the problem. Developing these representations cannot be rushed, and adequate time has to be provided for students to use, discuss, and make connections among representations.

4. Facilitate meaningful mathematical discourse.

Teachers should engage students in productive discussions about how they solved a problem, including the reasoning and representations that they used. A classroom environment that positions discussions of student thinking as central is essential. Moreover, teachers need to carefully orchestrate the discourse so that progress is made toward the mathematical goals of the lesson.

5. Pose purposeful questions.

Teacher questioning is central in advancing students' mathematical understanding. Rather than focusing on low-level questions that require factual responses, teachers should strive to use open-ended questions that promote depth of the students' thinking. Moreover, teachers should avoid "funneling" students to certain desired answers, instead asking questions that focus on making student thinking visible. Providing adequate wait time is essential.





6. Build procedural fluency from conceptual understanding.

Many well-meaning teachers provide lessons that promote students' conceptual understanding, but then have their students work on developing procedural skill without connection to their conceptual understanding. The result is that the conceptual understanding may be seen by students as irrelevant. Moreover, students cannot easily access their conceptual understanding when unable to recall procedural skills. True procedural fluency is built on and connected to deep conceptual understanding.

7. Support productive struggle in learning mathematics.

All too often, students feel that success in mathematics is tied to how quickly they can find the answer to a problem, and when the method to find an answer is not readily apparent, they may quickly give up. Teachers need to actively encourage their students to persevere in solving problems, giving them adequate time to struggle rather than providing cues or hints that undercut their persistence. Teachers need to help their students see that effort is an essential component in mathematical success and that there is more to mathematics than finding answers.

8. Elicit and use evidence of student thinking.

Eliciting and using evidence of student thinking helps teachers assess their students' progress in reaching their mathematical goals and to adjust instruction as needed to support their students' learning. Informal assessments can provide valuable information, including observations of student discussions, brief interviews with specific students, exit slips asking students to summarize their current thinking, and journal writing.

How to Use This Book

The overall goal of this book is to help teachers more deeply understand the mathematical meaning behind the high school standards, including a general understanding of the domains within each of the conceptual categories, as well as more specific understanding of the clusters and the standards within those domains. The intent of the resource is to use it as your personal toolkit for teaching of your mathematics standards. Blank space on numerous pages has been left for you to take notes, add ideas, and reference other resources that may be helpful.

The book is organized by the conceptual categories described in the high school standards. Each of these sections begins with a general description, along with suggested materials and overarching vocabulary. This is followed by a general discussion of each domain within the conceptual category, including key vocabulary organized by domain. The discussion of each domain continues with a description of each cluster within the domain, and how the Standards for Mathematical Practice can be incorporated into your teaching of the cluster follows. Because the standards are intentionally designed to connect within and across domains and grade levels, a list of related standards is included in the cluster overview. As you prepare work on a cluster, we suggest you look at these standards to have a better idea of the mathematics students learned in previous grades and where they are going in the future. A list of all the standards is found in the Quick Reference Guide at the beginning of the book.

Each standard within a cluster is explained with a section called *What the TEACHER does* followed by a description of *What the STUDENTS do*. It is important to note that most standards will take several days, and you should be connecting conceptual understandings across standards and domains as you teach for understanding. Addressing student misconceptions and common errors in developing student understanding of a concept concludes the contents for each standard.

At the end of each domain, you will find a sample planning page based on one standard for that domain.

In the resource section, you will find reproducible key materials. These are designed to be samples, and we encourage you to use them or redesign them to best meet the needs of your students. Also included is a planning page template for you to duplicate and use for planning. You will find downloadable versions of the reproducibles at resources.corwin.com/yourmathcompanion9-12. Finally, you'll find a list of our favorite resource books and high quality online resources that are particularly useful for developing mathematical ideas in high school.

Most important, we have aligned all these materials with the updated standards from most states not identified as a Common Core State. If you teach in a state that is not using the Common Core Standards, you can find the standards listed for your state and





your grade in the beginning of this book. Read across the table to see how any given standard relates to the original Common Core version and to see where to find the guidance in this book to help you more deeply understand how a mathematical concept or skill should be developed with your students. You'll note that in a few cases there are uncorrelated standards that we list but do not address in this book.

We believe that this can become your mathematics standards bible. Read it, and mark it with questions, comments, and ideas. We hope that this resource will help you use these standards and good teaching practices to lay the essential foundation that will ensure your students' success not only in your grade but in all their future study of mathematics.

