Part 1

Ratios and Proportional Relationships

Ratios and Proportional Relationships

Domain Overview

GRADE 6

Sixth graders are introduced to ratio, a relationship or comparison of two quantities or measures. Students represent ratios in various forms and compare types of ratios. At this level, they use reasoning about multiplication and division to solve ratio and rate problems about quantities. Students learn how and where ratios and rates are used in the real world.

GRADE 7

Continuing to develop an understanding of operations with rational numbers, seventh graders describe situations in which opposite quantities combine to make zero and determine the absolute value for a given number. Students estimate solutions, then add, subtract, multiply, and divide integers in the context of real-world problems. Given a real-world context, students simplify an expression using four integer operations and the order of operations.

SUGGESTED MATERIALS FOR THIS DOMAIN

6	7	
✓	✓	Common objects such as tennis shoes, cereal boxes, etc.
	✓	Copies of restaurant menus
✓		Counters (two-color, chips, etc.)
✓	✓	Graph paper
✓		Newspapers or grocery ads
✓		Percent wheel

KEY VOCABULARY

6	7	
✓	✓	**commission** a percentage of sales paid to a salesperson
✓		**complex fraction** a fraction with a fraction in the numerator and/or a fraction in the denominator
✓	✓	**constant of proportionality** same as unit rate
	✓	**coordinate plane** a plane formed by the intersection of a horizontal number line (called the *x*-axis) with a vertical number line (called the *y*-axis). The number lines intersect at their zero points, called the origin
	✓	**covariance** a measurement of how related the variances are between two variables. The extent to which any two random variables change together or vary together.
✓	✓	**discount** amount a store takes off of the original price of an item. It is usually expressed as a percent or fraction.
✓		**double number lines** two number lines used when quantities have different units to easily see there are numerous pairs of numbers in the same ratio
✓	✓	**equation** a mathematical statement of the equality of two mathematical expressions. An equation uses a sign stating two things are equal (=).
✓		**equivalent ratios** ratios that have the same value
✓		**gratuity** tip
✓	✓	**markdown** a reduction in price
✓	✓	**markup** the difference between the wholesale price and the selling price
✓	✓	**origin** on a coordinate plane, the point (0, 0)
✓	✓	**percent** a ratio per 100 such as 25% is 25 parts of 100

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KEY VOCABULARY

6	7	
✓	✓	**percent error** the ratio of the error compared to the exact value. For example, my estimate was off by 7. The exact value was 35, so the percent error is $\frac{7}{35} \times 100\% = 20\%$.
✓	✓	**percent increase/decrease** the amount of increase or decrease expressed as a percent of the original amount
✓	✓	**proportion** two equal ratios
✓	✓	**proportional reasoning** multiplicative reasoning as opposed to additive reasoning. It is often used when finding the better buy, sharing two items with three students, adjusting calculations of travel time with different speeds, or calculating a sale price when everything is 40% off.
✓		**rate** ratio that compares two quantities of different units such as 3 ft per second
✓	✓	**ratio** comparison of two quantities
✓		**ratio language** language used to describe a ratio relationship in number or quantity between two things such as "*For every vote candidate A received, candidate C received nearly three votes*"
	✓	**ratio table** a table that shows the relationships between different ratios and/or a comparison of two or more quantities
✓	✓	**simple interest** the formula is $I = prt$, where I is interest, p is principle, r is rate, and t is time
✓		**simplify a ratio** divide each number in the ratio by its greatest common factor; $\frac{2}{6}$ simplifies to $\frac{1}{3}$ by dividing 2 and 6 by 2
✓		**tape diagram** a drawing that looks like a segment of tape, used to illustrate number relationships; Also known as a strip diagram, bar model, fraction strip, or length model
✓	✓	**unit rate** ratio comparing an amount to one

Notes

Ratios and Proportional Relationships
6.RP.A*

Cluster A

Understand ratio concepts and use ratio reasoning to solve problems.

STANDARD 1

6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

STANDARD 2

6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]

[1]Expectations for unit rates in this grade are limited to non-complex fractions.

STANDARD 3

6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

*Major cluster

Ratios and Proportional Relationships 6.RP.A

Cluster A: Understand ratio concepts and use ratio reasoning to solve problems.

Grade 6 Overview

The focus for this cluster is the study of ratio concepts and the use of proportional reasoning to solve problems. Students learn how ratios and rates are used to compare two quantities or values and how to model and represent them. Sixth graders find out how ratios are used in real-world situations and discover solutions to percent problems using ratio tables, tape diagrams, and double number lines. Students also convert between standard units of measure.

Standards for Mathematical Practice

SFMP 1. Make sense of problems and persevere in solving them.

Sixth graders interpret and solve ratio problems.

SFMP 2. Reason abstractly and quantitatively.

Students solve problems by analyzing and comparing ratios and unit rates in tables, equations, and graphs.

SFMP 4. Model with mathematics.

Students model real-life situations with mathematics and model ratio problem situations symbolically.

SFMP 6. Attend to precision.

Students communicate precisely with others and use clear mathematical language when describing a ratio relationship between quantities.

SFMP 7. Look for and make use of structure.

Sixth graders begin to make connections between covariance, rates, and representations showing the relationships between quantities.

Related Content Standards

4.OA.2 5.NF.3 5.G.1 5.G.2 5.MD.1 6.EE.9 7.RP.A.1

Notes

STANDARD 1 (6.RP.A.1)

Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

In this standard, students learn to compare two quantities or measures such as 6:1 or 10:2. These comparisons are called ratios. Students discover that ratios can be written and described in different ways. For instance, 6:1 uses a colon to separate values. Ratios can also be stated with words such as 6 to 1, or as a fraction such as $\frac{6}{1}$. Standard 1 focuses on understanding the concept of a ratio, however, students should use ratio language to describe real-world experiences and use their understanding for decision making.

What the TEACHER does:

- Help students discover that a ratio is a relationship or comparison of two quantities or measures. Ratios compare two measures of the same types of things such as the number of one color of socks to another color of socks *or* two different things such as the number of squirrels to birds in the park. Ratios compare parts to a whole (part:whole) such as 10 of our 25 students take music lessons. Ratios can also compare a part of one whole to another part of the same whole (part:part) such as the ratio of white socks in the drawer to black socks in the drawer is 4:6. Ratios are expressed or written as *a* to *b*, *a*:*b*, or $\frac{a}{b}$.
- Compare and model ratios with real-world things such as pants to shirts or hot dogs to buns. Ratios can be stated as the comparison of 10 pairs of pants to 18 shirts and can be written as $\frac{10}{8}$, 10 to 18, or 10:18 and simplified to, $\frac{5}{9}$, 5 to 9, or 5:9. Ensure that students understand how the simplified values relate to the original numbers.
- Ask students to create or find simple real-world problems to use in their learning such as, "*There are 2 Thoroughbred horses and 6 Appaloosa horses in the field. As a ratio of Thoroughbreds to Appaloosas it is:* $\frac{2}{6}$ *or 2 to 6 or 2:6 or simplified as* $\frac{1}{3}$*, 1 to 3, or 1:3. Or, there are 14 girls and 18 boys in our math class. As a ratio of girls to boys it is:* $\frac{14}{18}$*, 14 to 18, or 14:18 or simplified as* $\frac{7}{9}$*, 7 to 9, or 7:9.*" Invite students to share their real-world examples of ratios and use ratio language to describe their findings such as, "*for every vote candidate A received, candidate C received nearly three votes.*" The problems students select or write can also be used as cyclical reviews with distributed practice throughout the school year.
- Focus on the following vocabulary terms: *ratio*, *compare*, and *simplify*.

What the STUDENTS do:

- Understand that a ratio is a comparison between quantities.
- Determine when a ratio is describing part-to-part or part-to-whole comparison.
- Describe ratio relationships between two quantities using ratio language.
- Use the different ratio formats interchangeably (4:5, 4 to 5, $\frac{4}{5}$).

Addressing Student Misconceptions and Common Errors

Some sixth graders may confuse the order of the quantities such as when asked to write the ratio of boys to girls in the sentence, "*There are 14 girls and 18 boys in our math class*." Instead of writing 18:14, some students may write 14:18. Other students may not recognize the difference between a part-to-part ratio and a part-to-whole ratio such as, "*There are 14 girls compared to 18 boys in the class (14:18 part-to-part); however, 14 of the 32 students in our class are girls (14:32 part-to-whole)*." To address these common misconceptions, ask students to label the quantities they are comparing such as 14 girls/18 boys.

STANDARD 2 (6.RP.A.2)

Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $\frac{3}{4}$ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."[1]

[1]Expectations for unit rates in this grade are limited to non-complex fractions.

This standard focuses student learning on the concept of a unit rate as a special kind of ratio. Students compare different units of measure such as the amount of money earned to the hours worked while babysitting and calculate unit rates by setting up ratios and simplifying them. Students understand a situation in ratio form and write the unit that describes the situation using appropriate rate language with words such as *per* and symbols such as / to compare different units or measures.

What the TEACHER does:

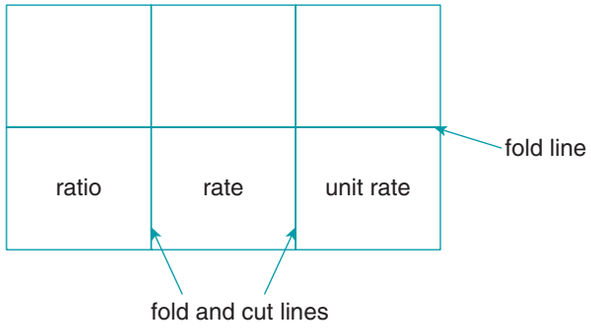
- Begin by exploring the difference between a ratio and a rate. Rate is a special ratio that compares two quantities with different units of measure. Share multiple examples for students to make sense of the concept for rate such as, "*LaShanda babysat for $35 for 7 hours*." Or, "*Dad's new truck got 400 miles on 20 gallons of gas*." Then explore the unit rate that expresses a ratio as part-to-one. Generate examples such as "*LaShanda is paid a unit rate of $5 per 1 hour for babysitting (5:1)*" and "*My dad's new truck gets 20 miles per gallon of gas (20:1)*."
- Ask students to locate and share real-world examples of cost per item or distance per time in newspapers, ads, or other media. (Note that in sixth grade, students do not work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.)
- Model how to convert rates from fraction form to word form using per, each, or @ such as 360 miles/12 gallons of gas = 30 miles per gallon of gas. Allow students to talk with each other and their teacher to make sense of what they are learning and then write and share several rate conversion examples of their own.
- Focus on the following vocabulary terms: *ratios*, *rates*, *unit rates*, *compare*, and *per*/@. Math journals or exit slips at the end of a math class with writing prompts such as, "*An example of a ratio and a problem that goes with it is. . . .* " provide closure.
- Provide cyclical, distributed practice over time to continually review simple unit rate problems.

What the STUDENTS do:

- Understand rate as a ratio that compares two quantities with different units of measure.
- Understand that unit rates are the ratio of two measurements or quantities in which the second term means "one" such as 60 miles per one hour.
- Interpret rate language with the @ symbol or with the words *per* and/or *each*.
- Solve unit rate problems.

Addressing Student Misconceptions and Common Errors

Students often confuse the terms *ratio*, *rate*, and *unit rate*. Try using a paper foldable with vocabulary definitions to help students with these confusing terms. To make the foldable, divide an $8\frac{1}{2}$ × 11-inch sheet of blank paper in half horizontally. Then fold it into thirds as if a letter is being folded to fit an envelope. Unfold and write a term on each of the sections. On the inside of the foldable, write the definitions that match each term. Students may want to cut on the vertical fold lines to flip up each section to practice the definitions.



STANDARD 3 (6.RP.A.3)

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. *Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.*

b. *Solve unit rate problems including those involving unit pricing and constant speed.* For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

c. *Find a* ***percent*** *of a quantity as a rate per 100 (e.g., 30% of a quantity means* $\frac{30}{100}$ *times the quantity); solve problems involving finding the whole, given a part and the percent.*

d. *Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.*

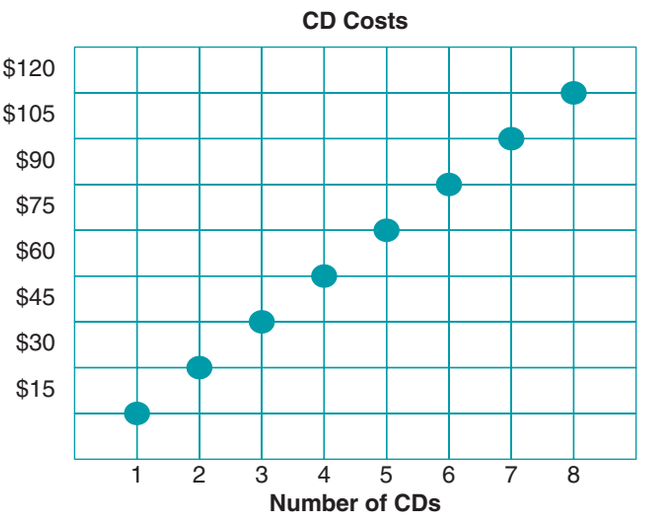
In these standards, students use reasoning about multiplication and division to solve a variety of ratio and rate problems about quantities. They make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. They use tables to compare ratios and solve unit rate and constant speed problems. Problems involving finding the whole given a part and the percent, such as 20% of a quantity means $\frac{20}{100}$, are also a focus. For these standards, students can use equivalent ratio tables, tape diagrams, double number lines, or equations. Students connect ratios and fractions.

What the TEACHER does:

- Explore ratios and rates used in ratio tables and graphs to solve problems. Pose a ratio situation problem with students such as "*3 CDs cost $45. What would 8 CDs cost? How many CDs can be purchased for $150.00?*" To solve the problem, students can use ratios, unit rates, and multiplicative reasoning by creating and filling in the missing values on a chart. They should note that if three CDs cost $45, one CD will cost $15. Every CD purchased is an additional $15. $15 times the number of CDs = the cost. They write an equation such as C = $15*n*.

# of CDs	Cost
3	$45
8	??

- Ask students to plot the points on a coordinate plane and draw conclusions about what is happening with the problem above. Students should reason that for every one movement to the right on the *x*-axis, the *y*-axis increases to 15*x*. Also, for every one movement to the left on the *x*-axis, the *y*-axis decreases by 15.



- Investigate unit rate problems, including unit pricing such as, "Quick Stop *has 12-oz. drinks for* $.99. Stop Here *has 16-oz. drinks for $1.19. Which drink costs the least per ounce?*" Assign students to create ratio and rate reasoning examples to compare and solve real-world problems. Students could use newspapers, store ads, or online ads to find the examples and make the comparisons. Ask students to use reasoning to determine the better buys.

- Explore finding a percent of a quantity as a rate per 100 such as 40% of a quantity means $\frac{40}{100}$ times the quantity. Noting that a percent is a rate per 100, model how a

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What the TEACHER does (continued):

percent can be represented with a hundreds grid by coloring in 40 units. Have students write this as a fraction ($\frac{40}{100}$), a decimal (0.40), and a percent (40%). Consider using a percent wheel (see Reproducible 1) or use double number lines and tape diagrams in which the whole is 100 to find the rate per hundred.

- Solve problems involving finding the whole, given a part and the percent such as, "*What is 40% of 60? 80% of what number is 300? Or 50 is 30% of what number?*"
- Examine the process of how to use ratio reasoning to convert measurement units such as, "*How many centimeters are in 5 feet?*" Use the information that 1 inch ≈ 2.54 cm. Represent the conversion of 12 inches = 1 ft as a conversion factor in ratio form, $\frac{12 \text{ inches}}{1 \text{ foot}}$.

$$\text{Then multiply } \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{5 \text{ ft}}{1} = 60 \text{ inches.}$$

$$\text{Then } 60 \text{ inches} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 152.4 \text{ cm.}$$

(Note that conversions can be made between units within a measurement system such as inches to feet or between systems such as miles to centimeters.)

- Allow students to talk with each other and their teacher to make sense of what they are learning.
- Focus on the following vocabulary terms: *ratios*, *rates*, *unit rates*, *equivalent ratios*, *percents*, *ratio tables*, and *tape diagrams*.
- Provide cyclical, distributed practice over time to continually practice unit rate problems.

What the STUDENTS do:

- Create and interpret a table of equivalent ratios.
- Plot pairs of values from a table to a coordinate plane.
- Use a table to compare ratios and find missing values using ratios.
- Explain the difference between a ratio and a unit rate.
- Understand that rate problems compare two different units, such as revolutions per minute.
- Solve real-world problems using ratios and rates.
- Reason to determine the better buy.
- Write a percent as a rate over 100, including percents greater than 100 and less than 1.
- Find the percent of a number using rate methods.
- Represent the relationship of part to whole to describe percents using models.
- Convert units by multiplication or division.

Addressing Student Misconceptions and Common Errors

Some sixth graders misunderstand and believe that a percent is always a natural number less than or equal to 100. To help with this misconception, provide examples of percent amounts that are greater than 100% and percent amounts that are less than 1%. Try using a percent wheel for developing this understanding. See Reproducible 1.

Notes

Sample PLANNING PAGE

6.RP.A

Ratios and Proportional Relationships

Cluster A: Understand ratio concepts and use ratio reasoning to solve problems.

Standard: 6.RP.A.1. *Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.* For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

Standards for Mathematical Practice:

SFMP 2. Reason abstractly and quantitatively.

Students solve problems by analyzing and comparing ratios and unit rates in tables, equations, and graphs.

SFMP 4. Model with mathematics.

Students model real-life situations with mathematics and model ratio problem situations symbolically.

SFMP 6. Attend to precision.

Students communicate precisely with others and use clear mathematical language when describing a ratio relationship between quantities.

Goal:

Students use real-world objects to compare two quantities such as the number of red candy pieces to the number of green candy pieces in the same bag (part to part) and the number of parts to a whole such as the number of red candy pieces compared to the total number of candy pieces in the entire bag (part to whole).

Planning:

Materials: plastic bags with approximately 35–40 pieces of M & M's™ candies or 1-inch color tiles, paper and pencil to write the ratios

Sample Activity

- Give each student a bag of M & M's™ to compare and model ratios. Divide students into partner pairs. Ask them to write a ratio comparing the number of M & M's™ they have to the number their partner has. Students will count and compare the number of M & M's™ each have in their own bags and then write the comparison such as $\frac{36}{40}$ or 36:40. Facilitate a discussion about how they just compared a whole to a whole.
- Next, ask students to compare the number of red M & M's™ in their bags to the number of brown M & M's™. Students will count and record comparisons such as 8 red/14 brown or 8:14. Facilitate a discussion about how this ratio compares a part of one whole to another part of the same whole (part to part). Ask students to create their own part to part ratios with their M & M's™ and record.
- Ask students to count the number of yellow M & M's™ and compare that number to the entire number of M & M's™ in the bag, such as 7 yellow compared to all 36 in the bag. Have students record the ratio such as $\frac{7 \text{ yellow}}{36 \text{ bag}}$ or 7:36. Facilitate a discussion leading students to reason that the ratio they just created is a part:whole ratio. This can be done by reviewing the other types of ratios created earlier in this lesson.

Questions/Prompts:

- Ask students to explain the comparisons of 8 red compared to 14 brown ($\frac{8}{14}$ and 8:14 vs. $\frac{14}{8}$ and 14:8).
- Ask students to compare other colors to show the relationship written as a ratio.
- Ask students to explain part-to-part versus part-to-whole ratios.

Differentiating Instruction:

Struggling Students: Some students may confuse the order of the quantities and may need to label, such as 8 red/14 brown or 14 brown/8 red. Have the students record the color order you request before they make the ratio. This will help them understand that the order matters.

Extension: Try other ratio scenarios. Direct students to look around the classroom to find the ratio of boys to the total number of students in the classroom. Have them compare the number of boys to the number of girls. Ask them to compare the number of students in their classroom to the entire sixth grade or find the ratio of sixth graders to seventh graders.

Notes

PLANNING PAGE

6.RP.A

Ratios and Proportional Relationships

Cluster A: Understand ratio concepts and use ratio reasoning to solve problems.

Standard:

Standards for Mathematical Practice:

Goal:

Planning:

Materials:

Sample Activity:

Questions/Prompts:

Differentiating Instruction:

Struggling Students:

Extension:

Notes