



PRE-K–GRADE 2

# NAVIGATING *through* GEOMETRY

## Chapter 4 Visualization, Spatial Reasoning, and Modeling

Visualization refers to a student's ability to construct a single image or a series of related images, whether the image is of an object or a set of directions. Sometimes visualization involves imaging what an object looks like from a different point of view, for example, from above rather than from the side. Other times, it involves predicting what an object will look like after it is turned or flipped or what the shadow of an object might look like. On other occasions, students are visualizing a path leading from one position to another or trying to determine where someone was standing to see a particular object from a given perspective. Visual memory—the ability to recall what a shape looked like or what has been changed about a shape or an arrangement—is prerequisite to the ability to manipulate mental images, an important part of spatial sense (Clements 1999).

Although visualization is usually considered in the context of geometry, visualization and spatial reasoning are significant in many other areas of mathematics, as well. For example, students who use dominoes gain experience in quickly recognizing sets of a particular size. Similarly, a student who can visualize, for example, that a set of two counters to the left and three to the right can also be viewed from the other side as three to the left and two to the right is in a much better position to make sense of the commutative property of addition. “Positive correlations have been found between spatial ability and mathematics achievement at all grade levels.... It is not difficult to see why this relationship exists for there are numerous concepts in mathematics that have an obvious visual dimension.” (Clements and Battista 1992, p. 443).

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It takes students time and experience to develop the ability to imagine an object from different points of view. Students can look at a cylinder from above and see a circle and from below and see a circle, but from the side, the cylinder looks like a rectangle. Students might build simple small structures with linking cubes and describe or draw what they look like from the four different sides and from above or below. The activity *Block Views* helps students image these perspective views.

The activity *Projector Math* offers students opportunities to develop their visual memory. Students might be asked to describe how an object or arrangement has been changed after quick looks at it before and after the change. They can look at an object briefly and then pick out what they saw from a group of possible objects. They can look at an object and simply describe its attributes.

In *Cutouts*, students have opportunities to predict what a folded sheet of paper will look like when part of it is cut away and it is then unfolded. Students need to cut and unfold the shape mentally if they are to cut a folded paper to create a given unfolded shape.

The activity *Skeletons* requires students to model three-dimensional shapes, focusing on the edges and vertices of the shape. Students often focus on the faces of a solid, so this activity serves to broaden their visualization skills.

## Shapes in the Environment

Teachers can employ many resources to encourage students to observe shapes in their environment. The activity *Projector Math* suggests that students consider where in their surroundings they would find examples of simple three-dimensional shapes. Children's picture books like the ones created by Tana Hoban (1973, 1986) are particularly attractive to children. Teachers should provide resources in which both two- and three-dimensional shapes are considered.

## Expectations for Students' Accomplishment

By the end of grade 2, students should be able to recognize images of a simple three-dimensional structure from different perspectives, create and interpret simple maps, navigate simple paths, and predict the result of unfolding a simple folded cutout. They should also be able to describe two- and three-dimensional shapes when seen briefly and to recognize them in the environment. Students should also be able to recognize changes made in arrangements of shapes.

# Block Views

## Prekindergarten–Kindergarten

### Summary

Given one or more pictures of a simple block structure, students build a structure that matches the pictures.

### Goals

- Identify three-dimensional objects from two-dimensional drawings
- Identify relationships among positions of blocks from drawings and recreate those relationships using blocks

### Prior Knowledge

- Naming and counting the faces of triangular right prisms (two triangles and three rectangles), right rectangular prisms (either six nonsquare rectangles or two squares and four nonsquare rectangles), and cubes (six squares)
- Using positional vocabulary: *above*, *below*, *between*, *next to*, *in front of*, *behind*

### Materials

- The following building blocks for each group of students: at least forty-five small cubes, ten larger cubes, two long and three short right rectangular prisms, seven cylinders, two equilateral triangular prisms, and four right triangular prisms
- A copy of the following blackline masters for each student: “Block Views A and B,” “Block Views C and D,” and “Bird’s-Eye Views”

### Activity

#### Engage

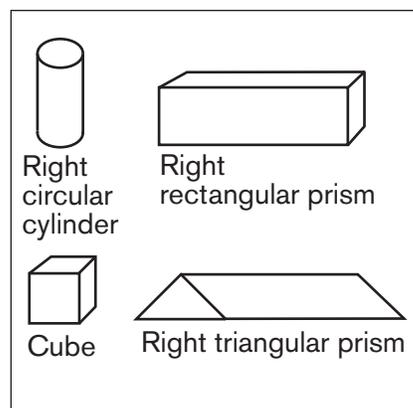
Hold up an example of each type of block, and have the students count the number of faces and identify the shapes of the faces. If the students do not know the names of the blocks, have them say the names in unison with you. You may want to make a display of the types of blocks with their names printed below each one for reference throughout the year. Although many students will not be able to read the names, constant exposure to the names in print and in conversation is important at this early stage. Call on students to find the blocks that have the following attributes:

- Some faces that are squares (cubes, some noncubic rectangular right prisms)
- Some faces that are rectangles (cubes, noncubic rectangular right prisms, triangular right prisms). *Note:* Squares are a special type of rectangle.
- All faces that are squares (cubes)

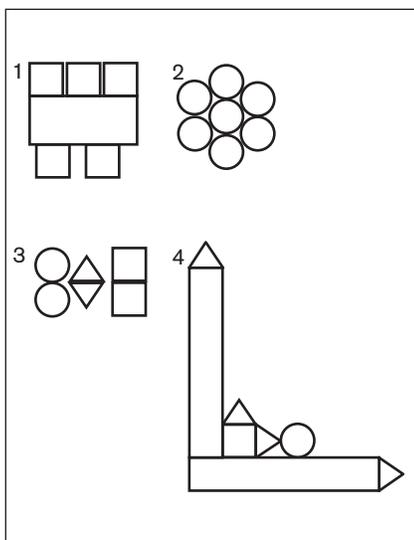
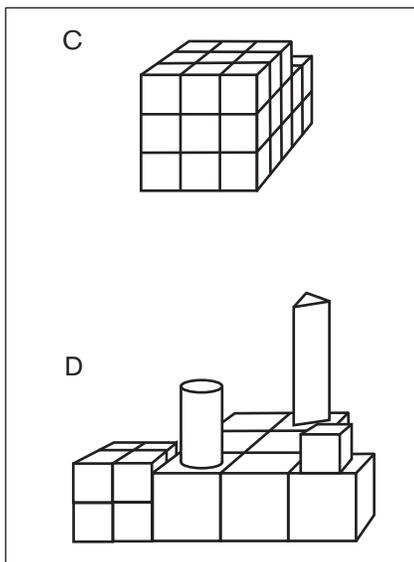
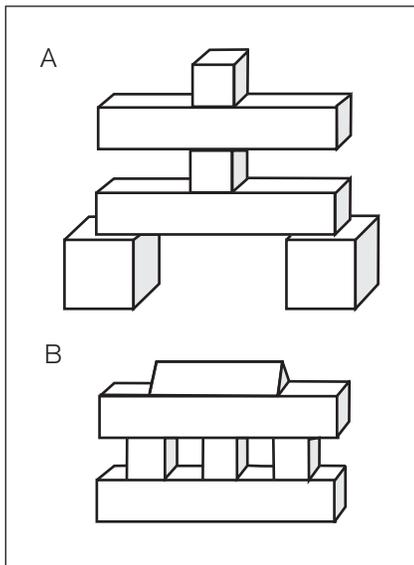


*Recognize and represent shapes from different*

*perspectives*



*p. 89, 90, 91*



- All faces that are rectangles (cubes and noncubic rectangular right prisms)

Build a simple structure with several blocks, and have the students duplicate it. Discuss with them the clues they used to help them create the duplicate.

### Explore

Show the students construction A on “Block Views A and B.” Tell them that their job is to make the construction using building blocks. Call on one student to find the blocks needed to make the construction and on another student to construct it. Then have the other children check the construction by comparing it to the picture. Be sure that the blocks are positioned correctly and match the picture.

Seat the students in groups of four, and give each student a copy of “Block Views A and B.” Have each student build structure A and then structure B. Walk around to check the constructions to see if any students are experiencing difficulty, that is, if they have used the wrong blocks or not enough blocks or if they have arranged the blocks incorrectly. If you observe any of these errors, have the students count the number of blocks they need, identify those that are the same shape, and point to the blocks that are *on top of*, *next to*, *between*, or *under* other blocks. Some students may gather all the required blocks before beginning construction; others may select the blocks as needed. Either method is fine.

Once the students have completed structure B, give them each a copy of “Block Views C and D” and have them build structure C and then D. Unlike for structures A and B, not every block needed for constructing C and D can be seen. For this reason, C and D are more complex than A and B.

### Extend

“Bird’s-Eye Views” shows a top view of four block constructions. The students should use blocks to build the structures that, when viewed from above, look like the pictures. You may want to have the students work in pairs to build the Bird’s-Eye structures so that they can check each other’s buildings.

Note that there are no depth clues in “Bird’s-Eye Views.” Thus, it is possible for the blocks to be at different heights. For this reason, more than one set of blocks could be used to make each building. For example, the small squares in view 1 could be faces of cubes or noncubic rectangular prisms and the prisms could be of different heights. All the square faces, however, must be congruent. Have the students talk about their constructions and describe the differences among them.

## Discussion

Block Views develops students’ abilities to view blocks and constructions from different perspectives and to identify relationships among positions of blocks. As they describe relative positions, students are developing their abilities to use spatial terminology correctly. They are also identifying the shapes of the faces of blocks. Most important, students are gaining familiarity with two-dimensional representations of three-dimensional shapes and constructions made from those shapes.