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Counting & Cardinality and Number & Operations in Base Ten

DIFFERENTIATED LEARNING activities in counting and cardinality and in number and operations in base ten are derived from applying standards for mathematical practice to the content goals that appear in the Counting & Cardinality domain for Kindergarten and in the Number & Operations in Base Ten domain for Kindergarten–Grade 5.

TOPICS

Before differentiating instruction in this aspect of the number strand, it is useful for a teacher to have a sense of how these topics develop over the grades.

Prekindergarten–Grade 2

Within this grade band, students move from counting and comparing very small numbers, usually 10 or less, to counting, comparing, modeling, and interpreting numbers up to 1000 using concepts that underlie the **place value system**. They move from counting to determine **sums** in simple joining situations and **differences** in simple subtraction situations to thinking more formally about adding and subtracting. Increasingly, as they move through this grade band, students use a variety of principles, strategies, and procedures with increasing efficiency to add and subtract and to solve problems requiring addition and subtraction.

Grades 3–5

Within this grade band, students begin to focus increasingly on multiplying and dividing **whole numbers** using a variety of strategies to calculate and estimate **products** and then **quotients**. They commit multiplication and related division facts to memory, become more fluent with **algorithms** for multiplying and dividing multidigit whole numbers, and solve problems that represent a variety of meanings of multiplication and division. Students learn to think of decimals as part of the place value system and use place value concepts to **round** and compare both whole numbers and decimals.

THE BIG IDEAS FOR COUNTING & CARDINALITY AND FOR NUMBER & OPERATIONS IN BASE TEN

In order to differentiate instruction in this content area, it is important to have a sense of the bigger ideas that students need to learn. A focus on these big ideas, rather than on very tight standards, allows for better differentiation.

It is possible to structure all learning in the topics covered in this chapter around these big ideas, or essential understandings:

1.1. Counting is fundamental for describing magnitude as well as for calculating.
1.2. Numbers often tell how many or how much.
1.3. Representing a whole number in different ways tells different things about that number and might make numbers easier to compare.
1.4. The place value system standardizes how whole numbers are **decomposed**. The system makes it easier to describe, compare, count by, and calculate with numbers.
1.5. Benchmark numbers are useful for relating numbers and estimating amounts.
1.6. It is useful to take advantage of the relationships between the operations in computational situations.
1.7. Decomposing whole numbers and recomposing them are critical skills for representing, comparing, and operating with whole numbers.

The tasks set out and the questions asked about them while teaching topics in counting and cardinality and in number and operations in base ten should be developed to reinforce the big ideas listed above. The following sections present numerous examples of application of open questions and parallel tasks in development of differentiated instruction in these big ideas across the Prekindergarten–Grade 2 and Grades 3–5 grade bands.

CHAPTER CONTENTS

OPEN QUESTIONS FOR PREKINDERGARTEN–GRADE 2

OPEN QUESTIONS are broad-based questions that invite meaningful responses from students at many developmental levels.

> Choose a number. Start counting. Make sure to say lots of numbers.

Counting & Cardinality: Kindergarten — **BIG IDEA:** 1.1
Number & Operations in Base Ten: Grade 1 — **PRACTICE STANDARD:** 6

Students who need to can start low, at 1 or 2, for example. But other students might want to show their counting prowess and begin with a higher number, such as 41 or even 48. All students are still practicing counting, but each is counting at his or her appropriate level.

Students who are "strong" might be encouraged to start counting at a place where transitions over decades are required, for example, counting 48, 49, and then realizing that 50 comes next.

Using the term "lots of numbers" allows for even more differentiation, with some students counting relatively few numbers and others doing more.

> Put out some forks and some spoons. Make sure there are just a FEW more forks than spoons.

Counting & Cardinality: Kindergarten — **BIG IDEA:** 1.1
PRACTICE STANDARD: 6

At this level, most students are working with only very small numbers. The idea in this problem is to give students an opportunity to count, but also to get them thinking about how numbers relate to one another. The question is open in that students can put out a lot of spoons and forks if they wish, or not many. In addition, students have some latitude in deciding what "just a few" means.

It is important with this question that teachers not define the word "few" as a particular number (e.g., three), which is sometimes done. Students should be allowed to see that there is some variation in what different people consider "a few."

> Samantha said it's just as easy to count by 5s as by 10s. Do you agree or disagree? Explain.

Number & Operations in Base Ten: Grade 2 — **BIG IDEA:** 1.1
PRACTICE STANDARD: 3

There is, of course, no correct answer to this question. But it does give students an opportunity to put forth an argument as to why a person might think that counting by 5s is as easy as or easier than counting by 10s. A student might argue that it is just as easy to count by 5s, since you use the same numbers as you would if you were counting by 10s, but you add extra numbers that end with 5s. Or students might suggest they are more accustomed to counting by 5s than 10s since they learned that earlier. The teacher might decide to have a **100-chart** or a large **rekenrek** available for students to use.

TEACHING TIP. Mathematical Practice Standard 3 (constructing viable arguments and critiquing the reasoning of others) can be accessed frequently by putting out an "opinion" statement and asking students to agree or disagree.

Arrange a group of stickers into a shape you know. Tell how many stickers you used. Tell what the shape is.

Counting & Cardinality: Kindergarten **BIG IDEA:** 1.1
PRACTICE STANDARD: 5

The objective in having students arrange the stickers to form a shape is partly to bring in some geometric considerations, but also to force students to use more than one or two stickers. This provides an opportunity for students to count as well as to think about shape creation.

Teachers might even make some suggestions to different students about how many stickers to use, for example, *Why don't you use more than 5 stickers?* or *Why don't you use more stickers than there are buttons on your shirt?*

➤ ***Variations.*** The use of stickers is clearly optional; students might arrange any items into a shape.

Choose a number and start counting from there. Will you have to say a lot of numbers to get to 50 or not very many? How do you know?

Counting & Cardinality: Kindergarten **BIG IDEAS:** 1.1, 1.2
Number & Operations in Base Ten: Grade 1 **PRACTICE STANDARD:** 7

This question is made open by allowing students to choose the number they will use as a starting point. If the student chooses a low number (e.g., 2 or 3), he or she should realize it will take a while to get to 50. If the student happens to choose a number greater than 50, it will be interesting to see if she or he is comfortable counting backwards as a way to reach 50. Some students will think of skip counting and, even though the numbers could be technically far apart, it might not take long to get to 50. This question should give the teacher insight into the flexibility students have with counting, while giving the students an opportunity to think about number size.

➤ ***Variations.*** The question can be varied by specifically asking students to skip count or to count backward, and the goal number 50 can be changed.

TEACHING TIP. One of the simplest strategies for differentiating instruction is allowing students to choose the numbers with which they will work.

Choose a number between 1 and 100. Tell why someone might call it a big number, but someone else might call it a small one.

Counting & Cardinality: Kindergarten
Number & Operations in Base Ten: Grade 1
BIG IDEA: 1.2
PRACTICE STANDARD: 3

An important mathematical concept for students to learn is that no number is inherently big or little; it always depends on context. The number 10 could be big if it describes the number of children in a family, but it would be small if it tells how many customers are in a large store.

➢ ***Variations.*** This kind of question is one that could be repeated over and over using numbers of the students' choice or the teacher's choice.

What makes 5 a special number?

Counting & Cardinality: Kindergarten
BIG IDEAS: 1.2, 1.3
PRACTICE STANDARD: 3

Posing this question to young students provides them an opportunity to participate in a mathematical conversation. Some students might think of the fact that there are 5 fingers on a hand, others that they are 5 years old, others that there are 5 people in their family, whereas others might think of something else, for example, that there is a special coin for 5 cents. The question helps students recognize that numbers are used to describe amounts in a wide variety of contexts.

➢ ***Variations.*** This question can be varied by using other numbers that students might view as special, for example, 0, 1, or 10.

Show the number 7 in as many different ways as you can.

Counting & Cardinality: Kindergarten
BIG IDEA: 1.3
PRACTICE STANDARD: 5

To respond to this question, some students will draw seven similar items, others will draw seven random items, others will use stylized numerals, others will write the word *seven,* and still others will show mathematical operations that produce an answer of 7, such as 6 + 1 or 4 + 3.

Because the question asks students to show as many representations as they can, even a student who comes up with only one or two representations will experience success. No matter what their developmental levels, all students will focus on the fact that any number can be represented in many ways. It is important to pose questions that focus students on how their own representations are alike and different.

For example, a teacher could ask:

- *Which of your representations are most alike?*
- *Which of your representations make it easy to see that 7 is less than 10?*
- *Which make it easy to see that 7 is an odd number?*
- *Which make it easy to see that 7 is 5 and 2?*

> Choose two of these numerals. How do they look the same? How do they look different?
>
> 1 2 3 4 5 6 7 8 9 0

Counting & Cardinality: Kindergarten **BIG IDEA:** 1.3
PRACTICE STANDARD: 3

An important part of mathematical development is the recognition that numbers can be used to represent quantities, but it helps if students recognize the numerals. By focusing on the form of the numerals, this question helps students learn to reproduce and read them.

By allowing students to choose whatever two numerals they wish, the question allows for virtually every student to succeed. For example, a very basic comparison might be that 1 and 4 both use only straight lines, but fewer lines are needed to form a 1 than to form a 4.

> The answer is 5. What is the question?

Counting & Cardinality: Kindergarten **BIG IDEA:** 1.3
Number & Operations in Base Ten: Grade 1 **PRACTICE STANDARD:** 3

This question has innumerable possible answers, and it supports the important concept that numbers can be represented in many ways. Some students will use addition and ask the question: *What is 4 + 1?* Other students will use subtraction and ask: *What is 6 – 1?* Yet other students will ask very different types of questions, for example: *What number comes after 4?* or even *How many toes are there on one foot?* Because of the wide range of acceptable responses, students at all levels are addressed.

➤ ***Variations.*** This task can be reassigned after changing the number required for the answer.

> How are the numbers 200 and 350 alike? How are they different?

Number & Operations in Base Ten: Grade 2 **BIG IDEAS:** 1.3, 1.4, 1.5
PRACTICE STANDARDS: 3, 7

A question like this one provides the opportunity to see what students know about numbers in the hundreds. They might observe similarities such as these:

- *They are both more than 100.*
- *They are both less than 400.*
- *You say both of them when you skip count by 10.*
- *They both have three digits.*
- *If they were an amount of cents, you could model both of them with quarters.*

200:

350:

The students might observe differences such as these:

- *350 is more than 300, but 200 is not.*
- *200 is an exact number of hundreds, but 350 is not.*
- *You can count from 0 to 200 by 20s, but you can't count by 20s to 350 if you start at 0.*

The question is suitable for a broad range of students because some students can choose simple similarities and differences, such as indicating that both numbers end in 0, whereas others can focus on more complex similarities, such as indicating that both numbers are part of the sequence when skip counting by 25. All students benefit from the larger discussion about number representations and meanings.

➢ ***Variations.*** The question can easily be varied by using other pairs of numbers.

> You can represent a number between 100 and 1,000 with just a few **base ten blocks**. What could the number be? How do you know?

Number & Operations in Base Ten: Grade 2 — **BIG IDEAS:** 1.3, 1.4, 1.7
PRACTICE STANDARDS: 5, 7

Using base ten blocks focuses students on the nature of the place value system. Students consider what "columns" there are in a **place value chart**, because each block type represents a different column, and they also consider relative size, for example, that the 100 is ten times as great as the 10.

There will be students who will realize that a relatively large number (e.g., 200) can sometimes be represented by many fewer blocks than a much smaller number (e.g., 17). Others will simply randomly select one, two, or three blocks from a set of blocks; their focus will then be on naming the number.

It would be interesting to observe whether students first choose blocks and then name their numbers or whether they realize that they want a number with zeroes in it with the other digits fairly small.

> Two numbers each have 7 as one of their digits. But one 7 is worth a LOT more than the other. What could the numbers be?

Number & Operations in Base Ten: Grades 1, 2 **BIG IDEA:** 1.4
PRACTICE STANDARDS: 1, 3, 7

To really understand the place value system, a student needs to realize that the placement of a digit matters a lot. If, for example, a 7 is in the ones place, it is clearly worth less than if it is in the tens or hundreds place. But the other digits are irrelevant. So students might choose, for example, 7 and 700, or 17 and 79, or 27 and 712.

Some students might benefit from access to a place value chart or base ten blocks.

> Two three-digit numbers have at least two of the same digits. The greater one is a little less than 300 more than the smaller one. What might the numbers be?

Number & Operations in Base Ten: Grade 2 **BIG IDEA:** 1.4
PRACTICE STANDARDS: 1, 6

This question, like some previous ones, allows you to emphasize what place value means, that is, that the placement of a digit in the number matters. But this question also has a problem solving twist, because students must meet the condition that one number be a little less than 300 more than the other.

The choice was made to ask for *a little less* so that a student could not just quickly write an answer without thinking hard (e.g., 127 and 427). Examples of solutions include 316 and 613, since 299 + 316 = 615, 613 shares three digits with the original 316, and both 613 and 615 are close to 616; or 475 and 764, since 289 + 475 = 764, and 764 is fairly close to 775.

You might help students notice, after different examples have been offered, how the values of the digits have changed based on their placement. You might also focus on the fact that for at least one position in the numbers, the digits would likely be 3 apart so that the numbers could be about 300 apart.

If, though, you wish to allow for a student to succeed with something simple, such as 315 and 615, you could allow for the greater number to be a little less than *or exactly* 300 more.

TEACHING TIP. By asking for values *a little more than, a little less than, close to,* etc., students work a little harder. Usually they first answer the question as though they had to be exact and then they do additional work in figuring out which of their values to change and in which direction.

> Choose three numbers. Two have to be pretty close together. The third has to be far from the other two. What might they be?

Counting & Cardinality: Kindergarten **BIG IDEAS:** 1.4, 1.5
Number & Operations in Base Ten: Grades 1, 2 **PRACTICE STANDARD:** 3

This question provides insight into what students think close together and far apart mean for numbers. It also gives them an opportunity to use numbers that are comfortable for them, making the question accessible to most students. While some students might choose numbers like 2, 3, and 9, others might choose numbers like 95, 100, and 2, or 500, 510, and 900. By asking students to write the numbers, it is more likely they will choose numbers that are not too large.

It would be interesting to discuss and compare answers. A teacher might ask:

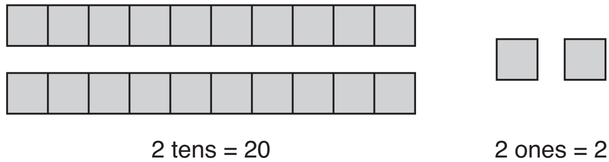
- *Do you think that 100 and 110 are close?*
- *Do you think that 5 and 8 are close?*
- *How did you decide which numbers to choose?*

> A two-digit number has more tens than ones. What could the number be? How do you know your number is correct?

Number & Operations in Base Ten: Grades 1, 2 **BIG IDEA:** 1.4
PRACTICE STANDARDS: 3, 7

Much of the work with numbers is built on the fact that numbers are written in such a way that the value of a numeral is dependent on its placement in the number. For example, the 2 in 23 is worth 20, but the 2 in 32 is worth only 2.

To help students work with the place value system, instruction often begins with models that show the difference. For example, base ten blocks show 20 as two rods, but 2 as two small cubes.



Students may or may not be provided with base ten blocks when the question above is posed. For most students, provision of the blocks would be helpful, but if students are more advanced, the blocks may not be essential. Some students may discover only one or two possible responses, whereas others might determine many possible responses (e.g., 31, 32, 43, 42, 41, etc.) or even all possible responses. By asking for only one number, the task seems less onerous to struggling students. In discussing responses, students will see that there were many possible values among which they could have chosen.

In showing how they know their number is correct, some students can use concrete models to support their answers, whereas others might use more symbolic arguments.

➢ ***Variations.*** Students who are ready to work with three-digit numbers can be given the option to do that if they wish.

Use a 100-chart. Choose two numbers to add. Both numbers must be on the top half of the chart. Show how to use the chart to add the two numbers without using pencil or paper.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

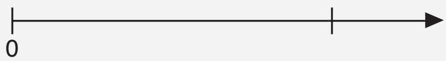
Number & Operations in Base Ten: Grades 1, 2

BIG IDEAS: 1.4, 1.7
PRACTICE STANDARD: 5

This question promotes students' use of visualization to help them with **mental math**. They can select simple numbers to add or more complicated ones. For example, students who start at 32 and add 1 realize that it is only necessary to move one space to the right. If the student adds 10, it is only necessary to move one space down. A student who adds something like 29, though, might realize that it is possible to go down three rows and back one space, a much more sophisticated mental math operation.

The question is useful for a variety of student levels because students can avoid adding to a number at the right side of the chart (e.g., adding 59 + 6) and having to go to the next line and can avoid adding complicated numbers if they find the maneuvers too difficult.

Choose a number for the second mark on the **number line**.



Mark a third point on the line. Tell what number name it should have and why.

Counting & Cardinality: Kindergarten
Number & Operations in Base Ten: Grades 1, 2
BIG IDEA: 1.5
PRACTICE STANDARD: 3

This question helps students see the value of using benchmarks to relate numbers. It is designed to be open by allowing students not only to choose which point to locate but also which benchmark to use.

A simple response might be to designate the second marked point as a 2 and to mark a halfway point as a 1. A more complex response might be to designate the second marked point as a 10 and attempt to locate a number like 2 or 3.

No matter which choice students make, they can participate in the conversation about how numbers are placed on a number line and how numbers relate to one another.

➤ ***Variations.*** The teacher can name the second point, for example as 5, then ask students to choose the third point.

The sum of □□ and □□ is closer to 80 than to 90. What could the two numbers be?

Number & Operations in Base Ten: Grades 1, 2
BIG IDEAS: 1.5, 1.7
PRACTICE STANDARDS: 1, 2, 5, 7

It would be interesting to see how students approach this question. Most likely, students will try to think of numbers that sum to 81, 82, 79, or some other number close to 80. But some students will realize that any sum less than 80 (even one as small as 20) is closer to 80 than to 90, and so they might use numbers such as 10 and 10. Some may push the limits to 84 for their sum.

It would be valuable to have a discussion about how students broke up the numbers 80 and/or 90 to help them answer the question.

➤ ***Variations.*** Instead of asking for a sum closer to 80 than to 90, values other than 80 and/or 90 could be selected.

You add two numbers that are almost 30 apart. The answer is almost 90. What might the numbers be?

Number & Operations in Base Ten: Grades 1, 2
BIG IDEAS: 1.6, 1.7
PRACTICE STANDARDS: 1, 3

A number of different strategies might be employed to solve this problem. Most students will use guess and check, but in different ways.

Whereas some students will try a combination, for example, 10 + 40, realize that the answer is too small, and just move up to 20 + 50 and then 30 + 60, others will realize right away, with 10 + 40, that they are 40 short and that each number has to go up by 20.

Other students might begin with a combination for 90, for example, 40 + 50, realize that the two **addends** are only 10 apart, and move the 40 down and the 50 up to get the appropriate difference. Both of these approaches involve valuable reasoning.

The fact that the question says "almost" means that students will probably take those answers and figure out how to adjust them. For example, the student might use 30 + 59, so that the numbers are not quite 30 apart and the answer is not quite 90.

➢ ***Variations.*** There are many ways to change this question. The numbers used could be changed or the word *almost* might be changed to *about* or *a bit more than.*

> Make up an addition question where there is a 2, a 3, and a 4 somewhere in the question or the answer.

Number & Operations in Base Ten: Grades 1, 2 **BIG IDEA:** 1.7
PRACTICE STANDARD: 1

All students who respond to the question will use addition, but different students can choose combinations with which they are more comfortable. For example, students might write:

$$\underline{2} + \underline{34} = \underline{3}6 \qquad \underline{2} + \underline{3} + \underline{4} = 9 \qquad \underline{23} + \underline{43} + \underline{2}5 = 91$$

An open question such as this promotes a rich discussion. As different individuals or pairs of students share their questions, other students repeatedly have the opportunity to learn new ideas about addition.

➢ ***Variations.*** The question can easily be varied by using different sets of three numbers, using two or four numbers, or using a different operation (e.g., subtraction).

OPEN QUESTIONS FOR GRADES 3–5

> Describe 100 thousand in as many ways as you can.

Number & Operations in Base Ten: Grades 4, 5 **BIG IDEAS:** 1.2, 1.3, 1.4, 1.5
PRACTICE STANDARD: 7

Typically this question would be presented using the numeral 100,000, rather than the written phrase *100 thousand.* The advantage of the suggested presentation is that the question becomes more accessible to students who struggle with numerals that represent large numbers. Although one might think that students need to have a concept of how much 100,000 is to answer the question, which would certainly be preferable, a student could simply say that it is more than 99 thousand.

Other students might refer to 100,000 by using place value ideas, for example, indicating that it is possible to represent 100,000 with 100 large base ten blocks, that it is $\frac{1}{10}$ of 1 million, or that it can be represented with 1,000 hundred blocks.

The class discussion would provide ample opportunity for exploration as different aspects of 100 thousand are raised.

> One number is a lot more than another one. What could the two numbers be?

Number & Operations in Base Ten: Grade 3

BIG IDEAS: 1.4, 1.5
PRACTICE STANDARDS: 3, 7

This question provides a great deal of latitude both in the choice of numbers and in the interpretation of the phrase *a lot more.*

For example, one student might choose 200 and 300, suggesting that being 100 apart means that 300 is a lot more. Another student might choose 10 and 90. Others might argue that those numbers are not far enough apart and might choose, for example, 1,000,000 and 101. None of these students are incorrect, but the door has been opened for a discussion about how some mathematical terms are more precise than other terms and how both types of terms can be useful in different situations.

In addition, there is an opportunity within the larger class community to repeatedly practice the concept of comparison.

> In a particular multidigit number, there are two 7s and two 5s. One 7 is worth 100 times as much as the other; one 5 is worth 10 times as much as the other. What could the number be?

Number & Operations in Base Ten: Grades 4, 5

BIG IDEA: 1.4
PRACTICE STANDARD: 7

When students really understand the place value system, they realize that a digit one column to the left of the same digit is worth 10 times as much as the one on the right. Similarly, a digit two columns to the left is worth 100 times as much as the one on the right. This question builds on that knowledge. There are many correct answers, for example, 71,755 or 55,727 or 55,797. But some students might use even larger numbers, such as 707,554 or 1,717,455; or others might use decimals, perhaps something like 55.727.

> You multiply two numbers and the product is of the form $\square 2, \square 4 \square$. What could the numbers and product have been?

Number & Operations in Base Ten: Grade 5 **BIG IDEA:** 1.4
PRACTICE STANDARDS: 1, 6

There are many solutions to this problem. Leaving the ones digit unknown might seem, at first glance, to make the question more difficult; often we use the ones digit to figure out what the ones digits of the **factors** might have been. In fact, though, leaving the ones digit open makes the problem more accessible. For example, putting a 0 in the ones digit allows students to use factors like 10×3254 or 20×3142.

Some students might simply choose a 2-digit number at random and multiply by other numbers until they hit the desired form. They will soon realize that in order to get a 5-digit answer, they are likely to be multiplying by a 3-digit or 4-digit factor; this helps build their number sense.

Other ideas that might come up that are worth discussing include looking at whether factors and the product are even or odd, whether they are **multiples** of particular numbers (such as 5), and so forth.

> You multiply two numbers and the product is almost 400. What could the numbers have been? Explain your answer.

Number & Operations in Base Ten: Grade 4 **BIG IDEAS:** 1.4, 1.7
PRACTICE STANDARDS: 3, 7

Estimation is an important aspect of mathematical calculation. Teachers often do not emphasize it enough. This open question allows students to work backward and think about what numbers might have been used to arrive at a particular estimate. They will also have to think about what "almost" means.

Some students might think of something fairly simple, for example, 1 and 399. Others might recognize that $20 \times 20 = 400$ and then use 19×19. Still others will consider other possibilities.

➢ ***Variations.*** Instead of using "almost," the question can be varied by using "a bit more than." The product can be changed from 400 to a different number. Other operations can also be allowed: for example, the question might be worded, "You add, subtract, multiply, or divide two numbers and the result is about 400. What could the numbers be? Explain."

A news item says that about 500 people were at a certain event. Exactly how many people might that have been?

Number & Operations in Base Ten: Grade 3 **BIG IDEA:** 1.5
PRACTICE STANDARDS: 2, 3

Notice that there is no indication of how the number 500 was obtained. Perhaps a count was rounded to the nearest 100 or to the nearest 25, or maybe it was not rounded at all. It is quite possible that the original number could have been something like 440, since 440 is not that far from 500. Or maybe it was greater, perhaps 530 or 600. Because there is no indication of how the estimate was obtained, this question is open.

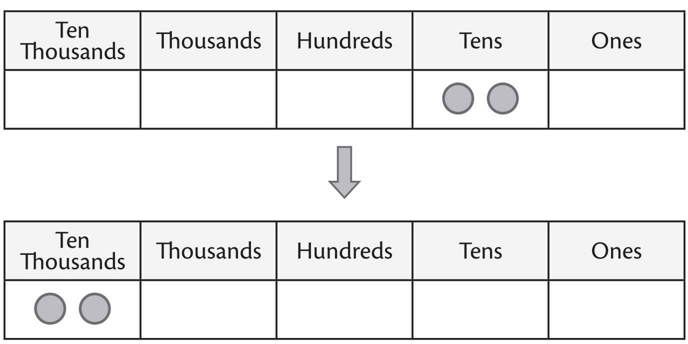
➢ ***Variations.*** The number 500 could be changed.

Choose a 4-digit factor and a 1-digit or 2-digit factor. Show how you could represent the product with models or diagrams in a way that would help you figure out the answer.

Number & Operations in Base Ten: Grades 4, 5 **BIG IDEA:** 1.5
PRACTICE STANDARD: 5

By giving students a choice of values, they can make the situation simpler (e.g., 1000 × 20) or more complicated (e.g., 3142 × 5).

To represent 1000 × 20, a student might use a place value chart and place two counters in the tens column to show 20; multiplying by 1000 has the effect of moving the counters three columns to the left.



To show 3142 × 5, a student might draw a table like this one:

×	3	1	4	2
5	15	5	20	10
Regrouped	15	7	1	0

> A number is about 400. What is the least you think it can be? What is the greatest?

Number & Operations in Base Ten: Grade 3 **BIG IDEA:** 1.5
PRACTICE STANDARDS: 2, 3

Although we often ask students to estimate or round numbers, we less frequently ask them to think about what a number might have been that has already been rounded. If you read a newspaper headline that says "About 400 people attended the meeting," you want to know what the actual number might have been.

Notice that there was no indication here whether the 400 was the result of rounding to the nearest 10 or the nearest 100 or whether rounding was even used, and that is what makes the question interesting. Different students will interpret it in different ways.

Questions a teacher might ask could include:

- *Do you think it could have been 398?*
- *Could it have been more than 400? How much more?*
- *Could it have been as low as 325? 350?*

> You have a decimal thousandth number. When you round it to the nearest tenth, you round down. But when you round it to the nearest hundredth, you round up. What might the decimal thousandth number be?

Number & Operations in Base Ten: Grade 5 **BIG IDEA:** 1.5
PRACTICE STANDARDS: 1, 7

Students learn the rules of rounding and may apply them, but often they don't actually think about them. This question forces students to think about the rules. Students will see that the only way it is possible to round down to the nearest tenth but up to the nearest hundredth is to have a decimal of the form $\square$.*abc*, where *b* is less than 5, but *c* is 5 or more. For example, 13.928 rounds to 13.9 (less than 13.928) or 13.93 (more than 13.928).

Students should be led to see that the values of the whole number and the tenths part of the decimal are irrelevant to this problem.

> You add two decimals and then multiply by 5. The result is between 4.5 and 5.0. What could the decimals have been?

Number & Operations in Base Ten: Grade 5 **BIG IDEAS:** 1.5, 1.7
PRACTICE STANDARDS: 1, 6

Because two operations are performed, there is significant latitude in coming up with an answer here. Some students might just do a guess-and-test and then revise their answers. For example, they might start with 1.2 + 3.4 to get 4.6. They quickly see that multiplying their sum by 5 will get take them beyond 5.0, so they reconsider. One possibility is to just start all over again; that's what most students will do. A more sophisticated response is to notice that their original sum is within the desired range. So, if they would divide their two original decimals by 5 and then multiply the new sum by 5 (as the problem requires), they would be in the right range. They readjust their original numbers to 1.2 ÷ 5 = 0.24 and 3.4 ÷ 5, which is about 0.7. Then, adding 0.24 + 0.7 = 0.94. And 0.94 × 5 = 4.7, a suitable result.

Other students might work backward. They might say, *I want an answer of 4.8 after I multiply by 5, so I want an answer of 4.8 ÷ 5 = 0.96 before I multiply.* Then they choose two decimals that add to 0.96. There are, of course, many possibilities.

Still other students might decide that they want to make the second decimal super small, so really they just need to find a first number that they can multiply by 5 and get between 4.5 and 5. Then when they add their tiny decimal to it, and multiply by 5, they will still be in the right range. They realize that 0.9 might work, so they choose 0.9 and 0.001 as their decimals to add.

> Choose a subtraction question you want to complete where the answer is in the hundreds. Show how you could figure out the answer to the problem efficiently either by adding or by subtracting.

Number & Operations in Base Ten: Grade 5 — **BIG IDEA:** 1.6
PRACTICE STANDARDS: 6, 7

A student might choose a calculation like 500 – 198. The student might then either add up from 198 to get to 500 or, for example, subtract 498 – 198 and then add 2.

Allowing students to choose the values they will work with empowers them and likely leads to greater success for more individuals.

> You divide two whole numbers and the quotient has 2 digits with no **remainder**. How many digits might the **dividend** and **divisor** have?

Number & Operations in Base Ten: Grade 5 — **BIG IDEAS:** 1.6, 1.7
PRACTICE STANDARDS: 6, 7

An important idea for students to learn is that with division, as with subtraction, you can end up with a relatively small answer when using either small numbers or large numbers. With subtraction, the result is 2 whether you subtract 2 from 4 or you subtract 30,002 from 30,004. Similarly, with division, the result is 2 whether you divide 4 by 2 or you divide 40,000 by 20,000. This open question addresses that idea in that knowing the answer has 2 digits does not require the use of large numbers, but it does allow for it.