2

Number & Operations—Fractions

DIFFERENTIATED LEARNING activities related to fractions and decimals are derived from applying the Common Core Standards for Mathematical Practice to the content goals that appear in the Number & Operations—Fractions domain for Grades 3–5.

TOPICS

Before differentiating instruction in work with fractions and decimals, it is useful for a teacher to have a sense of how fraction and decimal topics develop over the grades.

Prekindergarten–Grade 2

The only work in fractions within the Prekindergarten–Grade 2 grade band appears in the Geometry domain, where students partition shapes into equal-sized pieces and name those pieces by using fraction language.

Grades 3–5

Within this grade band, students work with fractions, beginning with fractions less than one and then moving on to **improper fractions** and mixed numbers. They learn to think of fractions as numbers on number lines, and as equal parts of a whole. In Grade 5, they learn why a fraction can also be thought of as the quotient when the **numerator** is divided by the **denominator**. They compare fractions in a variety of ways, including using **equivalent fractions**.

Students explore addition, subtraction, and multiplication of fractions in real contexts and learn the algorithms for those operations. They also learn how to divide **unit fractions** (those with a numerator of 1) by whole numbers and whole numbers by unit fractions. They learn how decimals are equivalent to fractions with denominators of 10 and 100 and learn about the equivalence of decimals and how decimals are compared. Operations with decimals are covered in Chapter 1 of this resource, in line with the organization of the Common Core Standards.

Grades 6–8

For Grades 6–8, continued work with fractions in covered under The Number System (Chapter 4).

THE BIG IDEAS FOR NUMBER & OPERATIONS—FRACTIONS

In order to differentiate instruction in fractions, it is important to have a sense of the bigger ideas that students need to learn. A focus on these big ideas, rather than on very tight standards, allows for better differentiation.

It is possible to structure all learning in the topics covered in this chapter around these big ideas, or essential understandings:

2.1. Representing a fraction or decimal in an alternate way might reveal something different about that number and might make it easier to compare with other fractions or decimals.
2.2. Fractions and decimals are useful for describing numbers that fall between whole numbers.
2.3. When a fraction or decimal is used to describe part of a whole, the whole must be known.
2.4. There are various strategies for comparing fractions.
2.5. Operations have the same meaning with fractions as they do with whole numbers.

The tasks set out and the questions asked about them while teaching fractions and decimals should be developed to reinforce the big ideas listed above. The following sections present numerous examples of application of open questions and parallel tasks in development of differentiated instruction in these big ideas across the Grades 3–5 grade band.

CHAPTER CONTENTS			

OPEN QUESTIONS FOR GRADES 3–5

OPEN QUESTIONS are broad-based questions that invite meaningful responses from students at many developmental levels.

Choose a fraction that is not $\frac{1}{2}$. Represent that fraction in at least three different ways. For each representation of the fraction, describe something that version makes it easier to see about the fraction than other representations do.

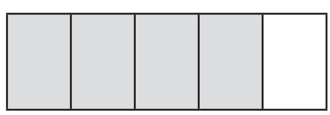
CCSS: Number & Operations—Fractions: 3.NF, 4.NF, 5.NF
Mathematical Practice: 5
BIG IDEA: 2.1

It is important that students realize that fractions can represent parts of areas, parts of lengths, parts of masses, or parts of capacities or volumes. Fractions are also numbers that happen to be the quotient of the numerator divided by the denominator.

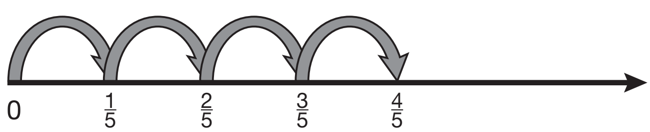
Allowing students to choose their own fractions makes the task more comfortable, although a teacher might encourage some students to pick "unusual" fractions like $\frac{3}{7}$ or $\frac{4}{9}$ and not just familiar ones like $\frac{2}{3}$ or $\frac{3}{4}$.

Discussion might include attention to many important concepts, for example, that in a part-of-set situation, the pieces need not be equal in area or volume; that in a part-of-area situation, the pieces need not be adjacent; that representing 2 of the 3 equal parts in a rectangle is essentially no different from representing 2 of the 3 equal parts in a square, and so forth.

The focus of the discussion should be on what representations show about the fractions being represented. For example, showing $\frac{4}{5}$ as below tends to make it very clear that $\frac{4}{5}$ is $\frac{1}{5}$ less than a whole.



But representing $\frac{4}{5}$ as shown below tends to make it very clear that $\frac{4}{5}$ is 4 groups of $\frac{1}{5}$.



➢ ***Variations.*** Once students are familiar with improper fractions, they might be requested to choose an improper fraction.

Open Questions for Grades 3–5

> In what situation might you want to use the idea that the fraction $\frac{5}{8}$ could be represented as 0.625? In what situation would you rather call it $\frac{5}{8}$?

CCSS: Number & Operations—Fractions: 5.NF
Mathematical Practice: 5
BIG IDEA: 2.1

Many students believe that it is always easier to work with decimals than with their fraction equivalents. But this is not always the case. For example, if you are trying to determine what $\frac{5}{8}$ of 24 is, you might prefer representing the amount as $\frac{5}{8}$ rather than as 0.625. But if you are comparing $\frac{5}{8}$ to $\frac{3}{5}$, you might prefer comparing 0.625 to 0.6

This question provides an opportunity for students to come up with situations on their own rather than being directed to consider a specific case.

➢ ***Variations.*** Fractions other than $\frac{5}{8}$, such as $\frac{2}{3}$, might also be proposed.

> You divide two numbers and the answer is a fraction a little less than $\frac{3}{4}$. What numbers might you have divided?

CCSS: Number & Operations—Fractions: 5.NF
Mathematical Practices: 5, 6
BIG IDEA: 2.1

Most students think of fractions as parts of wholes and that is, of course, useful. But it is even more useful for students to realize that a fraction is a number that could be conceived as the quotient of its numerator divided by its denominator. This question gets to the heart of that notion.

If students really understand this, they might quickly create a fraction slightly less than $\frac{3}{4}$, for example, $\frac{7}{10}$, and say that they had divided 7 by 10.

Alternatively, students might realize that if you divide, for example, 9 by 12 you get $\frac{3}{4}$, so they could divide by something slightly more to get a smaller fraction. In this case they might respond that they divided 9 by 13.

Any two numbers where the first is slightly less than $\frac{3}{4}$ of the second will do.

➢ ***Variations.*** The fraction $\frac{3}{4}$ can easily be altered.

> Tell about a time when you would use the number $\frac{1}{2}$.

CCSS: Number & Operations—Fractions: 3.NF
Mathematical Practice: 4
BIG IDEA: 2.2

All students who address this question will think about the fraction $\frac{1}{2}$. Allowing them to come up with their own contexts lets each one enter the mathematical conversation at an appropriate level. Often, when a teacher preselects a context for students, it is meaningful to only some of them.

If students need a stimulus to get started, the teacher can provide drawings that might evoke ideas, for example, a picture showing a sandwich cut in two, a glass half full of juice, or a **hexagonal** table with a line dividing it in half and two people sitting at opposite sides of the table. It is likely that most students will only think of halves of wholes, but some might think of half of a set of two.

TEACHING TIP. Scaffolding tasks by providing models for students reduces the likely breadth of responses that will be offered and may inhibit some students who feel obligated to follow the models.

A number between 3 and 4 is slightly closer to 4 than to 3. What could the number be?

CCSS: Number & Operations—Fractions: 3.NF, 4.NF, 5.NF
Mathematical Practices: 3, 6
BIG IDEA: 2.2

For some students, there are no numbers between 3 and 4. For others, there are a few numbers, like $3\frac{1}{2}$ and maybe $3\frac{1}{4}$ and $3\frac{3}{4}$. This question forces students to think about the **density** of the number line; there are always more numbers, even more fractions, between any two existing ones.

Some students will consider numbers in decimal form, and others in fraction form. Possible responses to the question include 3.51, 3.501, $3\frac{5}{8}$, $3\frac{51}{100}$, and so forth. Students should be required to prove that their values make sense.

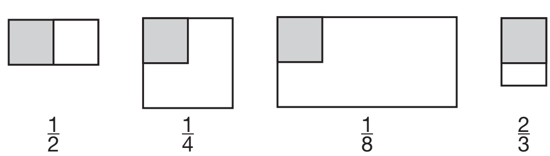
➢ ***Variations.*** Students might be asked to determine a fraction (or decimal) between two fractions (or decimals).

Draw a small rectangle. Draw a bigger rectangle that the smaller one is part of. Tell what fraction of the big rectangle the small one is.

CCSS: Number & Operations—Fractions: 3.NF
Mathematical Practices: 3, 4
BIG IDEA: 2.3

In teaching fractions, students are usually asked to identify a fraction given a partitioned whole. The question above has students thinking the other way—what is the whole if a fractional part is known? Such reversible thinking is an important mathematical process to develop.

To answer a question phrased in this more open way, a struggling student might use a simple fraction such as $\frac{1}{2}$, whereas other students might suggest more complex fractions. Several possibilities are shown in the diagram below:



> Describe a situation where $\frac{1}{3}$ is actually more than $\frac{1}{2}$.

CCSS: Number & Operations—Fractions: 3.NF, 4.NF, 5.NF **BIG IDEA:** 2.3
Mathematical Practices: 3, 6

Normally when we compare fractional values, we assume that the whole is 1. But when a fraction is part of a whole and the wholes are different, a smaller fraction, applied to a greater whole, might in fact be greater. This could be true when talking about portions of objects; for example, $\frac{1}{3}$ of a watermelon might have more volume than $\frac{1}{2}$ of a lemon. But it can also be true for numbers. For example, $\frac{1}{3}$ of 30 is more than $\frac{1}{2}$ of 10.

➢ ***Variations.*** Instead of phrasing the question as presented above, a teacher might be more direct and ask students to describe a pair of wholes such that $\frac{1}{3}$ of one whole is *more than* $\frac{1}{2}$ of the other, another pair where $\frac{1}{3}$ of one whole is *equal to* $\frac{1}{2}$ of the other, and a third pair where $\frac{1}{3}$ of one whole is *less than* $\frac{1}{2}$ of the other.

> Which two fractions would you find easiest to compare? Why those?
>
> $\frac{4}{5}$ $\frac{4}{10}$ $\frac{2}{9}$ $\frac{1}{12}$ $\frac{8}{9}$

CCSS: Number & Operations—Fractions: 3.NF, 4.NF **BIG IDEA:** 2.4
Mathematical Practices: 3, 5, 7

This question provides the opportunity to hear what students are thinking about as they compare fractions. Some may only be comfortable comparing fractions with the same denominator; others may be comfortable comparing fractions with the same numerator as well. Yet other students might think about fractions that are close to 0 as compared to those that are clearly closer to 1.

Questions a teacher might ask could include:

- *If you put these fractions on a number line, which would be closest to 0 and which would be closest to 1?*
- *How might you know $\frac{4}{5}$ is more than $\frac{4}{10}$ without even drawing a picture?*
- *Is it possible for $\frac{1}{12}$ of something to actually be more than $\frac{4}{5}$ of something?*

> Choose two fractions with different denominators. Tell how to compare them.

CCSS: Number & Operations—Fractions: 4.NF **BIG IDEA:** 2.4
Mathematical Practices: 2, 5

Students can use many different strategies to compare fractions. Some students might choose a fraction less than 1 and a fraction greater than 1 by appropriately manipulating the numerators and denominators.

Others might use other benchmarks to make the comparisons simple. For example, they might choose a fraction very close to 0, such as $\frac{1}{1,000}$, and another very close to 1, such as $\frac{99}{100}$. Still others might think of a fraction less than $\frac{1}{2}$, such as $\frac{1}{3}$, and a fraction greater than $\frac{1}{2}$, such as $\frac{3}{4}$.

And other students, perhaps even most students, are likely to select two arbitrary fractions and then use equivalent fractions with a common denominator to compare them.

As students tell how to compare the fractions, they are free to use whatever strategy they wish. This sort of choice is important to allow all students to succeed. It is also helpful to all students, as the comparisons are discussed, to hear about the many different strategies they could have accessed.

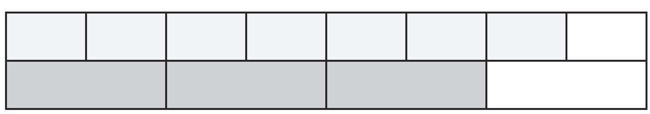
➢ ***Variations.*** The teacher can require that one or both of the numerators of the two fractions not be 1, because often students tend to use only unit fractions.

> Two fractions are close to 1, but one of the fractions is closer to 1 than the other fraction is. What might the two fractions be and how could you show or tell why the one closer to 1 is indeed closer?

CCSS: Number & Operations—Fractions: 3.NF, 4.NF **BIG IDEAS:** 2.1, 2.4
Mathematical Practices: 3, 5, 6

While some students might begin with a picture, others might begin with a symbolic representation. For example, some students realize that $\frac{7}{7} = 1$, so $\frac{6}{7}$ must be close to 1. Similarly, $\frac{5}{5} = 1$, so $\frac{4}{5}$ is close to 1.

The most important part of this question is the explanation: a student describing why, for example, $\frac{7}{8}$ is closer to 1 than, say, $\frac{3}{4}$. Many students are likely to focus on to how far each is from 1 (e.g., $\frac{1}{8}$ away from 1 is closer than $\frac{1}{4}$ away, since $\frac{1}{8}$ is less). But other students will simply appeal to a visual.



➢ ***Variations.*** The question could be changed to the fractions needing to be close to $\frac{1}{2}$, or close to 0, for example.

> You multiply two fractions and the result is just slightly less than one of them. What fractions could you have multiplied?

CCSS: Number & Operations—Fractions: 5.NF **BIG IDEA:** 2.5
Mathematical Practices: 3, 5, 6

Students with good fraction sense realize that when you multiply by a **proper fraction** less than 1, the result is less than the other factor. To be slightly less, you want to multiply by a proper fraction fairly close to 1. So students should realize that in this case one fraction should be a proper fraction close to 1, such as $\frac{9}{10}$ or $\frac{95}{100}$, and the other fraction can be anything at all.

> You multiply two fractions. The result is $\frac{24}{60}$. What numbers might you have been multiplying? How would you model the multiplication?

CCSS: Number & Operations—Fractions: 5.NF
Mathematical Practice: 5
BIG IDEA: 2.5

Although many students will simply look for two numbers to multiply to 24 to be numerators and two to multiply to 60 to be denominators (e.g., creating the fractions $\frac{2}{15}$ and $\frac{12}{4}$), others will simplify $\frac{24}{60}$ to $\frac{2}{5}$ and use $2 \times \frac{1}{5}$, and others will implicitly put conditions on their fractions, for example, deciding both fractions must be proper fractions or in **simplest form**. This allows the question to be appropriately challenging to different groups of students.

Some other possible solutions are $\frac{1}{2} \times \frac{4}{5}$, $\frac{1}{2} \times \frac{24}{30}$, $\frac{2}{3} \times \frac{12}{20}$, etc.

➢ ***Variations.*** Additional problems could be created by changing the resulting value or the operation used.

> You are solving a story problem where the sum is a mixed number a little more than 1. What could the story be?

CCSS: Number & Operations—Fractions: 4.NF, 5.NF
Mathematical Practice: 2
BIG IDEA: 2.5

Since the problem requires addition, students must recognize that there has to be some sort of combining. They can then think about what might be combined to make the sum a little more than 1. Many students will first think of two fractions to make exactly 1 and then increase one of them. Possible solutions are $\frac{1}{2} + \frac{5}{8}$, $\frac{9}{10} + \frac{1}{9}$, $\frac{3}{4} + \frac{1}{3}$, etc. They might create stories involving things like pizza, where someone has eaten one fraction of a pizza and then some more, or they might use a more unusual story line.

TEACHING TIP. By asking for values *a little more than, a little less than, close to,* etc., students work a little harder. Usually they first answer the question as though they had to be exact and then they do additional work in figuring out which of their values to change and in which direction.

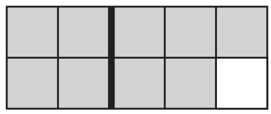
> You add two fractions and the sum is $\frac{9}{10}$. What could the fractions be?

CCSS: Number & Operations—Fractions: 3.NF, 4.NF
Mathematical Practices: 5, 6
BIG IDEA: 2.5

Typically, a teacher provides two fractions and asks students for the sum. If, instead, the sum is given and students are asked for the fractions, a broader range of students can respond successfully.

For example, the objective might be to sum two fractions to get $\frac{9}{10}$. A simple response would be $0 + \frac{9}{10}$. An interesting discussion could ensue as students debate whether 0 does or does not have to be written as $\frac{0}{10}$ to satisfy the instruction or to perform the addition.

Other fairly straightforward solutions are $\frac{8}{10} + \frac{1}{10}$, $\frac{7}{10} + \frac{2}{10}$, and so on. These become more obvious to students if they draw a diagram to represent $\frac{9}{10}$ and simply break up the parts. For example, the diagram below shows how $\frac{9}{10}$ can be represented as $\frac{4}{10} + \frac{5}{10}$:



Looking at the representation, some students might even see $\frac{1}{2} + \frac{4}{10}$ if they recognize that the top row is $\frac{1}{2}$ of the whole and shaded boxes in the bottom row represent $\frac{4}{10}$ of the whole.

Yet other students will work symbolically, either looking for fractions with denominators of 10 to add or by writing $\frac{9}{10}$ as an equivalent fraction, for example, $\frac{18}{20}$ or $\frac{27}{30}$, and adding fractions with denominators of 20 or 30, respectively.

Some students are likely to choose only one pair of addends; others may seek many pairs. In this way, a broader range of students is being accommodated. All of the students will recognize that adding is about putting things together.

➢ ***Variations.*** It is easy to vary the question by using a different sum or by asking for a particular difference rather than a particular sum.

> Create a story problem that you could solve by subtracting $2\frac{1}{3}$ from $4\frac{1}{2}$.

CCSS: Number & Operations—Fractions: 5.NF
Mathematical Practices: 2, 4
BIG IDEA: 2.5

With whole numbers, students learned that subtraction can describe a take-away situation, a comparison situation, or a missing addend situation. The same is true with fractions.

A take-away fraction problem might sound something like this:

I started with $4\frac{1}{2}$ cups of flour but used $2\frac{1}{3}$ cups for a recipe. How much flour do I still have?

A comparison problem involving fractions might sound something like this:

I used $4\frac{1}{2}$ cups of flour and $2\frac{1}{3}$ cups of sugar in my cake. How much more flour than sugar did I need?

A missing addend fraction problem might sound something like this:

I already put in $2\frac{1}{3}$ cups of flour, but I need $4\frac{1}{2}$ cups. How much more do I need to put in?

PARALLEL TASKS FOR GRADES 3–5

PARALLEL TASKS are sets of two or more related tasks that explore the same big idea but are designed to suit the needs of students at different developmental levels. The tasks are similar enough in context that all students can participate fully in a single follow-up discussion.

Choice 1: Two fractions are equivalent. If you add the numerators, the result is 22 less than if you add the denominators. What could the fractions be?

Choice 2: Draw a picture to show two equivalent fractions for $\frac{2}{8}$.

CCSS: Number & Operations—Fractions: 3.NF, 4.NF
Mathematical Practices: 2, 5

BIG IDEA: 2.1

The fact that a part of a whole can be represented in many ways is fundamental to students' ability to add and subtract fractions. Although many students learn the rule for creating equivalent fractions, some use it without understanding why it works; others generalize the rule inappropriately, for example, adding the same amount to both numerator and denominator instead of only multiplying or dividing by the same amount.

The choice of tasks shown for this exercise allows the student who is just beginning to understand equivalence to show what he or she knows about why two fractions might be equivalent. It also allows the more advanced student to work in a more symbolic way to solve a problem involving equivalence. Because there are many solutions to ***Choice 1*** (e.g., $\frac{2}{4} = \frac{20}{40}$ and $(40 + 4) - (20 + 2) = 22$; $\frac{6}{10} = \frac{27}{45}$ and $(10 + 45) - (6 + 27) = 22$; $\frac{8}{10} = \frac{80}{100}$ and $(10 + 100) - (8 + 80) = 22$), students must recognize that they should try more than one combination of values.

Questions that could be asked of both groups include:

- *What two equivalent fractions did you use?*
- *How did you know they are equivalent?*
- *What kind of picture could you draw to show that they are equivalent?*
- *How did you solve your problem?*

➤ ***Variations.*** Instead of talking about the sums of numerators and denominators, Choice 1 could ask for two fractions where the differences between the respective numerators and denominators are 4 in one case and 44 in the other.

Choice 1: Choose a denominator $\square$ that is more than 4. Draw a picture that helps explain why $\frac{4}{\square}$ is what $4 \div \square$ turns out to be.

Choice 2: Choose a denominator $\square$ that is less than 4. Draw a picture that helps explain why $\frac{4}{\square}$ is what $4 \div \square$ turns out to be.

CCSS: Number & Operations—Fractions: 5.NF
Mathematical Practice: 5
BIG IDEA: 2.1

Some students will find it easier to relate an improper fraction to the concept of division. For example, a student might think of $\frac{4}{2}$ as $4 \div 2$ since $4 \div 2$ asks how many 2s are in 4, and that is exactly what you mean when you think of how many wholes 4 halves make.

Other students will be comfortable with proper fractions and thinking of division as sharing. They might think of $\frac{4}{10}$ as the amount an individual would get if 4 items are shared fairly among 10 people. (Each person gets $\frac{1}{10}$ of each of the 4 items, for a total of $\frac{4}{10}$.)

Still other students might be comfortable with proper fractions by thinking of, for example, $\frac{2}{3}$ as asking how much of a 3 fits in a 2 (instead of how many 3s fit in a number).

TEACHING TIP. When working with fractions, switching between proper and improper fractions is often an automatic and valuable way to differentiate.

Choice 1: List three fractions between 4 and 5. Explain how you know you are right.

Choice 2: List three fractions between $\frac{4}{8}$ and $\frac{5}{8}$. Explain how you know you are right.

CCSS: Number & Operations—Fractions: 3.NF, 4.NF
Mathematical Practice: 5
BIG IDEA: 2.2

Some students might be more comfortable determining fractions between 4 and 5 (simply using mixed numbers and then changing them to improper fractions), while others might be just as comfortable using equivalent fractions for $\frac{4}{8}$ and $\frac{5}{8}$ to determine fractions between these two (e.g., $\frac{9}{16}$ or $\frac{45}{80}$).

In either case, it is valuable to reinforce the notion that there are always more fractions that could have been selected.

Choice 1:	20 is _______ of _______. Fill in the blanks to make this statement true in four different ways so that the number in the first blank is a fraction with a numerator that is NOT 1 and the number in the second blank is any number you choose.
Choice 2:	8 is _______ of _______. Fill in the blanks to make this statement true in four different ways so that the numbers in each blank are fractions and the numerators are NOT 1.

CCSS: Number & Operations—Fractions: 3.NF, 4.NF
Mathematical Practices: 2, 5, 7
BIG IDEA: 2.3

Because fractions can be applied to different wholes, students can explore the notion that the same value (whether 20 or 8) can be different fractions of different wholes. For example, 20 is $\frac{2}{10}$ of 100, but it is also $\frac{2}{5}$ of 50. The second choice might be more difficult for some students, but it certainly can be handled by many students. For example, 8 is $\frac{40}{3}$ of $\frac{3}{5}$ or is $\frac{16}{4}$ of $\frac{8}{4}$.

Students who realize that they can apply the routine for multiplying fractions in ***Choice 2*** will probably experience more success.

Not allowing for unit fractions makes the task slightly more challenging.

➤ ***Variations.*** Unit fractions (with numerator 1) could be allowed as fill-ins.

Choice 1:	Model two fractions with the same numerator. Tell which is greater and why.
Choice 2:	Model two fractions with the same denominator. Tell which is greater and why.

CCSS: Number & Operations—Fractions: 3.NF
Mathematical Practices: 3, 5
BIG IDEA: 2.4

Students are given a great deal of flexibility in deciding which fractions to use. They could use simple denominators such as 2 or 4, or more complex ones. They are free to use proper or improper fractions, whichever they choose. Because many students find it simpler to compare fractions with the same denominator rather than the same numerator, ***Choice 2*** is provided. However, students ready for ***Choice 1*** should be encouraged to take it.

Questions that suit both tasks include:

- *Is either of your fractions really close to 0? Really close to 1?*
- *Is either of your fractions greater than 1? Is the other one?*
- *Do you need to rename the fractions to decide which is greater, or is it easy to tell without renaming?*
- *Which of your two fractions is greater? How could you convince someone?*

TEACHING TIP. Students should occasionally be encouraged to explain why they selected the choices they did.

Choice 1:	$\frac{\square}{3}$ is more than $\frac{*}{4}$. List four pairs of values for $\square$ and *, but don't use the same number for both $\square$ and *.
Choice 2:	$\frac{\square}{4}$ is less than $\frac{*}{8}$. List four pairs of values for $\square$ and *.

CCSS: Number & Operations—Fractions: 3.NF, 4.NF
Mathematical Practice: 6
BIG IDEA: 2.4

There are many possible solutions for each choice. For example, $\frac{2}{3}$ is more than $\frac{1}{4}$, or $\frac{10}{3}$ is more than $\frac{3}{4}$ for ***Choice 1***. And $\frac{3}{4}$ is less than $\frac{10}{8}$, or $\frac{9}{4}$ is less than $\frac{30}{8}$ for ***Choice 2***.

The use of 4ths and 8ths in ***Choice 2*** might make that task more attractive to some students. They might write $\frac{\square}{4}$ as an equivalent fraction with a denominator of 8 and then adjust the numerator.

Students might use a variety of strategies to compare their fractions and should be encouraged to explain their strategies.

Choice 1:	You add two fractions. The sum is an improper fraction with a denominator of 20. What might you have added? Think of a few possibilities.
Choice 2:	You add a fraction with a denominator of 6 to a fraction with a denominator of 9. What could the denominator of the answer be? Why?

CCSS: Number & Operations—Fractions: 5.NF
Mathematical Practices: 1, 3, 5
BIG IDEA: 2.5

In both instances, students consider the fact that when we add fractions, we normally use equivalent fractions with a common denominator. In ***Choice 1***, students are told the new denominator and asked to determine fractions; in ***Choice 2***, it is the reverse.

Students responding to the first task might consider fractions with denominators that are both 20, or one that is 4 and one that is 5, or one that is 4 or 5 and one that is 20. They also must ensure that the result is greater than 1. Students responding to the second task might realize that, depending on the numerators, the new denominator might be 9 or 18 or even 36.

Questions that suit all students include:

- *Are you free to choose any numerators you want?*
- *What is usually true about the denominators of the fractions you add compared to the denominator of the sum? Does that have to be true?*

Choice 1:	A rectangle has a length between 3" and 4", but closer to 3". It has a width between 1" and 2", but closer to 2". What could its area be?
Choice 2:	The area of a rectangle is $2\frac{1}{2}$ square inches. What could the length and width be?

CCSS: Number & Operations—Fractions: 5.NF **BIG IDEA:** 2.5
Mathematical Practices: 4, 6

In either instance, students use the notion that the area of a rectangle is the product of its length and width. In the first situation, the **linear dimensions** are given, but in the second situation, it is the product that is given and students are likely to work backward.

Students can choose the values with which they are comfortable in ***Choice 1***, for example, $3\frac{1}{4}$ rather than $3\frac{7}{16}$ or $1\frac{3}{4}$ rather than $1\frac{9}{10}$. In ***Choice 2***, there is still flexibility in choosing two values that multiply to $\frac{5}{2}$ (or an equivalent to $\frac{5}{2}$), for example, $\frac{5}{3} \times \frac{6}{4}$.

Questions that suit all students include:

- *What operations do you use when you are determining the area of a rectangle? Why?*
- *Could the number or numbers you are looking for get really high or not?* [**Note:** A length or width could be high if the other dimension is very small for ***Choice 2***. The area is limited in ***Choice 1***.]
- *How could you model that your calculations are correct using grid paper?*

Choice 1:	You fill 8 drinking glasses $\frac{3}{4}$ full. How many full-to-the-top glasses could have been filled instead?
Choice 2:	You fill some drinking glasses $\frac{5}{6}$ full. If you rearranged the water, you could have filled whole glasses instead with nothing left over. How many glasses could have been filled $\frac{5}{6}$ full?

CCSS: Number & Operations—Fractions: 4.NF **BIG IDEA:** 2.5
Mathematical Practices: 2, 3, 5

Both of the problems posed involve multiplication of a whole number by a fraction or **repeated addition** of a fraction, but the student must recognize that. In the first situation, the student knows exactly the values he or she has to work with, but the total is not known in ***Choice 2***, making it more challenging for some students. Working on either problem could help students realize that $c \times \frac{a}{b}$ is a whole number only if b is a factor of $c \times a$.

Whereas there is only one answer to ***Choice 1***, 6 glasses, there are multiple answers to ***Choice 2***, including 6 glasses, 12 glasses, 18 glasses, and so forth.

Whichever task is attempted, students could be asked:

- *Could it have been 2 glasses? Why or why not?*
- *Could it have been 3 glasses? Why or why not?*
- *Could it have been 4 glasses? Why or why not?*
- *How could thinking about 3 glasses or 4 glasses have helped solve the problem?*

Choice 1:	You divide a fraction of the form $\frac{1}{\square}$ by a whole number. The result is less than 0.1. Describe several possible fraction/whole number combinations and draw pictures to show why you are right.
Choice 2:	You divide two whole numbers and the result is a fraction less than 0.1. Describe several possible pairs of whole numbers and draw pictures to show why you are right.

CCSS: Number & Operations—Fractions: 4.NF
Mathematical Practice: 5
BIG IDEA: 2.5

Some students will gravitate toward ***Choice 2*** because they are attracted to working with whole numbers. Others will select ***Choice 1*** because they feel that one of the numbers they need to use is more apparent; it must be of the form $\frac{1}{\square}$.

In the first instance, the students will be looking for a denominator and a whole number with a product greater than 10, so the result will be less than 0.1, or $\frac{1}{10}$ (e.g., $\frac{1}{5} \div 4$). In the second instance, the students will be looking for a numerator that is less than $\frac{1}{10}$ of the fraction's denominator (e.g., $3 \div 50$, or $\frac{3}{50}$). In this case, students must show their understanding that a fraction is the quotient of the numerator and the denominator.

Questions a teacher might ask could include:

- *Could you be dividing by the whole number 2? Explain.*
- *Could you be dividing by a whole number less than 10? Explain.*
- *What kind of picture did you use to show your division? Why that kind?*
- *Once you got a first solution, how did you get a second one?*

Choice 1:	You multiply a fraction by $\frac{4}{3}$. What do you know, for sure, about the result?
Choice 2:	You multiply a fraction by $\frac{2}{3}$. What do you know, for sure, about the result?

CCSS: Number & Operations—Fractions: 5.NF
Mathematical Practices: 2, 3
BIG IDEA: 2.5

It is important for students to learn that multiplying by a fraction greater than 1 results in an answer greater than the number being multiplied, but multiplying by a fraction less than 1 results in a smaller answer. There are students who are less comfortable with improper fractions, so ***Choice 2*** is available for them.

In the proposed tasks, no particular numbers to multiply are provided. Students either will have to try a lot of different numbers or will have to think about the general effect of the multiplication.

A particular fraction, with a denominator of 3, was chosen as one of the multipliers to encourage an even greater diversity of responses. For example, some students will assume that if you multiply by thirds, the denominator of the result has to be a multiple of 3 (which may not be true after simplification, but will be true initially) or that if you multiply using a numerator of 2 or 4, the numerator of the result must be even (which also may not be true after simplification).

Questions that might be asked no matter which task is completed include:

- *What sort of model would you use to show the multiplication?*
- *What do you know about the numerator of the resulting fraction?*
- *What do you know about the denominator?*
- *Will your answer be more or less than the number you multiplied by? Why?*

SUMMING UP

MY OWN QUESTIONS AND TASKS

Lesson Goal: **Grade Level:** ______

Standard(s) Addressed:

Underlying Big Idea(s):

Open Question(s):

Parallel Tasks:

Choice 1:

Choice 2:

Principles to Keep in Mind:

- All open questions must allow for correct responses at a variety of levels.
- Parallel tasks need to be created with variations that allow struggling students to be successful and proficient students to be challenged.
- Questions and tasks should be constructed in such a way that will allow all students to participate together in follow-up discussions.

The five big ideas that underpin work with fractions and decimals were explored in this chapter through more than 20 examples of open questions and parallel tasks, as well as variations of them. The instructional examples provided were designed to support differentiated instruction for students in the Grades 3–5 grade band.

Fractions are a difficult concept for many students to grasp. It is important that they meet success in understanding fractions to prepare them for later success in the secondary grades.

The examples presented in this chapter are only a few of the possible questions and tasks that can be used to differentiate instruction in work with fractions. Other questions and tasks can be created by, for example, using alternate operations or alternate numbers (e.g., different numerator/denominator relationships). A form such as the one shown here can be a convenient template for creating your own open questions and parallel tasks. Appendix B includes a full-size blank form and tips for using it to design your own teaching materials.