Part 1

Operations and Algebraic Thinking

Operations and Algebraic Thinking

Domain Overview

GRADE 3

The major work of this domain in Grade 3 is to develop students' conceptual understanding of multiplication and division by using concrete materials to model multiplication and then relate their understanding of multiplication to division. Multiplication problem situations provide a context for understanding multiplication as finding the total number of items given a number of equal groups and the number of items in each group. Division problem situations develop the meaning of division and how it is related to multiplication. When you know the total number of items and the number of groups, you can determine how many items in a group, or, when you know the total number of items and the number of items in a group, you can find the number of groups. All of these activities culminate in the expectation that students will demonstrate fluency with multiplication and division within 100 using single-digit factors.

GRADE 4

Students in Grade 4 continue to solve problems using the four operations with whole numbers. New to this grade level are problem situations that involve multiplicative comparisons. Students become familiar with factors and multiples and how they relate to prime and composite numbers. They work in a variety of contexts to generate and analyze patterns.

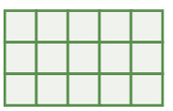
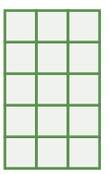
GRADE 5

In preparation for the Expressions and Equations domain in grades 6–8, fifth graders begin to explore, interpret, and evaluate numerical expressions. Work with patterns that began in Grade 4 extends to generating patterns, forming ordered pairs, graphing on a coordinate plane, and then analyzing the graphical representations.

SUGGESTED MATERIALS FOR THIS DOMAIN

3	4	5	
✓	✓	✓	Hundreds chart (Reproducible 1)
✓	✓	✓	Chips, counters
✓	✓	✓	Cups, containers, other objects to represent "groups"
✓	✓	✓	Place value chart to hundreds (Reproducible 2)
✓	✓	✓	Square tiles
✓	✓	✓	Grid paper (Reproducible 3)
✓	✓	✓	Pattern blocks
✓	✓	✓	Number cards (such as a deck of playing cards)

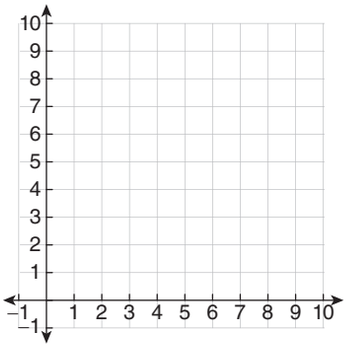
KEY VOCABULARY

3	4	5	
✓	✓	✓	**add** to combine or join together related words: *add, and, plus, join, put together, (+)*
✓	✓	✓	**addend** any of the numbers added to find a sum
✓	✓	✓	**area model** a concrete model for multiplication or division made up of a rectangle. The length and width represent the factors and the area represents the product.  3×5  5×3
✓	✓	✓	**array model** a concrete model for multiplication in which items are arranged in rows and columns. Each row (or column) represents the number of groups and each column (or row) represents the number of items in a group.  $5 \times 4 = 20$ 5 rows of $4 = 20$
✓	✓	✓	*** associative property of multiplication** an extension of the commutative property; to change the order and group two factors to find convenient products (such as 10) in order to make the multiplication easier. Students may begin to use parentheses at this level. $7 \times 8 \times 5 = 7 \times (8 \times 5) = 7 \times 40 = 280$
✓	✓	✓	*** commutative property of multiplication** reversing the order of the factors does not change the product $8 \times 5 = 40$ and $5 \times 8 = 40$ therefore the product of $8 \times 5 = 5 \times 8$

(Continued)

(Continued)

KEY VOCABULARY

3	4	5	
	✓	✓	**comparison model** a multiplication or division situation in which one number is a multiple of the other *Example:* Maya has 5 marbles. Alexa has 3 times as many. How many marbles does Alexa have?
	✓	✓	**compose** put a number together using other numbers 1 + 9, 2 + 8, 3 + 7, 4 + 6, 5 + 5, 1 + 2 + 3 + 4 are ways to compose 10
	✓	✓	**composite number** a number that has more than two factors
		✓	**coordinate plane** a plane determined by a horizontal number line, called the *x*-axis, and a vertical number line, called the *y*-axis, intersecting at a point called the origin. Each point in the coordinate plane can be specified by an ordered pair of numbers. 
✓	✓	✓	**decompose** separate a number into parts using other numbers 8 can be decomposed into 4 + 4, 3 + 5, 2 + 2 + 2 + 2
✓	✓	✓	*** distributive property** multiplying a sum by a given number is the same as multiplying each addend by the number and then adding the products 6 × 9 = 54 6 × (5 + 4) = (6 × 5) + (6 × 4) = 30 + 24 = 54 The distributive property says that if a, b, and c are real numbers, then: $a \times (b + c) = (a \times b) + (a \times c)$
✓	✓	✓	**division** sharing a number into equal groups and finding the number of groups or the number of items in each group
✓	✓	✓	**equal groups model (measurement division)** a division model in which the total number of items and the number of items in each group is known and the number of groups that can be made is the unknown. *Example:* I have 48 peanuts. I want to put 8 peanuts in a cup. How many cups will I need?
✓	✓	✓	**equation** a mathematical sentence in which one part is the same or equal to the other part 3 + 5 = 8 12 − 7 = 5 11 = 8 + 3 6 = 9 − 3
✓	✓	✓	**estimate** to make an approximation or calculate using closer or easier numbers
	✓	✓	**evaluate** find the numerical value of mathematical expression
✓	✓	✓	**expression** one or more mathematical symbols that represent a number or quantity examples of expressions 3 × 6 4 + 7 × 3 8
✓	✓	✓	**fact family** a set of related mathematics facts, such as 3 × 5 = 15 15 = 5 × 3 15 ÷ 3 = 5 3 = 15 ÷ 5
✓	✓	✓	**factor** one of the numbers multiplied to find a product

KEY VOCABULARY

3	4	5	
	✓	✓	**factor pair** a pair of numbers that when multiplied give a product; for example, 1 and 15, 3 and 5 are factor pairs for 15
✓	✓	✓	**fair share model (partitive division)** a division model in which the total number and the number of groups is known and the number of items in each group is unknown *Example:* I have 48 peanuts and want to put them into 6 cups. If I put the same number of peanuts into each cup, how many peanuts will be in each cup?
✓	✓	✓	*** identity property of multiplication** any number multiplied by 1 equals the number $3 \times 1 = 3$ $1 \times 3 = 3$
✓	✓	✓	**measurement division (equal groups model)** a division model in which the total number of items and the number of items in each group is known and the number of groups that can be made is the unknown *Example:* I have 48 peanuts. I want to put 8 peanuts in a cup. How many cups will I need?
✓	✓	✓	**missing factor** the unknown factor when a product and one factor are known $4 \times \square = 32$ The missing factor is 8.
	✓	✓	**multiple** the result of multiplying a whole number by other whole numbers multiples of 5 are 0, 5, 10, 15, 20, 25, 30
✓	✓	✓	**multiplication** a mathematical operation in which a number is added to itself a specific number of times; one factor tells the number of groups or sets, the other factor tells the number of items in a group or set and the result, or product, tells the total number of items $3 \times 5 = 15$ 3 groups with 5 in each group would give a total of 15
✓	✓	✓	**number line** a line used to show the position of a number in relation to zero and other numbers
		✓	**ordered pair** a pair of numbers that gives a location on a coordinate plane. The first number is the *x* coordinate and the second number is the *y* coordinate.
✓	✓	✓	**partitive division (fair share model)** a division model in which the total number and the number of groups is known and the number of items in each group is unknown *Example:* I have 48 peanuts and want to put them into 6 cups. If I put the same number of peanuts into each cup, how many peanuts will be in each cup?
✓	✓	✓	**pattern** set of numbers or objects that can be described by a specific rule
	✓	✓	**prime number** a number that has exactly two factors
✓	✓	✓	**product** the result when two numbers are multiplied
✓	✓	✓	**quotient** the result when two numbers are divided; the missing factor
✓	✓	✓	**remainder** amount left when two numbers are divided
✓	✓	✓	**round** to change a number to a less exact number that is more convenient for computation
✓	✓	✓	**strategy** a plan to find an answer or solve a problem that makes sense
✓	✓	✓	**sum** the result when two numbers are added
✓	✓	✓	**unknown** the quantity you are finding in a mathematics problem
✓	✓	✓	*** zero property of multiplication** any number multiplied by 0 equals 0 $8 \times 0 = 0$ (8 groups of 0 is 0) $0 \times 8 = 0$ (0 groups of 8 is 0)

*Students are not responsible for these vocabulary words; however, they should understand the mathematical concept.

Operations and Algebraic Thinking
3.OA.A.*

Cluster A

Represent and solve problems involving multiplication and division.

STANDARD 1

3.OA.A.1: Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5×7.

STANDARD 2

3.OA.A.2: Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as* $56 \div 8$.

STANDARD 3

3.OA.A.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

[1] See Table 2 in the Resources, page 256.

STANDARD 4

3.OA.A.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations* $8 \times ? = 48$, $5 = \_ \div 3$, $6 \times 6 = ?$

*Major cluster

Operations and Algebraic Thinking 3.OA.A

Cluster A: Represent and solve problems involving multiplication and division.
Grade 3 Overview

Third grade students explore the meaning of multiplication as finding the total number of objects (product) when they know the number of groups (factor) and the number of items in each group (factor). The relationship between multiplication and division helps students understand that when dividing, they are finding the number of groups (missing factor) when they know the total count (product) and the number of items in a group (factor), or finding the number of items in a group (missing factor) when they know the number of groups (factor) and the total count (product). Problem solving situations and activities that include a variety of representations showing equal-sized groups, arrays, and area models lay the foundation for multiplication and division of whole numbers.

Note that these Standards are not linear. It is important for students to understand the meaning of multiplication and division (3.OA.A1, 3.OA.A.2) through the use of problem situations (3.OA.A.3). As students demonstrate understanding they begin to relate models to symbolic notation (3.OA.A.4). The use of symbols for easier facts and relating the symbols to fact families should be happening as students continue to use models to solve problems with the more difficult facts.

Standards for Mathematical Practice
SFMP 1. Make sense of problems and persevere in solving them.
SFMP 2. Use quantitative reasoning.
SFMP 3. Construct viable arguments and critique the reasoning of others.
SFMP 4. Model with mathematics.
SFMP 5. Use appropriate tools strategically.
SFMP 6. Attend to precision.
SFMP 7. Look for and make use of structure.
SFMP 8. Look for and express regularity in repeated reasoning.

Students solve a variety of problems as contexts for learning what it means to multiply or divide. They use quantitative reasoning to determine what is happening when they multiply (given the number of groups and the number of items in a group, they find the total number of items) and divide (given the total number of items and the number of groups, they find the number of items in a group *or* given the total number of items and the number of items in a group, they find the number of groups). Constructing mathematical arguments to justify their reasoning and comparing their strategies with those of classmates helps students to make connections among ideas and between concrete models and numerical notations (expressions and equations). They use a variety of tools to model multiplication and division including sets, arrays, area models, and the number line to represent what is happening when they multiply or divide. Developing the mathematical vocabulary of multiplication and division (factor × factor = product and product ÷ factor = missing factor) helps students to explain their thinking not only about the individual operations but also how they are related to each other. The commutative, associative, and distributive properties lay the foundation for fluency with basic facts through looking at the structure of multiplication and division and provide students with strategies for solving problems. Students use patterns and repeated reasoning (multiplication by 0, 1, 5, and 10) to help them identify patterns and become fluent with basic facts.

Related Content Standards

2.OA.C.3 2.OA.C.4 4.OA.A.1 4.OA.A.2

Notes

STANDARD 1 (3.OA.A.1)

Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.

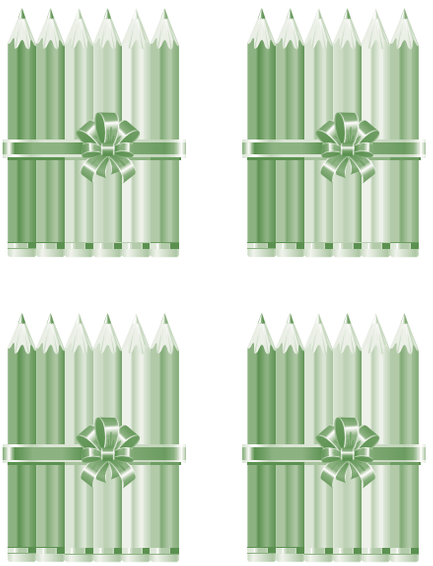
Students develop an initial understanding of multiplication of whole numbers by modeling situations in which there are a specific number of groups with the same number of items in each group. Unlike addition, in which each addend represents a certain number of items, in multiplication one factor represents the number of groups and the other factor represents the number of items in each group. The product represents the total number of items in all of the groups. Problem situations provide students contexts for using concrete materials.

New vocabulary includes *factor*, *equal groups*, *product*. The symbol × means groups of (or times) and 3 × 5 can initially be read as "3 groups of 5."

Example:

Mrs. Flack has 4 packages of pencils for the class. Each package contains 6 pencils. How many pencils does Mrs. Flack have?

This can be expressed as 4 × 6 or 4 groups of 6.



What the TEACHER does:

- Provide students with a variety of multiplication situations to model using concrete materials such as chips, counters, straws to represent the items, and cups, egg carton, paper to represent groups.
- Ask students to identify the number of groups and the number of items in each group and then the total number of items.

  There are 3 seats in the van and each seat can hold 4 people. How many people can ride in the van?

3 groups of 4 people

3 × 4 = 12

- Introduce multiplication terminology and symbols as students are ready. Add these terms and symbols to the class multiplication and division word wall.
  - factors, product, groups, times
  - ×
- Introduce students to numerical representations by writing equations that represent their work.

  3 × 4 = 12
- As students show understanding and can identify the number of groups, the number of items in a group, and connect that to the symbolic representation, progress to situations with pictures, numbers, and words.

What the STUDENTS do:

- Use concrete materials to model various multiplication situations.
- Identify the number of groups and the number of items in each group.
- Explain how they determined the total number of items.
- Connect representations to numeric expressions.
- Use pictorial representations for multiplication situations.
- Use appropriate vocabulary (*factor*, *product*, *times*, *groups of*) to describe their work.
- Write expressions and equations for their models and drawings.

Addressing Student Misconceptions and Common Errors

In previous work with addition, both addends represented the count or number of items that are joined for a total count. For example, 6 markers and 3 more markers give a total of 9 markers. In multiplication, one factor represents the number of groups, sets, or collections, and the other factor represents the number of items in each group, set, or collection. Students need multiple experiences identifying which factor represents the number of groups and which factor represents the number of items in each group. Early experiences with concrete models and pictures and explicit connections to the symbolic notation will not only help students to identify multiplication situations but will also support student understanding of division.

Notes

STANDARD 2 (3.OA.A.2)

Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Once students understand the meaning of multiplication in terms of finding the total number of items given the number of groups and the number of items in a group, division can be understood by thinking in terms of finding a missing factor (either the number of groups or the number of items in a group).

There are two distinct meanings of division. The first is the partitive (or fair share) meaning.

Example 1:

John has 32 crayons and 4 bags. If he wants to put the same number of crayons in each bag, how many crayons will he put in each bag?

In this case, John knows the total number of items (product) and the number of bags or groups (factor) and he is looking for the number of items to put in each bag (missing factor). This can be written as 4 × ____ = 32 or as the division expression 32 ÷ 4 = ____.

The other meaning of division is the measurement (or repeated subtraction) meaning.

Example 2:

John has 32 crayons. He wants to give 8 crayons to each person in his group. How many people are in John's group?