Preface

This book is about becoming a teacher of mathematical modeling. Did you pick up this book because you know something about mathematical modeling in classrooms, or maybe because you are interested in learning more about the process? Our goals with this book are to help you better understand mathematical modeling and its place in your classroom and to set the course for your journey toward becoming a teacher of mathematical modeling.

What do we mean by "mathematical modeling"? *Mathematical modeling* is the process of using mathematics to understand and make decisions about the world. When we have worked with teachers who are beginning to teach mathematical modeling, they have wondered "What makes a good modeling task?" and "How do I know that my students are modeling?" Throughout the book, you will have the opportunity to explore both of these questions as we introduce teaching practices and classroom vignettes that highlight different parts of the process of teaching modeling. A good modeling task is one that addresses a question that is meaningful to students and allows for multiple, rich mathematical approaches. It is one that engages students in making decisions that require them to use their judgment. You will know that students are modeling when they are using their mathematical knowledge to make sense of and make decisions about a situation.

You have probably used the word *modeling* in connection with teaching and with mathematics in a few different ways. In this book, we refer to classroom mathematical modeling as a process that begins and ends with an authentic situation that is meaningful to students, and of course that is different from some of the other meanings of the phrase *mathematical modeling*. To some, modeling might mean using manipulatives or graphing software to illustrate mathematical processes. Although using tools like these might be part of the modeling process, it is not the focus of this book. Others may understand modeling as connected to the mental models that describe students' mathematical thought processes. Again, mental modeling can be a part of engaging students in modeling, but it is not the focus here. Still others might connect modeling to teacher modeling—that is, demonstrating thinking processes or enacting appropriate, student-centered teaching practices. Although techniques like these are valuable and might be useful as a part of teaching modeling, when we say *mathematical modeling* we mean students using their mathematical skills to improve their understanding of a situation or problem that is based on their day-to-day experiences.

You may have noticed that we refer to mathematical modeling as a process. Processes typically have multiple steps for getting to an end goal. The modeling process begins when modelers encounter authentic problems and formulate these problems mathematically. They choose variables and make assumptions. From there, the modelers devise and implement mathematical solution strategies. Modelers interpret and then validate solutions in light of the original problem. If modelers are satisfied with their solution, they issue some kind of a report. If not, they revise and refine their model in pursuit of a better outcome. The revision step is why modeling is typically conceptualized as a cycle. Although a variety of depictions of modeling cycles

are available from different authors, most represent the same general process. (You will see our depiction in figure 2.1 of chapter 2.)

We offer a bit of caution here. If you are too focused on the components of a cycle, it can be easy to get lost in the cycle's phases. Do not be tempted to envision modeling as a sequence of steps to be checked off while marching toward an answer. Remember that each and every student brings their own knowledge and perspectives to a problem, and it is likely that the modeling process will unfold in different ways for different problems. To help navigate this complexity, rather than focusing on modeling as a series of steps, we ground our conceptualization of modeling in four big ideas that underpin the process of mathematical modeling, and we discuss these ideas throughout the book.

We have been inspired by watching mathematical modeling unfold in K–5 classrooms. We have seen students excited to share their ideas and perspectives when faced with a problem they care about. We have seen them tackle new mathematical content in order to make sense of the situation. We have seen students use creative problem-solving strategies and form robust classroom communities as they share and refine their solutions. Importantly, we have witnessed joy in mathematics and empowerment as students envision themselves as knowers and doers of mathematics.

In requiring students to be creative as they tackle mathematical problems, mathematical modeling empowers students. In the book *Mathematics for Human Flourishing*, Francis Su (2020, p. 143) describes power and empowerment in relation to mathematics:

> To use, teach, or learn math effectively requires careful thinking about power dynamics: how people interact, who has authority, who has freedom and who is constrained, who is encouraged and who is shut down, who is included and who is excluded, implicitly or explicitly. These are all questions of power. If there is any simple criterion to guide your actions with respect to power, it is this virtue: creative power elevates human dignity. What does it mean to "empower" someone? It means to affirm their dignity as a creative human being.

Mathematical modeling in classrooms affirms students' dignity as creative—and mathematical—human beings.

Mathematical modeling requires students to understand the ideas and perspectives of others. It is in seeking to understand others' points of view that human beings engage in empathy. As we think about our role in education, we take to heart our responsibility to use education to leave the world better than we found it. In today's era of political and social discord, we are troubled by how many issues are framed as though they are dichotomous, with one right answer being fought over by two opposing sides. One role of education is to teach students critical thinking skills, and we see in teaching modeling the opportunity to teach *empathetic* critical thinking skills. We see in modeling a way for issues to be understood beyond right or wrong, and even beyond "agree to disagree." We see in modeling a way for students—human beings—to use their mathematical skills to examine different solutions to authentic problems based on different perspectives. The modeler

emerges from the modeling process knowing that different perspectives of a problem highlight different points of view that different people hold. A complex problem does not have a right or wrong answer; instead, it has a problem statement and a proposed solution that capture something important about what another human being values. Two different models of the same situation can be useful, and these different models capture the perspectives of different human beings. A student who understands this tenet has learned empathetic critical thinking skills.

In writing this book, we have drawn on our experiences of working with teachers in elementary classrooms; our experiences leading professional development about mathematical modeling; our experiences teaching mathematical modeling to undergraduates studying to be elementary or secondary teachers; our experiences as classroom teachers in elementary and secondary schools; and our experiences as mathematical modelers in university departments of mathematical sciences. Although the modeling experiences in our collective past stretch across decades, our recent experiences with future teachers at Montana State University and our professional development partnership with the Bozeman School District are particularly influential on our current knowledge about teaching mathematical modeling, and we are incredibly grateful to all those students and teachers who have helped us gain this knowledge.

This book is a companion to *Becoming a Teacher of Mathematical Modeling, Grades 6–12*, also published in 2021. Teaching practices for modeling in the elementary and secondary grades overlap, and the content of both books overlaps as well. In 2016, we published a chapter in NCTM's *Annual Perspectives in Mathematics Education* called "A Case for Mathematical Modeling in the Elementary School Classroom" (see chapter 5), in which we articulated what mathematical modeling could look like in elementary classrooms. We argued that students—each and every one—no matter of their age or current mathematical understanding, should have the experience of modeling. The four big ideas about modeling that form the foundation of our teaching recommendations span the elementary and secondary grades. Our description of the four big ideas and broad descriptions of teaching mathematical modeling are very similar in both books.

This book describes a journey, segmented into three parts, and we will travel alongside you as you explore what it means to teach modeling. In part I, we help you understand the journey and what it entails. We discuss mathematical modeling broadly and allow you to explore the student practices and perspectives involved, how modeling can empower students, and classroom communities that support modeling. In part II, we launch you on your journey of becoming a teacher of modeling by illustrating features of modeling that are evident in the classrooms of teachers who engage their students in modeling and by sharing specific strategies for making modeling a part of your own classroom practices. In part III, we share wisdom from other teachers who have engaged in this journey and invite you to join in the modeling community.

We have included classroom vignettes throughout the book that illustrate different features of mathematical modeling. These vignettes are based on contexts we have experienced in K–5 settings, but often the vignette is an amalgam of different teachers, students, and classrooms, so these are hypothetical examples rather than anonymized case studies. We have named the teachers who are the characters in the vignettes.

We do not intend for these vignettes to provide a collection of modeling tasks for you to use directly with the students in your classroom. Instead, the vignettes will help you envision the process of integrating modeling into your own classroom. As you read through the vignettes, reflect on the features of modeling highlighted there. At the end of each chapter, we have posed reflection questions, and as you complete those, you will be able to plan a modeling task that you can implement in your classroom. We have also provided some teachers' modeling lesson plans in more4u (see www.nctm.org/store/more4u).

Equity-minded teaching is always our aim, and we invite you to think about the integration of access, equity, and empowerment in mathematical modeling throughout your journey. In our work, we have come to realize that mathematical modeling inherently provides opportunities for access, equity, and empowerment for each and every student. To share our insights with you, we have "Spotlight on Access, Equity, and Empowerment" callout boxes throughout the book where we highlight ways that teachers and students address these ideas.

Spotlight on Access, Equity, and Empowerment

Callout boxes like these are featured throughout the book to highlight ways that teachers and students address issues of access, equity, and empowerment.

We invite you to begin the journey of becoming a teacher of mathematical modeling!

ACKNOWLEDGMENTS

We wish to thank the many individuals who have helped us all develop these ideas about mathematical modeling and teaching modeling, especially the classroom teachers and students with whom we have worked. We would not have been able to write this book without their willingness to try new things, their insights into the modeling process, and their generosity in inviting us into their classrooms. We would especially like to thank Barb Bolte, Brian Brown, Jenny Cade, Ruth Carr, Roxi Cook, Erin Farrell, Kelly Fulton, Kristi Gaines, Lindsay Hall, Kimberly King, Marilyn King, Dacia Lackey, Gia LaForge, LeAnne Lorenz, Christine O'Shea, and Whitney Todd.

This material is based, in part, upon work supported by the National Science Foundation under Grant No. 1441024 and Grant No. 1810992. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.