

Contents

Preface	viii
Preface to Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics	ix
Acknowledgements	x
1. Introduction.....	1
Purpose of This Guide	1
Purpose of Curriculum Focal Points.....	1
Impact of Focal Points on Curriculum, Instructions, and Assessment	2
Using This Guide in Study Groups or Learning Communities	2
Focal Points and the Common Core State Standards for Mathematics	3
Bringing Focus into the Classroom: Classrooms That Build Understanding and Fluency.....	4
Using mathematics drawings.....	5
Learning phases	5
Moving to mathematically desirable methods.....	5
The standard algorithmic approach.....	6
Conceptual prerequisites	6
In-depth instructional conversations	7
Differentiating instruction within whole-class activities	7
Mathematizing across number and operations, geometry, and measurement.....	7
Effective Teaching-Learning Practices	8
Organization of This Book.....	9
2. Number and Operations.....	15
Overview	15
The Number Core.....	18
The number core developed in preschool and kindergarten.....	18
Difficulties in English number words for 11 to 19	18
Difficulties in English number words for 20 to 29	19
A kindergarten advance: seeing/making ten and some ones for a teen number	21
Learning the number-list to 100	24
Written work for triads to ten and for teens.....	25
The number core developed in grade 1	26

Contents — Continued

The number core developed in grade 2	29
The Relations (More Than/Less Than) Core	32
The relations (more than/less than) core developed in preschool, kindergarten, and grade 1	32
The relations core (more/less) developed in grade 2.....	33
The Operations (Addition and Subtraction) Core: Single-Digit Addition and Subtraction	33
Overview.....	33
Single-digit addition and subtraction developed in preschool and kindergarten.....	34
Types of addition/subtraction situations	34
Solving by modeling.....	35
Kindergartners also represent situations with equations.....	35
Language learning.....	36
Partners as embedded numbers	37
Roles of worksheets in kindergarten.....	37
Single-digit addition and subtraction developed in Grade 1	38
Levels in addition/subtraction solution methods	38
More difficult problem types in grade 1: Solving algebraic problems.....	44
Additive comparison situations	47
Levels of thinking and word-problem subtypes	49
Number lines are not appropriate for children before grade 2.....	50
Fluency with single-digit addition and subtraction.....	53
Single-digit addition and subtraction developed in grade 2	53
Solving more difficult problems	53
Variability in representing and solving	54
The importance of representing problem situations with numerical drawings related to equations	57
Representing and solving two-step problems.....	58
Fluently add and subtract within 20.....	58
The Operations (Addition and Subtraction) Core: Multidigit Addition and Subtraction	59
Overview of the learning progression for multidigit addition and subtraction across grade levels 1 to 4.....	59

Contents — Continued

Multidigit addition and subtraction developed in grade 1	64
Multidigit addition and subtraction developed in grade 2	72
Multidigit addition in grade 2	72
Multidigit subtraction in grade 2	75
Multidigit addition and subtraction and length	78
Understanding monetary values in kindergarten, grade 1, and grade 2	82
Understanding time in kindergarten, grade 1, and grade 2	82
Number and Operations in Grade 3	85
The Importance of Discussing Student Reasoning	86
3. Geometry, Spatial Reasoning, and Measurement	91
Geometry	91
Shape and structure	94
Spatial Relations	101
Compositions and decompositions in space	104
Measurement	107
Length	108
Concepts in linear measurement	108
Development of length measurement concepts	109
Area measurement and spatial structuring	112
Concepts of area measurement	112
Volume and capacity	114
Geometry, Spatial Reasoning, and Measurement: Final Words	114
4. Standards for Mathematical Process and Practice in	
Second Grade	117
The NCTM Process Standards	117
The Standards for Mathematical Practice of the Common Core State Standards in	
Mathematics	118
Final thoughts on mathematical processes and practices	119