MATHEMATICAL lens

The Three Stooges Meet the Conic Sections

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Expansion of the Solution to Question 4(a)

Estimating $y_1{}'$ and $y_2{}'$ in equation (4) yields

$$2m\left(x_0'\right)^2\left(\frac{2hr^2s}{r^2-d^2}-2y_0'\right)+x_0'\left(\frac{2hr^2s}{r^2-d^2}-2y_0'\right)^2=2mr^2s^2\left(\frac{2hr^2s}{r^2-d^2}-2y_0'\right)+x_0'\left(\frac{2hr^2s}{r^2-d^2}\right)^2,$$

implying that

$$\begin{aligned}&2m\left(x_0'\right)^2\left(r^2-d^2\right)\left(2hr^2s-2y_0'\left(r^2-d^2\right)\right)+x_0'\left(2hr^2s-2y_0'\left(r^2-d^2\right)\right)^2\\&=2mr^2s^2\left(r^2-d^2\right)\left(2hr^2s-2y_0'\left(r^2-d^2\right)\right)+x_0'\left(2hr^2s\right)^2.\end{aligned}$$

Letting $R=r^2$, we obtain the following:

$$\begin{aligned}&2m\left(x_0'\right)^2\left(R-d^2\right)\left(2hRs-2y_0'R+2y_0'd^2\right)+x_0'\left(2hRs-2y_0'R+2y_0'd^2\right)^2\\&=2mRs^2\left(R-d^2\right)\left(2hRs-2y_0'R+2y_0'd^2\right)+4x_0'h^2R^2s^2\end{aligned}$$

or

$$\begin{aligned}&2mRs^2\left(R-d^2\right)\left(2hRs-2y_0'R+2y_0'd^2\right)-2m\left(x_0'\right)^2\left(R-d^2\right)\left(2hRs-2y_0'R+2y_0'd^2\right)\\&+4x_0'h^2R^2s^2-x_0'\left(2hRs-2y_0'R+2y_0'd^2\right)^2=0\end{aligned}$$

or

$$\begin{aligned}&2m\left(R-d^2\right)\left(2hRs-2y_0'R+2y_0'd^2\right)\left(Rs^2-\left(x_0'\right)^2\right)\\&+x_0'\left[\left(2hRs-\left(2hRs-2y_0'R+2y_0'd^2\right)\right)\times\left(2hRs+\left(2hRs-2y_0'R+2y_0'd^2\right)\right)\right]=0\end{aligned}$$

or

$$2m\left(R-d^2\right)\left(2hRs-2y_0'R+2y_0'd^2\right)\left(Rs^2-\left(x_0'\right)^2\right)+x_0'\left[\left(2y_0'R-2y_0'd^2\right)\left(4hRs-2y_0'R+2y_0'd^2\right)\right]=0$$

or

$$2m\left(R-d^2\right)\left(2hRs-2y_0'R+2y_0'd^2\right)\left(Rs^2-\left(x_0'\right)^2\right)+2x_0'y_0'\left(R-d^2\right)\times 2\left(2hRs-y_0'R+y_0'd^2\right)=0$$

or

$$m\left(2hRs-2y_0'R+2y_0'd^2\right)\left(Rs^2-\left(x_0'\right)^2\right)+2x_0'y_0'\left(2hRs-y_0'R+y_0'd^2\right)=0 \text{ if } R-d^2\neq 0$$

or

$$\begin{aligned}&mhs^3R^2-mh\left(x_0'\right)^2sR-my_0's^2R^2+m\left(x_0'\right)^2y_0'R+my_0'd^2s^2R-m\left(x_0'\right)^2y_0'd^2\\&\quad+2hx_0'y_0'sR-x_0'\left(y_0'\right)^2R+x_0'\left(y_0'\right)^2d^2=0\end{aligned}$$

or

$$\begin{aligned}&R^2\left(mhs^3-my_0's^2\right)+R\left(-mh\left(x_0'\right)^2s+m\left(x_0'\right)^2y_0'+my_0'd^2s^2+2hx_0'y_0's-x_0'\left(y_0'\right)^2\right)\\&\quad+\left(x_0'\left(y_0'\right)^2d^2-m\left(x_0'\right)^2y_0'd^2\right)=0\end{aligned}$$

or

$$R^2\left[ms^2\left(hs-y_0'\right)\right]+R\left[m\left(x_0'\right)^2\left(y_0'-hs\right)+y_0's\left(md^2s+hx_0'\right)+x_0'y_0'\left(hs-y_0'\right)\right]+\left[x_0'y_0'd^2\left(y_0'-mx_0'\right)\right]=0$$

or

$$R^2\left[ms^2\left(hs-y_0'\right)\right]+R\left[\left(hs-y_0'\right)\left(x_0'y_0'-m\left(x_0'\right)^2\right)+y_0's\left(md^2s+hx_0'\right)\right]+\left[x_0'y_0'd^2\left(y_0'-mx_0'\right)\right]=0.$$