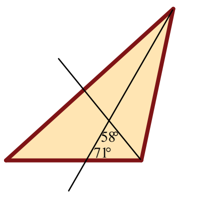
MAY

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

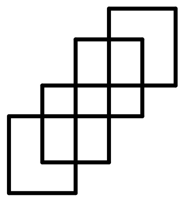
A quotation by a famous mathematician is encrypted in this month's problems. The answer to each problem is a number that corresponds to a letter of the alphabet (1 = A, 2 = B, . . . , 26 = Z). After you find the quote, identify the mathematician who said these words.

An angle bisector makes an angle of 71° with the opposite side and intersects another angle bisector at 58°. In degrees, what is the measure of the smallest angle of the triangle? (The figure is not drawn to scale.)



1

What is the total number of rectangles (of any dimension) in the figure?



2

A convex n-gon has 5 times as many diagonals as sides. Find the value of n.

3

Originally, five statements were listed below. Unfortunately, the last two were erased! How many of the remaining statements are true?

(a) Statement b is true.
(b) At most, one of these five statements is true.
(c) All five statements are true.
(d)
(e)

4

When a quintic and a cubic polynomial are graphed in the same coordinate plane, what is the maximum number of possible intersection points?

5

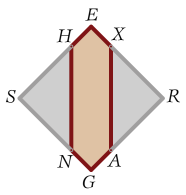
Three hundred students attend a local school. Each student takes 6 classes, and each teacher teaches 4 classes. If each class consists of 1 teacher and 25 students, how many teachers are there at the school?

6

Each of ten judges for a beauty pageant ranks his or her top three choices from the pool of contestants. A first-place ranking earns 5 points, a second-place ranking earns 3 points, and third-place ranking earns 1 point. What is the maximum number of judges that could rank a contestant who scores a total of 37 points?

7

Square $SGRE$ has perimeter 24. Points H and N lie on $\overline{SE}$ and $\overline{SG}$, respectively, such that $SH = SN$. Segment $\overline{XA}$ is the reflection of $\overline{HN}$ in $\overline{EG}$. Find the perimeter of $HEXAGN$ to the nearest integer if its area is half that of $SGRE$.



8

Find the area of the region in the fourth quadrant bounded by the coordinate axes and the lines $y = x - 3$ and $2x - 2y = 14$.

9

How many positive integers from 1 to 400, inclusive, have exactly three positive integral factors?

10

Determine the minimum value obtained by

$$f(x, y) = 4x^2 + 9y^2 - 12(x - y) + 18.$$

11

How many ways are there to write 18 as the sum of three distinct positive integers?

12

How many paths will spell GEOMETRY? Begin at G; descend one row and go to the immediate left or right to select the next letter. Repeat, ending with the leftmost (red) Y.

```
          G
        E   E
      O   O   O
    M   M   M   M
  E   E   E   E   E
T   T   T   T   T   T
  R   R   R   R   R
    Y   Y   Y   Y
```

13

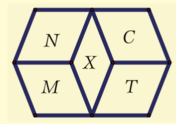
Let $a_1 = 0$, and for $n > 0$, $a_{n+1} = a_n^{\,2} + i$, where $i = \sqrt{-1}$.. If

$$|a_{2015}| = \sqrt{k}$$

for some positive integer k, find k. (Recall that $|a + bi| = \sqrt{a^2 + b^2}$ for any complex number $a + bi$.)

14

As arranged, four congruent rhombi N, C, T, and M each have area $\sqrt{99}$.. How many different integral areas are possible for quadrilateral X?



15

If

$$x + \frac{1}{x} = \sqrt{22},$$

find the exact value of

$$x^2 + \frac{1}{x^2}.$$

16

For real numbers x and y, define $x \oplus y = xy - x - y$. Calculate

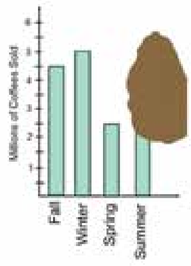
$$(((4 \oplus 3) \oplus 2) \oplus 2).$$

17

A king died, leaving his gold bars to his 6 sons. The youngest son received a certain number of gold bars, the second youngest twice as many, the third youngest three times as many, and so on. The queen ordered each son to give 2 gold bars to every son younger than himself. If, after this, all the sons had the same number of gold bars, how many did each now have?

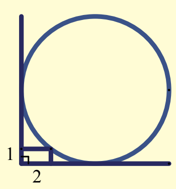
18

The CEO of a coffee company spilled coffee on the yearly report. If summer sales were 20% of the total annual sales, how many millions of coffees were sold in the summer?



19

The intersection of two lines forms one vertex of a 1 × 2 rectangle. A circle tangent to the two lines passes through the opposite vertex of the rectangle. If the interiors of the circle and rectangle do not overlap, how many units is the radius of the circle?



20

The first three figures of a sequence are shown. Figures 1, 2, and 3 are composed of 1, 5, and 13 unit squares, respectively. If this pattern continues, which figure will contain 421 unit squares?

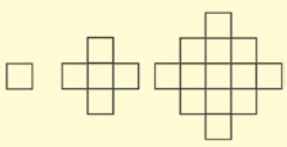


Fig. 1 Fig. 2 Fig. 3

21

How many more 1s are in the representation of 2015 in base 2 than there are 3s in the representation of 2015 in base 4?

22

Suppose that $\sec t = 4\sqrt{15}/15$, where t is the measure of an acute angle. Find the exact value of

$$\log_{1/4} \cos t + \log_{1/4} \tan t.$$

23

Find the exact area of the triangle whose side lengths are the roots of

$$f(x) = x^3 - 18x^2 + 105x - 200.$$

24

The mathematics department consists of 9 men and 11 women, including a married couple, Mr. and Mrs. Lebesgue. When a committee of 3 men and 3 women is chosen at random, the probability that the couple is part of the committee can be written as p/q in simplest form. Find $p + q$.

25

Let $f(x) = \ln(6 - x)$ and let $g(x) = |x^2 - 10x + 15|$. The domain of $(f \circ g)(x)$ can be written in interval notation as $(a, b) \cup (c, d)$ for values a, b, c, d. Find $a + b + c + d$.

26

Find the y-intercept of the perpendicular bisector of the segment whose endpoints are (1, 1) and (7, 9).

27

A cube has a volume of 432 units^3. What is the volume of the pyramid formed by slicing the cube with a plane passing through the midpoints of three intersecting edges?

28

For an unfair six-sided die with pips showing the whole numbers 1 through 6, the probability of rolling any odd number is twice that of rolling any even number. Let p be the probability of rolling a sum of 6 on two rolls of this die. Find the value of $81p$.

29

Each boy in a family has as many sisters as brothers, but each girl has twice as many brothers as sisters. How many children are in the family?

30

At Pizza Pi Restaurant, every sixth customer receives a free drink, and every eighth customer receives a free slice of pizza. How many of the day's 79 customers received a free drink or a free pizza slice?

31