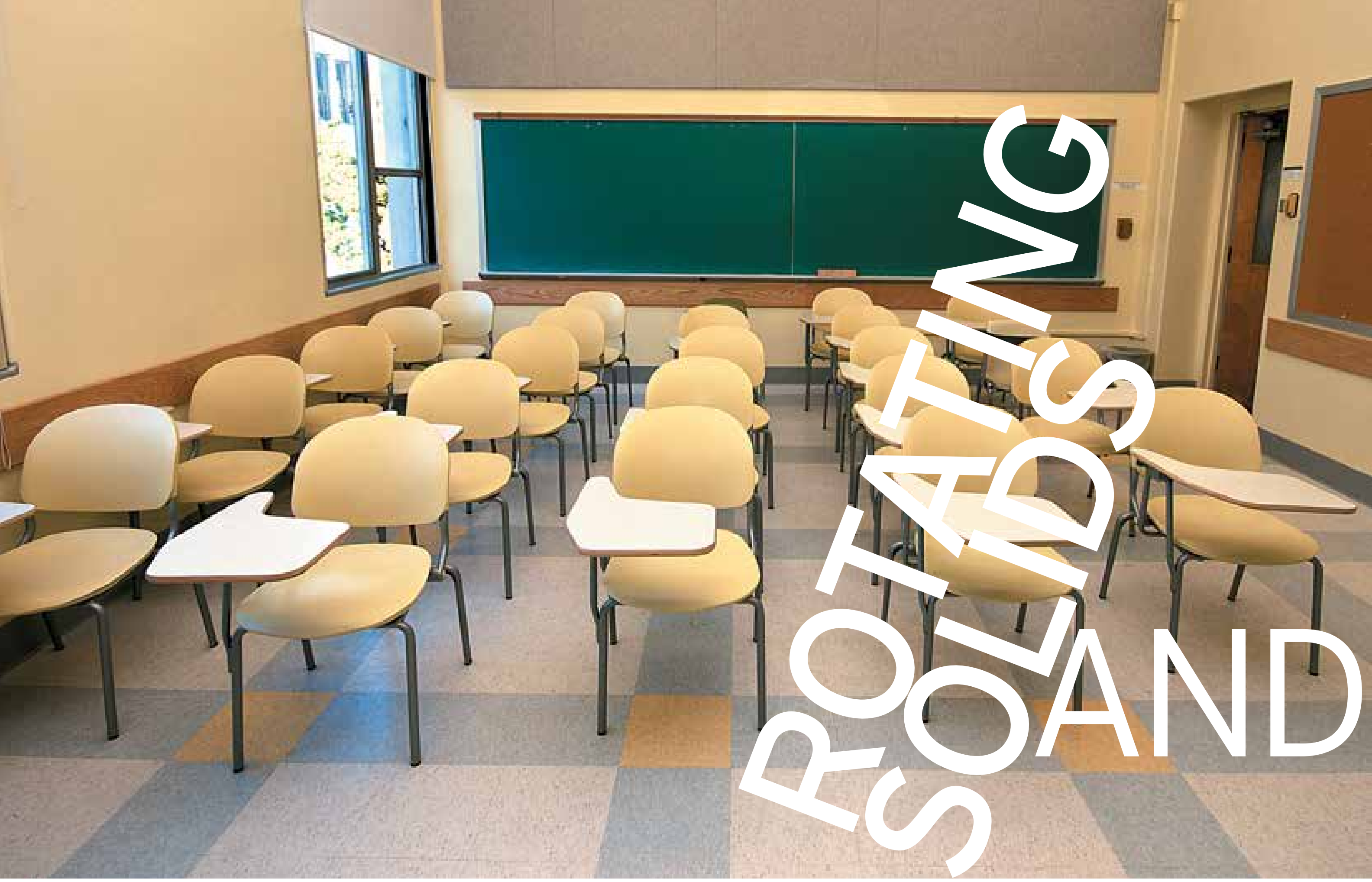


ROTATING SOLIDS AND

Lynette Grypp and Jennifer Luebeck

Action research helped one classroom teacher make thoughtful, data-based

Technology is causing us to rethink the entire notion of classroom learning, not only with respect to what learning should take place but also where it should take place. Walk down a high school hallway, and you will see teachers and students accessing myriad forms of technology. Teachers guide lessons using SMART Boards™ and document cameras; students take notes on laptops and tablets; data and feedback are gathered using smartphones and clickers. In this fast-changing environment, how can teachers make thoughtful decisions about using new technologies? By the time formal research findings are available, an innovation may be old news.

One such innovation is flipped instruction, broadly defined by Staker and Horn (2012) as an instructional model in which students learn partly through online delivery and partly through face-to-face interaction in a school setting. Staker and Horn describe flipped instruction as a teaching approach in which content and instruction are largely delivered online, whereas face-to-face, teacher-guided practice takes place during the

FLIPPING INSTRUCTION

decisions about embracing new strategies and technologies.

school day. With the growing accessibility of technology for flipping the classroom and the positive results that some teachers and researchers assign to this practice, "flipping" is definitely worthy of further investigation.

EXAMINING CLASSROOM PRACTICE

But how to investigate? Many teachers are eager to explore technology innovations but lack the framework necessary to document and analyze the results of those explorations. We have found that action research provides an efficient and reliable option for teachers who want to experiment with technology and also test its effectiveness. In this article, we blend research and practice as we describe how action research helped us examine a broadly accepted phenomenon with limited research on its effectiveness: flipping the classroom.

The Teacher's Perspective

Over a period of four years, co-author Grypp experimented with several aspects of flipped instruction in mathematics. Using Camtasia™, she created lessons for students to view at home, thus allowing

additional time in the classroom for group problem-solving activities. These lessons proved beneficial, mainly because students could work through foundational material at their own pace and she could facilitate more complex activities in class. Flipped instruction, however, involves more than simply prerecording lessons, and Ms. Grypp wanted to test its manageability and efficacy for both herself and her students. Encouraged by the positive results reported by other schools in flipping mathematics classrooms (Fulton 2012), Ms. Grypp decided to fully embrace flipped instruction in the final unit for the twenty-one students in her AP Calculus AB class at St. Xavier High School in Cincinnati. This article describes that experience as viewed through the lens of action research.

The Researcher's Perspective

As a mathematics teacher-educator, co-author Luebeck frequently mentors teachers through action research as they explore instructional innovations. The action research model guides teachers to be systematic in applying a new technique and enables meaningful assessment of its value and potential for improving student learning. There are many approaches to action research in education, but most generally follow a three-phase process. In phase 1, teachers define a focus problem; read literature to better understand the problem and formulate a research question; and develop a research plan. Phase 2 involves implementing the plan in a deliberate manner while collecting and analyzing data in the classroom or school setting. Finally, in phase 3, teachers use their evidence-based results to inform instructional decisions or to support changes in practice.

RESEARCH IN ACTION

This action research study followed a slightly modified version of Mertler's nine-step framework (2013). The *preparation* phase included identifying a focus problem and research question; gathering background and contextual information; reviewing related literature; and developing a research plan, which in our model had two components. The first component described the intervention to be carried out. *Interventions* are classroom experiments in which a teacher implements new tools or strategies (e.g., flipping the classroom). Another teacher conducting action research might opt for an *investigation* to examine the effectiveness of an existing strategy or explore what students know and can do (e.g., strategically selecting tools when solving problems). The second component described how data would be collected—in our case, through teacher observations and journal entries, student surveys, and traditional assessments.

In the *research* phase, we implemented both components of the research plan simultaneously. Then we analyzed and interpreted the collected data using qualitative analysis methods. Finally, in what might be called the *action* phase, we reflected on our results and developed a plan for future action. Often this phase involves a change in classroom practice or provides further questions to explore with another cycle of action research. Well-structured action research produces trustworthy findings regarding the viability and usefulness of a technology innovation—in this case, flipping the classroom in calculus.

The students would critically compare their experiences earlier in the year with this flipped unit.

Phase 1: Defining the Question

Is flipping worth the time and effort? Proponents answer with a resounding yes, citing benefits to students that include increased one-on-one time with teachers, opportunities for self-pacing, greater equity in learning, and more parental involvement (Grafton 2012). But are these claims backed by evidence? We decided to seek evidence to answer three specific questions about flipped instruction:

- Can mathematics teachers create meaningful lessons and assignments using flipped instruction and still meet curriculum goals?
- Can students successfully learn mathematics content through flipped instruction?
- Do students feel that their mathematics learning is supported or even enhanced using this medium?

Our first step was to define flipped instruction in a way that would focus the research questions and guide planning and implementation of instruction. We settled on a two-part definition: (1) new content is presented primarily through at-home assignments that include taking notes from teacher-created screencasts and preparatory reading from the textbook; and (2) application of content is developed through hands-on activities and group problem-solving assignments in the classroom, where the teacher is available to offer guidance and individualized instruction.

The second step was to establish some degree of certainty that our context for investigating flipped

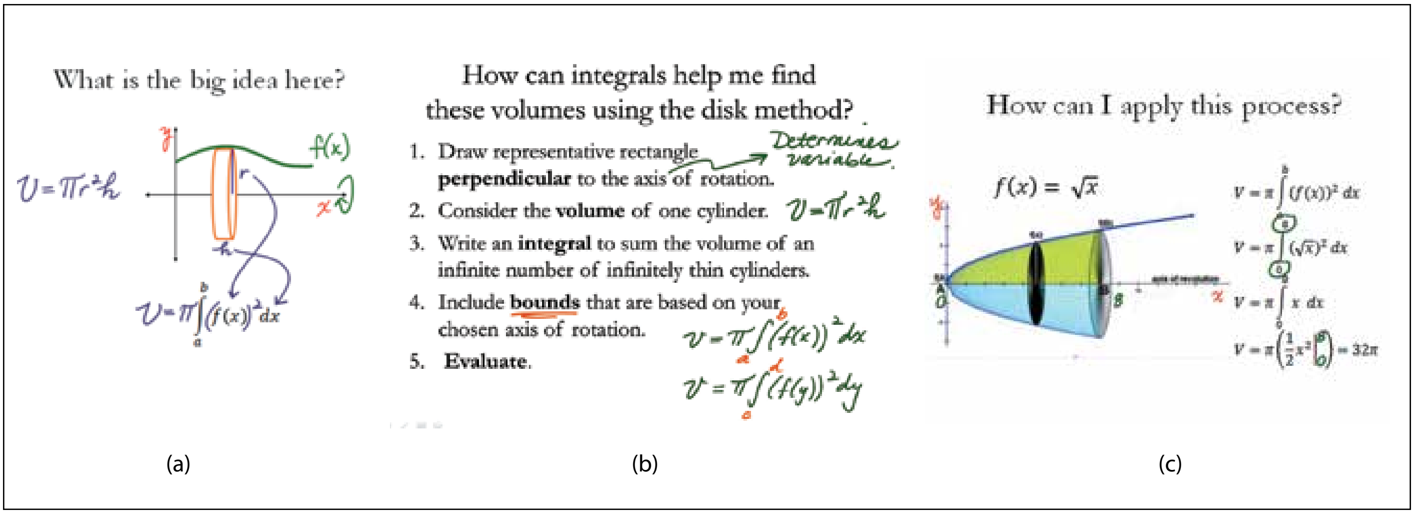


Fig. 1 Screencast notes may be used to interpret graphs and sketches (**a**); to supplement and clarify instructions (**b**); and to link graphical and symbolic representations (**c**).

instruction would produce authentic and valid results. With five years' experience teaching Calculus AB, Ms. Grypp was well qualified to adapt lessons to the flipped environment. Because the students in this class had used screencasts earlier in the year, the integration of at-home lessons was not entirely new to them. Finally, these students had shown themselves to be motivated, engaged, and mature, so we expected that they would thoughtfully reflect on their learning and engagement and critically compare their experiences earlier in the year with this flipped unit.

Phase 2: The Flipping Experiment

The three-week unit that Ms. Grypp chose for this experiment focused on applications of integration, primarily to find bounded area, rotational volume, and cross-sectional volume. Before this unit, students had learned techniques of integration and applied integration as an accumulation process; the new unit extended this knowledge. Although this unit traditionally included many graphs and illustrations in two-dimensional space, the students' work with volume in particular lent itself to hands-on modeling in three dimensions, an ideal use of the additional class time.

Daily homework focused on preparation for the next day's class and relied on five teacher-created screencasts (six to ten minutes each) and occasional preparatory reading from the textbook. PowerPoint® presentations created in previous years formed the basis for the screencasts. Using the tablet feature of her laptop and Camtasia software, Ms. Grypp scripted demonstrations and narrated PowerPoint presentations in real time. Audio, video, and real-time notations using the tablet stylus offered complementary modes of learning, whereas typed text provided students with concrete steps to include in their notes. Examples with animations from Calculus in Motion™ also helped students visualize the process of constructing solids. Despite the countless tutorials and videos available online for each topic in this unit, Ms. Grypp wanted to create personal screencasts that would align with her daily goals and lead smoothly into the following class period's assignment.

Daily in-class lessons relied largely on group problem solving. After a brief introduction and opportunity for clarifying questions at the start of each class, students worked in groups of four to apply and extend the ideas introduced the previous evening. Handouts guiding each day's work were completed by individual students but submitted as a group packet. Ms. Grypp then chose one student's mathematical work at random from each group for review, comment, and assignment of a shared grade. This approach motivated students to work collectively and make sure that all group members could capably express their thinking. For practical purposes, it also cut down on daily grading.

The third lesson in the unit, for example, focused on using the disk method to calculate rotational volumes. Students were already comfortable with the notion of summing up an infinite number of paper-thin rectangles to approximate area. The screencast for this lesson made the jump to summing the volumes of an infinite number of paper-thin cylinders. As with every lesson, the screencast began and ended with a goal statement—in this case, to "establish a conceptual understanding of rotational volume and use integrals to compute volumes." Screen shots from this screencast (see **fig. 1**) highlight some of the main elements of this lesson.

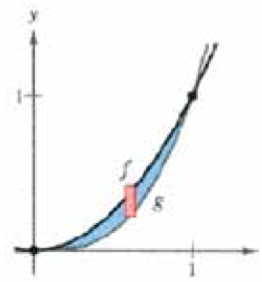
Activity: Using Integrals to Calculate Volume

1. Consider the graph of $y = x^2$.
   - *Describe in words* what the following integral represents in terms of volume:

     $$\int_0^1 x^2 dx = \frac{1}{3}$$

   - Using Play-Doh, *create a model* representing this volume. Slice the solid into four cylinders using dental floss.
   - How does slicing this Play-Doh model into cylinders illustrate the disk method?

2. Now consider the shaded region on the graph below. Here, $f(x) = x^2$ and $g(x) = x^3$.



   - In problem 1, you considered the rotation of $f(x)$ about the x-axis using the integral $\int_0^1 x^2 dx$.
   - What does the integral $\int_0^1 x^3 dx$ represent?
   - Evaluate $\int_0^1 x^3 dx$.
   - What does the integral

     $$\int_0^1 \left(x^2 - x^3\right) dx$$

     represent?
   - Can you create this solid out of Play-Doh? Give it a try! What do you discover?

Fig. 2 Hands-on explorations such as this class activity are used to both extend and foreshadow concepts introduced in the screencasts.

The screencast provided students with a working understanding of the big idea behind rotational volumes, supported by handwritten notes on the steps involved and mathematical notation required. Students came to the following class period armed with this knowledge plus five worked examples, three that Ms. Grypp presented step by step and two that they stopped the screencast to complete on their own. Building on prior knowledge, Ms. Grypp and her students spent class time creating rotational solids out of Play-Doh® and then slicing these volumes into disks using dental floss; they concluded with a problem that introduced the washer method, the topic for the upcoming screencast (see **fig. 2**). In this lesson and throughout the unit, flipping instruction allowed nearly half the class time to be used for creative constructions and other opportunities for sense making and application of concepts.

As a result of flipping instruction, Ms. Grypp found that her preparation time increased. Revising each PowerPoint presentation and recording and editing the accompanying screencast took an average of forty-five minutes. Preparing the daily lesson handouts and activities for class took even more time. Even though Ms. Grypp was using past presentations as the basis for each lesson, all this preparation struck her as a significant time investment. By the end of the unit, noting her own fatigue and lack of energy, she assigned a reading on finding volumes using cross sections for the final at-home introduction assignment rather than creating another screencast. Not surprisingly, additional explanation was necessary the following day in class to establish a clear conceptual framework for students before they began the group activity.

Phase 3: Results and Recommendations

A daily teacher journal and a student survey conducted at the conclusion of the unit provided primary data for our teaching experiment. Analyzed collectively, the journal entries indicate that pairing preclass homework with in-class exploration did enable students to meet the unit learning goals. In other words, student learning and achievement using the flipped instruction format were at least equivalent to past performance in the AP Calculus class.

With the flipped model, we also saw evidence of increased depth and equity in group interactions. Quieter students were more actively engaged in discussions than was typical in a lecture-based class. Participation was more equally distributed among stronger and weaker students, with no indication that one person was completing the work for the entire group. Students' daily assignments demonstrated consistent understanding of how integration is used to compute both area and volume; their errors generally indicated problems with notation rather than the big ideas of the lesson.

At the conclusion of the unit, twenty students completed a survey to provide their perspective on the strengths and weaknesses of the flipped approach. They compared their confidence and grasp of the calculus applications in the flipped unit to the confidence they felt using more traditional

instruction. Besides revealing an overall positive reception to flipped instruction, the students provided specific data. When asked, "How many of the five screencasts did you watch in advance of the class period?" twelve students reported having viewed all five recordings before class, and all twenty students viewed at least three in advance. Although 100 percent preparation for each class would be ideal, we were encouraged to learn that most students took the at-home work seriously. Most students (all but eight) also chose to watch certain screencasts multiple times and found value in being able to review these lessons on occasion.

Students named three ways in which the screencasts helped them learn and understand the unit material: establishing a conceptual sense of each topic, offering step-by-step instruction through sample problems, and previewing topics before expanding on them the following day. We view these as evidence that flipped instruction did indeed successfully support student learning of single-variable calculus concepts. Students specifically valued the visual nature of the screencasts, the use of audio explanation to expand on graphs and images, and the in-class explorations with three-dimensional models.

At least seven students suggested improvements regarding the types of examples included in the screencasts. They felt that more demonstration would have further reinforced their learning even though conceptual foundations were clearly presented. Some students simply asked for more examples, others for more challenging and varied applications, and still others for problems similar to those on the AP exam.

Most students said that they would recommend flipped instruction, but there was no overwhelming consensus about which mode of instruction they preferred.

Students overwhelmingly acknowledged the integral role of group processing with respect to the daily in-class activities. They gained clarity from doing mathematics together and noted the "expanded brain power" that emerges from collaboration. Most students felt that the in-class activities supported equal participation and "helped get everyone on the same level of understanding." They appreciated how the handouts raised relevant

questions and made use of the concepts presented in the preceding screencasts. They found working together to apply concepts more beneficial than working alone on a homework assignment or taking notes during a lecture.

Students clearly desired additional teacher involvement with in-class activities. They recommended that the teacher recap the screencast before beginning an activity, check in with the groups more frequently during the activity, and facilitate a whole-class review following completion of the activity. Students also suggested improvements to the group process to ensure better communication, greater efficiency, and broader diversification of group members. It appeared that even these academically advanced students needed further instruction on how to work together productively and maximize the benefits of this new learning model.

Despite these positive results, the survey data do not suggest that students found that flipped instruction necessarily improved their learning of calculus concepts when compared with traditional presentation methods. Most students said that they would recommend flipped instruction for use in other mathematics classes, but there was no overwhelming consensus about which mode of instruction they preferred. This result is worth considering when determining whether to adopt a flipped instruction model.

DECIDING WHETHER OR NOT TO FLIP: RECOMMENDATIONS

Our results indicate positive answers to the three questions that were posed. Yes, it is possible to create meaningful calculus lessons and at-home assignments using flipped instruction while meeting curriculum goals. Yes, students can successfully learn challenging material through this medium. And, yes, they feel supported as learners along the way. Does this mean that flipped instruction is the right strategy for AP Calculus? That depends. Whether "possible" is the same as "feasible" or whether a teaching approach that indicates "success" but not necessarily "improvement" is worth the investment are questions for the individual teacher.

Action research is highly personal, and results are not always transferable; similar research in a different classroom might tell a different story with different outcomes. However, when action research is carefully executed, based on data, and thoughtfully interpreted, one teacher's findings may inform next steps for other teachers in similar circumstances. As a result of our experience, we offer three recommendations for flipping instruction.

Mathematics teachers should explore these innovations from the perspective of cautious consumers who both embrace and critically question innovation.

1. Embrace Change

Because students study the lesson content at home, time spent in a flipped classroom can be used in new ways. Creatively using this extended time, rare in a world of forty-five-minute classes where homework review and traditional lecture are often the norm, can have significant benefits for students but may seem daunting to teachers. Before contemplating a shift to flipped instruction in mathematics, you must first embrace the inherent value of this new structure and explore new uses of class time. Investigate resources available to support project-based learning or in-depth problem-solving activities. Assess your confidence level with the content and curriculum and your ability to devise meaningful explorations. Embrace innovative ideas and be ready to involve students in doing mathematics in creative ways.

2. Commit Time

Consider the practicality of this time investment and balance it against other commitments before jumping into flipped instruction. In this one-unit experiment, a substantial amount of teacher time was spent organizing and formulating instruction relating to pacing, potential projects, necessary screencasts, and assessments. Although flipped instruction may seem similar to how one would typically plan a unit, it took far more time because of the steep learning curve, the need for additional background planning, and the necessity of more frequent grading. Feeling confident and capable of meeting these time requirements will enable you to begin and maintain the process with enthusiasm.

3. Start Small

Experiment in small ways with the advantages of flipped instruction before attempting it for a full year or even a full unit. Investigate the technology and software available to create screencasts; try recording and editing a lesson; or have students watch the lesson in advance and use the freed-up time to implement a rich mathematical task or collaborative project. Collaboration may benefit you as well; investigate whether colleagues might be interested in flipping a course as a team effort. Even if you are not ready for a full-blown action research study, treat the lesson as an experiment and collect

data through observation and student feedback. Taking one step at a time will help novice researchers determine the benefits and challenges of technology innovations pertinent to you, your students, and your school community.

TECHNOLOGY AND ACTION RESEARCH

Development will likely continue to outpace research as more and more technology-enhanced modes of teaching and learning appear on the horizon. We believe that mathematics teachers should explore these innovations from the perspective of cautious consumers who both embrace and critically question innovation. Action research allows teachers to do both by designing data-producing experiments around technology. The findings from action research provide a thoughtful and informed knowledge base for teachers to adopt or adapt new technology practices in ways that ensure effective instruction and more meaningful learning for students.

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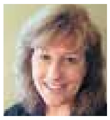

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